Three slices of Jay Forrester's general theory of economic behavior: An interpretation*

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Abstract

This paper defines the unique contributions Jay Forrester made to economics through his system dynamics models that emphasize pursuit of operational goals in every day decisions rather than meeting the abstract criteria of rational expectations. In particular, the paper attempts to reconstruct his distinctive explanations of economic cycles over the course of his National Modeling Project, whose details appear in PhD theses, occasional papers, and internal records of the MIT system dynamics group but have not been published in a succinct form and are not widely known especially by the newer vintages of system dynamics practitioners.

Key words: Jay Forrester, System Dynamics, Business Cycles, Economics

Introduction

A few years ago, in a casual conversation, I asked Professor Forrester about the progress on publication of the work on the National Modeling Project that many of our colleagues in the system dynamics group at MIT participated over late 1970s and 1980s. He said he was working on a volume that would be titled "A general theory of economic behavior", which will articulate the intellectual contributions of the project. Since the book is not yet out, I cannot say much about its content, but I do have specific views on Forrester's rethink on economics and how his work on the National Model factored into it, which I shared with Jay in a recent communication. I am happy to say that he encouraged me to articulate these views. I'd like to posit at the outset that the

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National model represents only a small part of Forrester's rethink on economics, which is demonstrated in all his models. However, given that it would be difficult to revisit all his models in this brief paper, I'll focus only on the unique contributions of the national model to macroeconomics. Hence the reference to slices in the title.

I must also add that I did not work on the National Modeling project and do not have access to Forrester's models. The models I present in this paper represent an informed outsider's interpretations of Forrester's Theory as inferred from the records of the project and in some of the publications and dissertations that arose from it. I, therefore, take full responsibility for any conceptual biases and errors that I might have introduced in his original concepts.

Forrester's rethink on economics

Forrester's rethink on economics started with Industrial Dynamics (Forrester 1961), which revisited microeconomics by expounding on how firms and customers really went about their business on an every day basis instead of pursuing abstract goal of balancing marginal costs and benefits. Forrester's realist structure created supply and demand chains with delays built into them that led to unexpected dynamics. His realist perspective continued in Urban Dynamics (Forrester 1969) that explored economic development as a renewal process instead of pursuit of growth in a supposedly infant economy posited in the prevalent models of the time. Next came World Dynamics (Forrester 1971), which challenged the micro-economic foundations of what was called environmental economics at the time. World Dynamics really represented a theory of environmental/ecological economics that addressed size and commons issues and the tipping points they created when environmental economics mainly focused on an assessment of marginal damage and control costs for determining an environmentally optimal mining quantity as prices continued to pour an unlimited flow of backstop resources into the economical stock. Last came the

National Model, which provided distinctive explanations of historically documented macroeconomic patterns of behavior that I'll discuss further in this paper.

In all cases, Forrester built decision rules into his models not on basis of prevalent abstract concepts of rational economic behavior and a priori assumptions about the existing systems but on how people actually made decisions with limited information in systems existing on ground. His vision of economics is thus distinctively different from any thing in the current theories in economics in that it builds not on abstract concepts of marginal values and rational expectations with perfect information driving the decisions in a hypothetical market system, but on how people go about their business on an every day basis and the irrational behavior that arises out of their interaction. It should be noted that while some of the mainstream economists have recognized the relationship between micro-structure and macro-behavior (Barro 1997) that Forrester emphasizes, the microeconomic decisions they subsume in such revisions are still based on economic agents pursuing abstract goals with perfect information, not on everyday decisions taken in a bounded rational framework as Forrester assumes in his models. I should add that my own model of income distribution discussed in Saeed (1994) conforms more to traditional economics than to Forrester's perspective.

I have described elsewhere my interpretation of Urban Dynamics that I view to contain a theory of economic development in a consumed society, which the developing countries truly are (Saeed 2010). I have also attempted in Saeed (1985), Saeed and Acharya (1996) and Saeed (2003) to address policy agendas concerning size considerations and intergenerational transfers that World Dynamics raised. I'll limit this paper to discussing Forrester's contribution to macroeconomics through his work on the National Model, which provided a unique explanation of three historically documented macroeconomic patterns -

the business cycle of 5-7 year periodicity, the Kuznets cycle of 28-25 year periodicity and the Kondratieff cycle of 50-70 year periodicity.

General perceptions about economic cycles

In a recent TV interview, the economics editor of Economist magazine posited that economic cycles represent short term endogenous trends, while sustained recessions like the one being currently experienced arise from autonomous events. A search of the keyword “economic cycles” on Google would return similar perceptions. The term economic cycle has been used interchangeably with business cycle and both are taken to imply 5-7 year cyclical trends observed in market economies, generally attributed to investment dynamics (Samuelson 1939), although as shown by Low (1980), capital formation lead times and capital output ratios existing in reality may in fact generate cycles of much longer periodicity. The real business cycle theory advanced by Lucas (1981) attempted to explain deviations from normal business cycle periodicity by attributing them to the rational responses of the economic actors to external events that might not appear to directly affect the periodicity. Historical records show, however, that endogenous trends of multiple periodicities are quite pervasive in industrialized countries with free market economies and appear with regularity that cannot be explained easily by external events or the chains of responses to them. Nobel Laureate economist Simon Kuznets discovered an 18-25 year cycle that is named after him while Nicholas Kondratieff observed a 50-70 years cycle, which now bears his. Few theories exist to explain such long cycles. Growth trends experienced over the course of such cycles are often attributed to good economic management and good governance, while unfortunate events are blamed for declines (Forrester 1977, van Duijn 1977).

Some of the findings of the Forrester’s National modeling project are documented in Mass (1975), Graham and Senge (1980) and a number of internal memoranda and Ph.D. theses listed in system dynamics literature archive that are available from the System Dynamics Society. Of particular interest among these memoranda are D-3573 by Graham (1984), D-2517-2 by Low and Mass (1980), D-3577 by Sterman (1984) and D-3712-1 also by Sterman (1985). Additionally, doctoral dissertations by Mass (1974), Low (1977), Runge (1976), Richmond (1979), Senge (1978) and Sterman (1981) address various aspects of the National Model. The findings of this extensive research led to unique causal explanations of endogenously generated short- and long- term economic cycles. According to these explanations, short term Business Cycles of 5-7 year periodicity are attributed to workforce adjustments policies, while Kuznets cycles of 18-25 periodicity are posited to arise from capital investment dynamics, and the Long Wave or the Kondratieff Cycle of 50-70 year periodicity from over-expansion of the capital goods production sector and the subsequent prevalence of underutilized infrastructure. The three simplified models discussed in the subsequent sections of this paper attempt to reenact the crux of each of these explanations.

Slicing complex reference mode of National Model into organized cyclical patterns

The National Model subsumed complex structure that could generate behavior simultaneously representing multiple modes creating by different segments of its structure. I am, however, of the view that complex behavior should be decomposed into its simpler components and simple models constructed to understand each component in our theory building effort (Saeed 1992). I will, therefore, outline three simple models that attempt to interpret Forrester's behavioral theory underlying these patterns. For this, the first task is to discern the organized cyclical patterns in complex historical time series.

The three organized cyclical patterns the national model explained might never be directly seen in the historical data. Figure 1 shows a graph of the unemployment rate in the US as documented by the bureau of labor statistics, which must be decomposed into its dominant harmonic components to get to the three cyclical patterns manifested in the Business, Kuznets and Kondratieff cycles. The complex behavior observed in the historical time series, in addition to the cyclical trends of the various periodicities, would also subsume random events, which are not of interest in a dynamic model as they may only serve to disturb the natural homeostasis to bring out endogenous trends created by the system structure.

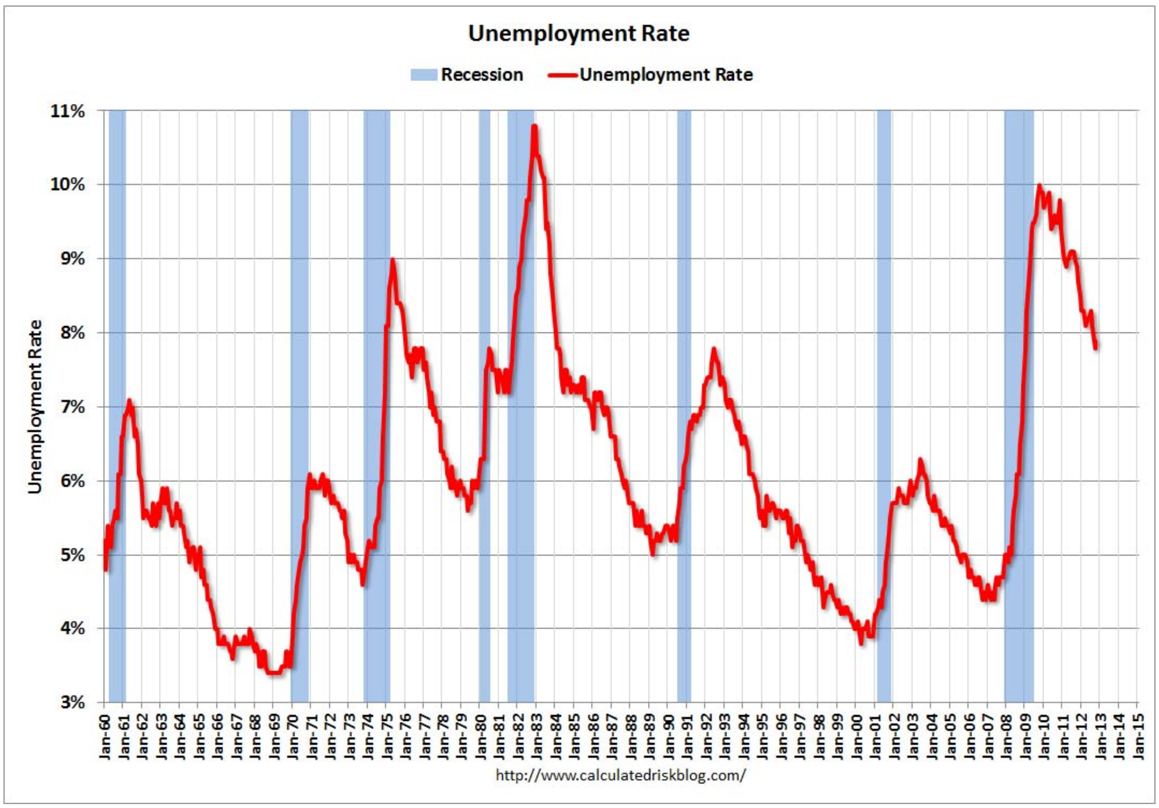

Figure 1 Long term unemployment rate trend in USA.

Source: US bureau of labor statistics

The pattern of Figure 1 can be smoothed to filter random noise and decomposed using complex mathematical tools that is outside of the scope of this paper. I'll instead demonstrate that a combination of the periodicities of business, Kuznets and Kondratieff waves will indeed create a pattern qualitatively similar to the historical trend of Figure 1.

Figure 2 shows the business (2), Kuznets (3) and Kondratieff (1) cycle trends represented by sin waves of respectively 5, 25 and 60 years using the following relationship:

Cyclical variable = A-B*SIN(2*PI*TIME/T), where T represents the periodicity, A is initial value and B the amplitude of the cyclical variable.

Figure 2 also shows an additive combination of above trends in Graph (4), which results in complex behavior that is qualitatively comparable to the long term historical trends. This combination is characterized by long periods of growth and short recessions over the upturn of the long wave cycle that might be attributed to good economic management, and short periods of growth and long recessions over its downturn that might be seen as an occasional deep recession attributed to external events and economic responses to them as well as to economic mismanagement by the leadership.

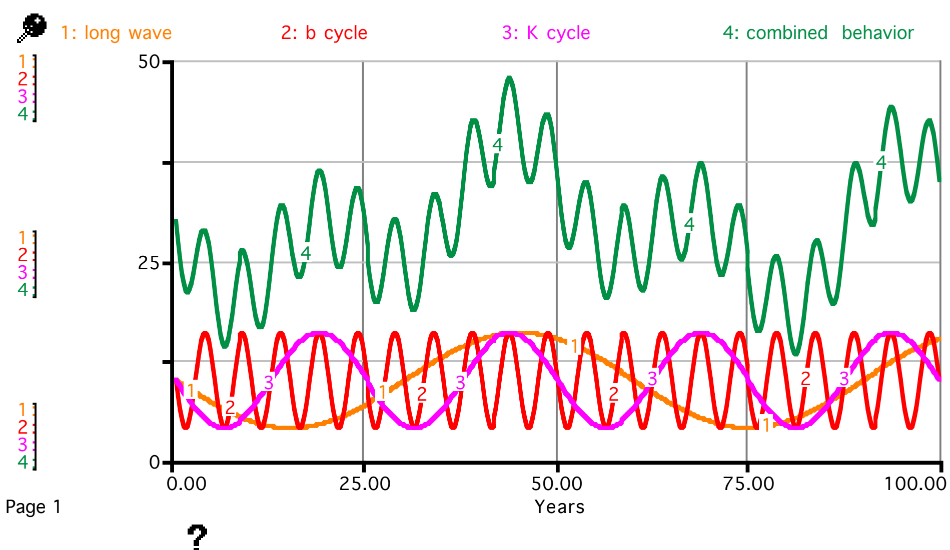

Figure 2 **Business cycle (1), Kuznets cycle (2) Long wave (3), an autonomous exponential growth trend (4) and their overlay (5) patterns shown with no particular variable and arbitrary scaling**

Point of departure

As I have advocated in Saeed (1992), I have attempted to construct rather parsimonious models addressing each pattern separately instead of attempting to replicate the National Modeling effort that led to a complex composite model. However, instead of reinventing the wheel by constructing these components from experiential information, I have drawn from the constructs found in the available public documents on the National Model.

The models I am presenting subsume the essential symbiotic relationships and replicate distinct patterns of complex economic histories, which have been experienced pervasively in market systems around the world. As demonstrated in the last section, these distinct patterns simpler components of complex time histories as suggested in Saeed (1992) and Saeed (2003). Both the patterns and their corresponding models are simple enough so the relationship between their structure and behavior can be understood, yet they are not so abstract that they lose sight of Forrester's basic tenet that macro-behavior arises out of the every day decisions of the role players (Forrester 1980). These role players do not have perfect information about the system contrary to the assumptions of the rational expectation models of the real business cycle theory (Lucas 1981). Instead, as Herbert Simon (1972) proposed, they act rather in a bounded rational framework and make satisficing decisions based on information available to them. This approach is also typical of decision rules in a system dynamics models (Morecroft 1985).

Also, my parsimonious models explain specific patterns they attempt to explain. They are not models of systems, which is an important philosophical point of departure that system dynamics adopts in dealing with complex systems (Sterman 2000). These models also differ from abstract economic concepts subsuming, rationality, marginal analysis and the working of the invisible hand in that they focus on every day behavior of real actors in the system As a result, their structure and parameters are identifiable.

A hypothetical steady state is a point of reference in these models rather than This steady state represents a dynamic equilibrium perceived by Mill (1848). It creates an internally consistent set of parameters for the homeostasis that is sought by the system but never achieved. This equilibrium is disturbed by changing a single parameter to invoke the search for a homeostasis that results in the patterns of interest. This pattern is therefore is entirely endogenously generated and must be explained entirely in terms of the structure of the respective model.

A simple model of labor hiring process leading to Business Cycle

Since the 5-7 year of business cycles could arise realistically only out of the relatively short lead times for hiring and firing workers in response to short term changes in aggregate demand and supply, business cycles in the National Model were attributed to managerial responses to changes in demand, maintenance of supply lines and adjustment of workforce to appropriate levels.

The stock and flow structure of the aggregate demand sector is shown in Figure 3. Aggregate demand is computed as average desired production, which represents expected value of desired production arising from average shipments and the inventory and backlog discrepancies. Note that the term aggregate demand is not used in the model since it is not clearly identifiable, whereas average desired production is the identifiable entity that the producers are aware of. The inventory goal is in turn a function of average shipments and backlog goal a function of average production. This assumes that production should not only fulfill perceived stream of demand, it should also maintain an appropriate level of inventory that is able to cover unanticipated shocks. Additionally, there must exist an appropriate backlog of orders, which is an integration of incoming orders representing instantaneous demand and shipments representing instantaneous supply, so production resources are fully employed for a foreseeable future. Any changes in the three components of desired production will alter the aggregate demand as seen by the producers, even with fixed prices.

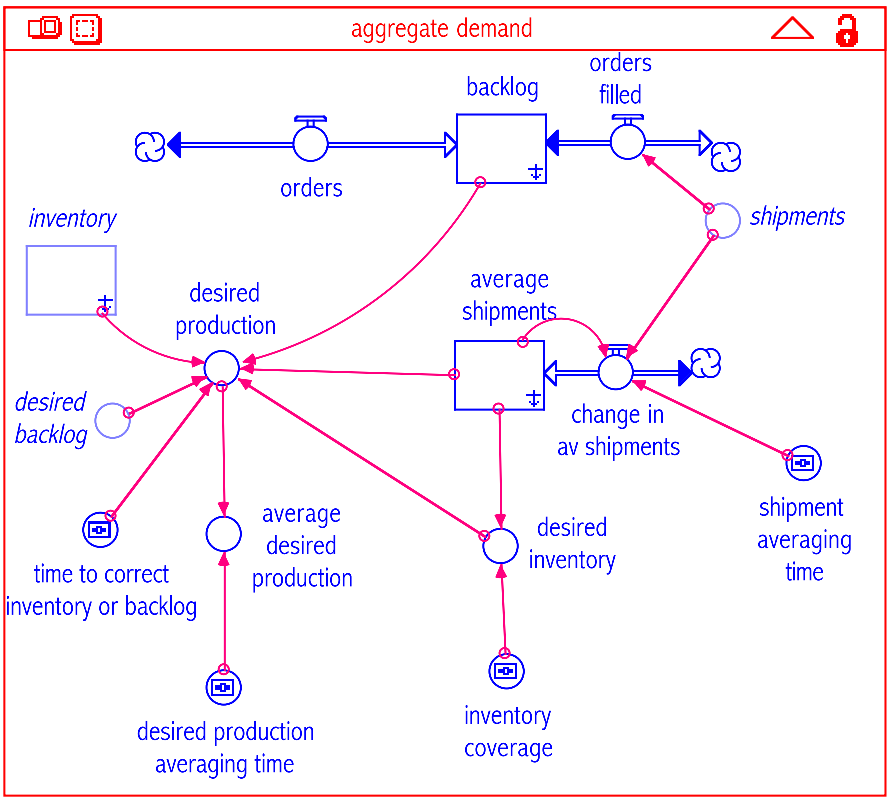

Figure 3: Creation of aggregate demand

The stock and flow structure of the aggregate supply sector is shown in Figure 4. While inventory is an integration of production and shipments, production depends on labor, overtime and labor productivity if we assume that the availability of capital as a production factor is completely elastic. Shipments on the other hand depend on how much is produced and the backlog of orders, since both are required for the sale to occur.

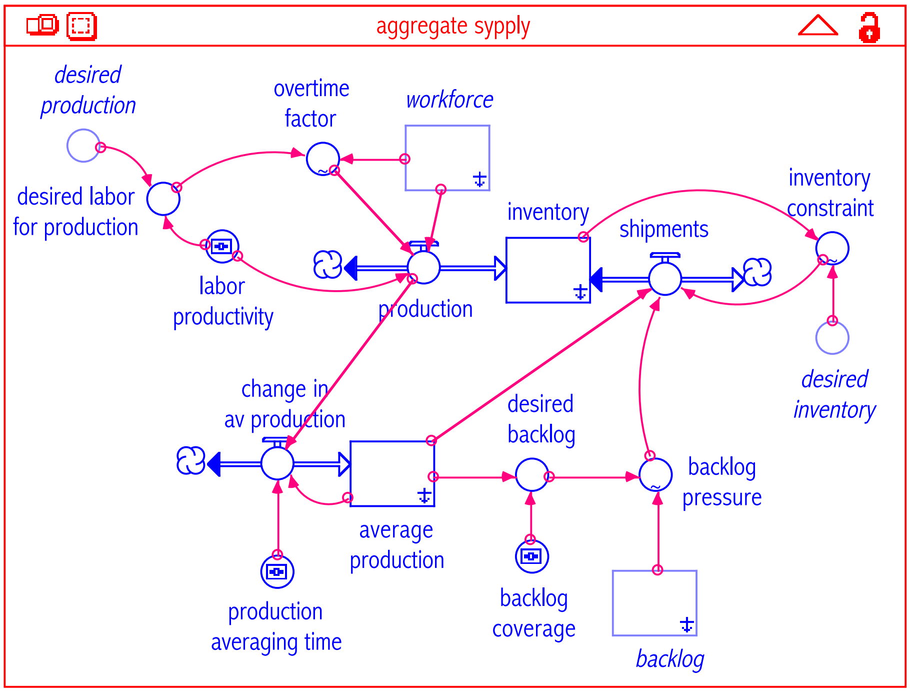

Figure 4: Determination of aggregate supply

The process of hiring and firing of workers is shown in Figure 5. Hiring is driven in the first instance by the past hiring tradition that essentially balances attrition in equilibrium. It is further modulated by the discrepancy between exiting workforce and desired workforce merited by the desired production volume as well as by the discrepancy between the in process pool and the needed in process pool. This system is initialized in equilibrium, which is disturbed by an exogenous step change in demand. The resulting behavior completely represents endogenous dynamics shown in Figure 6.

The time constants of the hiring subsystem are relatively short. The delays involved in hiring are of the order of 6 months - 1 year, while the median length of employment in the United Stated, where labor mobility is high, is 3-5 years (US bureau of labor statistics website). Thus, the periodicity of the cycle generated by the interaction of workforce adjustment process and the aggregate demand and supply subsystems is of the order of 5 years.

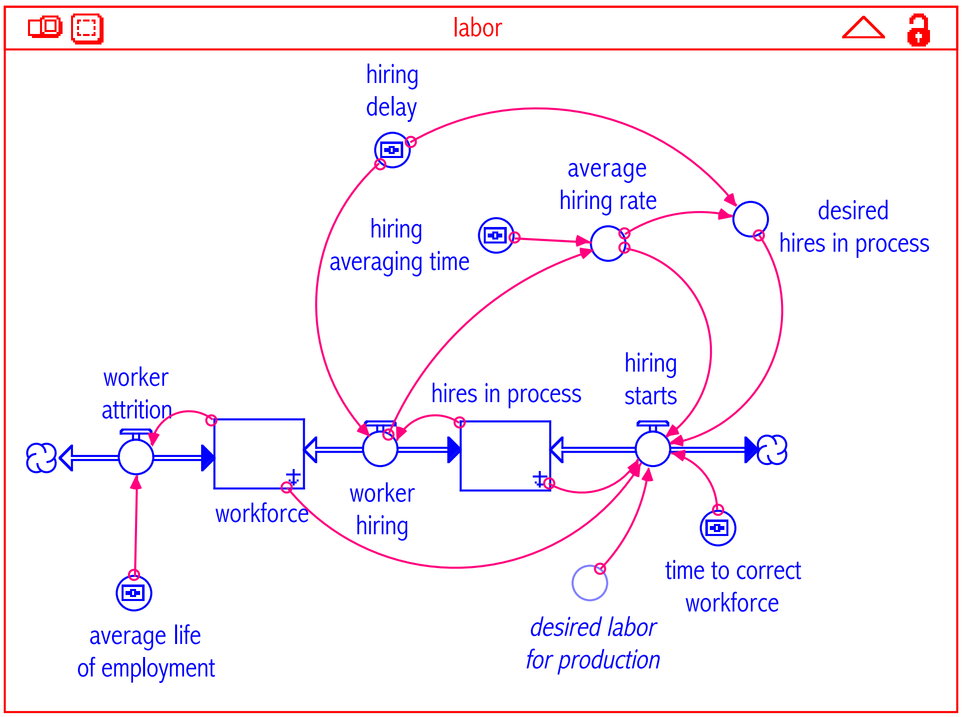

Figure 5: Workforce adjustment

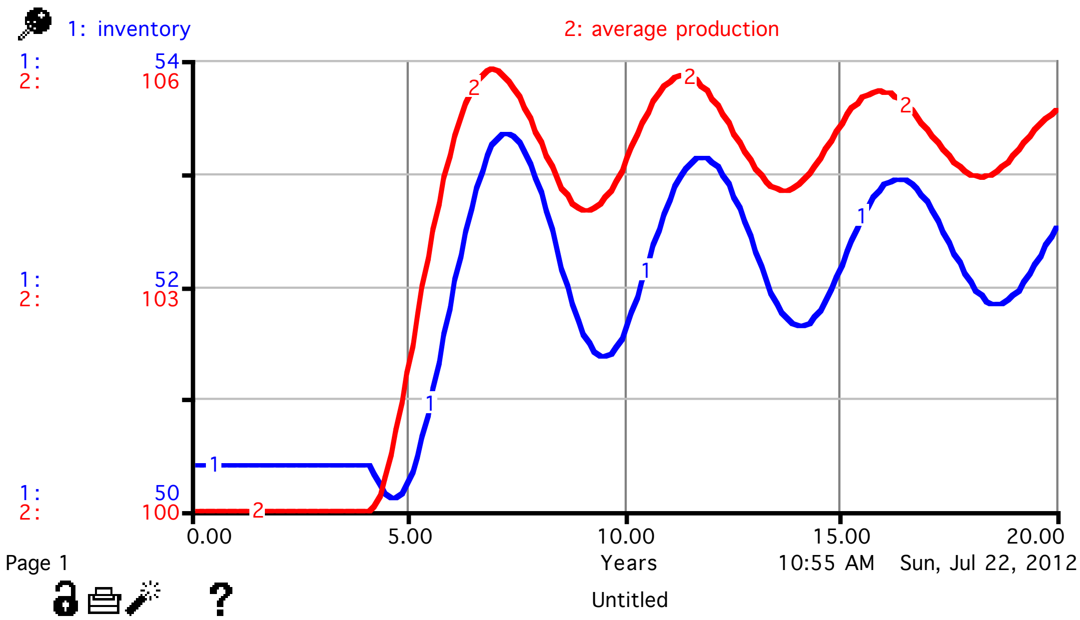

Figure 6: Business cycle periodicity generated by managerial actions involved with maintenance of supply, response to demand and workforce adjustment

Figure 7 shows how interaction between backlog, production, inventory and workforce would create negative major feedbacks that cause instability due to delays in perception of information and adjustment of the production resource, which is workers in this case.

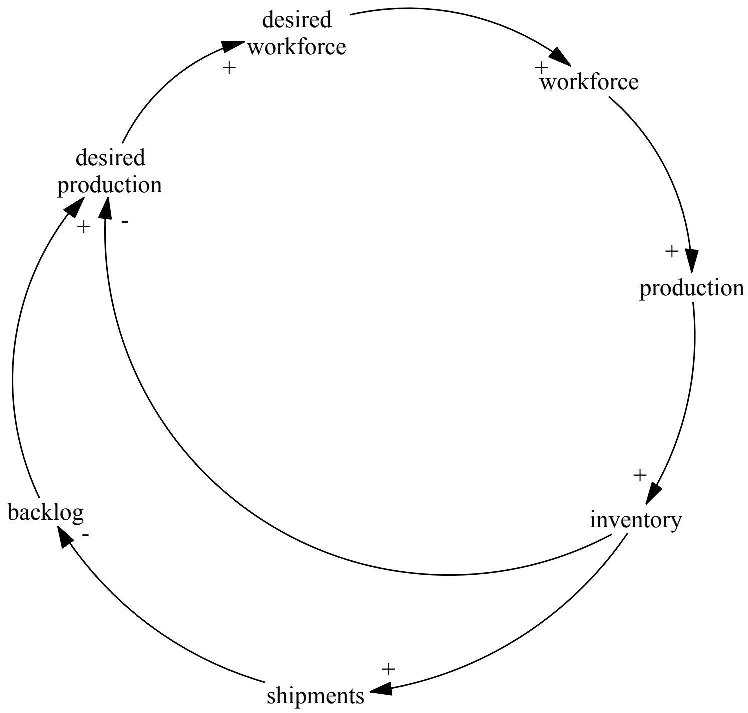

Figure 7: Feedback loops in the business cycle model creating instability

Increases in production will not only diminish backlog in the aggregate demand sector, it would also accumulate inventories that would reduce desired production. Thus, aggregate supply actually reduces aggregate demand due to managerial actions even when the abstract concept of price as a market clearing mechanism is dispensed with. An increase in aggregate demand on the other hand will increase backlog and fuel increase in production first through use of overtime and then through hiring of additional workers, both actions reducing the demand for labor. Thus, both labor – production and aggregate production backlog interaction will create major negative feedback loops that contribute to instability. In simple terms, the delays in workforce adjustment lead to a slight overexpansion with subsequent overproduction, which depletes backlog and expands inventory that calls for layoffs (Forrester (1968), Mass and Senge (1975)).

A simple model of capital plant management process leading to Kuznet's cycle.

An economic cycle of periodicity ranging between 18-25 years observed in real estate sector and is sometimes held responsible for occasional deepening of the business cycle. Often referred to as Kuznets cycle, it is attributed in the National Model to the interactions involved with managing investment into capital plant. Forrester postulated that this cycle arises out of mechanisms similar to those leading to the business cycle but since the capital ordering and formation lead times for durable capital are much longer (of the order of a few years) than those for adjusting workforce (of the order of several months), this cycle has a proportionately longer periodicity.

The management decisions pertaining to capital formation are shown in Figure 8. These decisions replicate those in Figure 5, except the variables have different names and capital adjustment takes longer than labor adjustment. Also, while it is possible to lay off workers in the model of Figure 5, capital stock at the level of an economy stays in the system until it is retired through the depreciation process. Thus capital orders are adjusted through an implicit stock adjustment process based on assessment of adequacy of capital and capital on order rather than through directly considering the discrepancy between their current and desired values. The aggregate supply and demand sectors are not changed in any way.

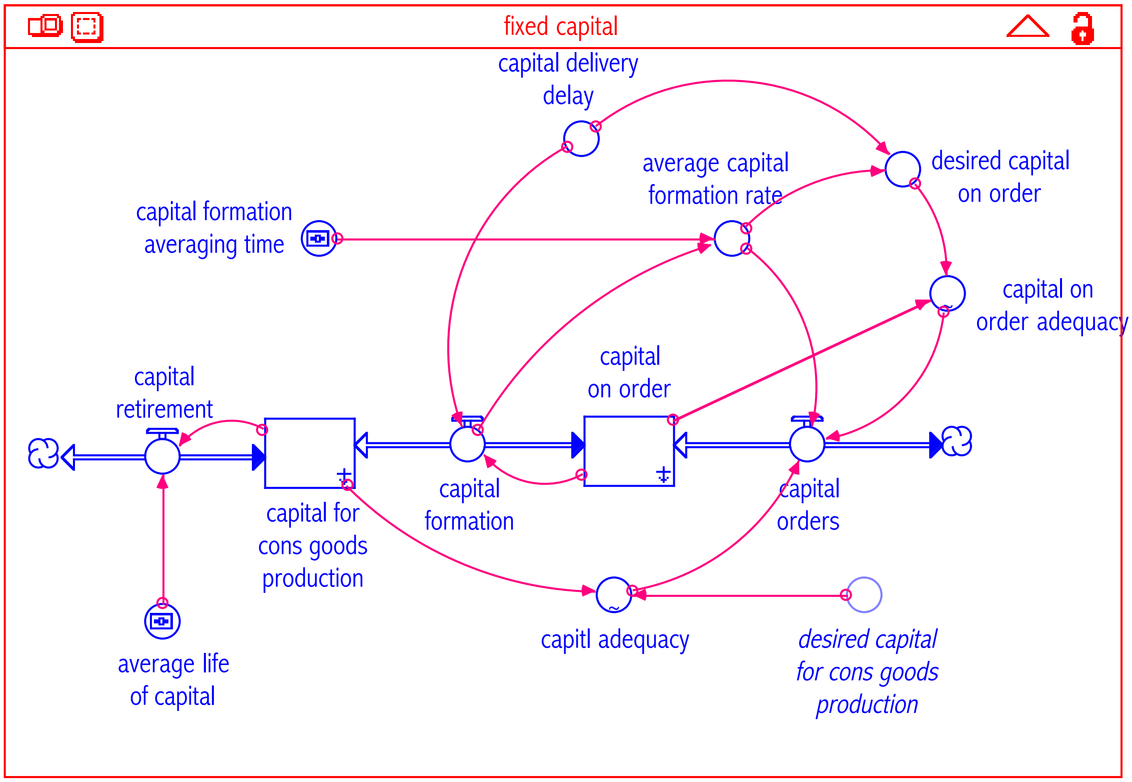

Figure 8: The structure of capital investment decisions

When demand is autonomously stepped up in this system, a cyclical trend of periodicity of about 27 years appears as shown in Figure 9, the exact periodicity being determined by the slopes of the adequacy functions. The delays in the expansion process lead to overexpansion of the capital and the subsequent piling up of inventories and depletion of backlog to an extended neglect of investment that creates recession much deeper than in a routine business cycle.

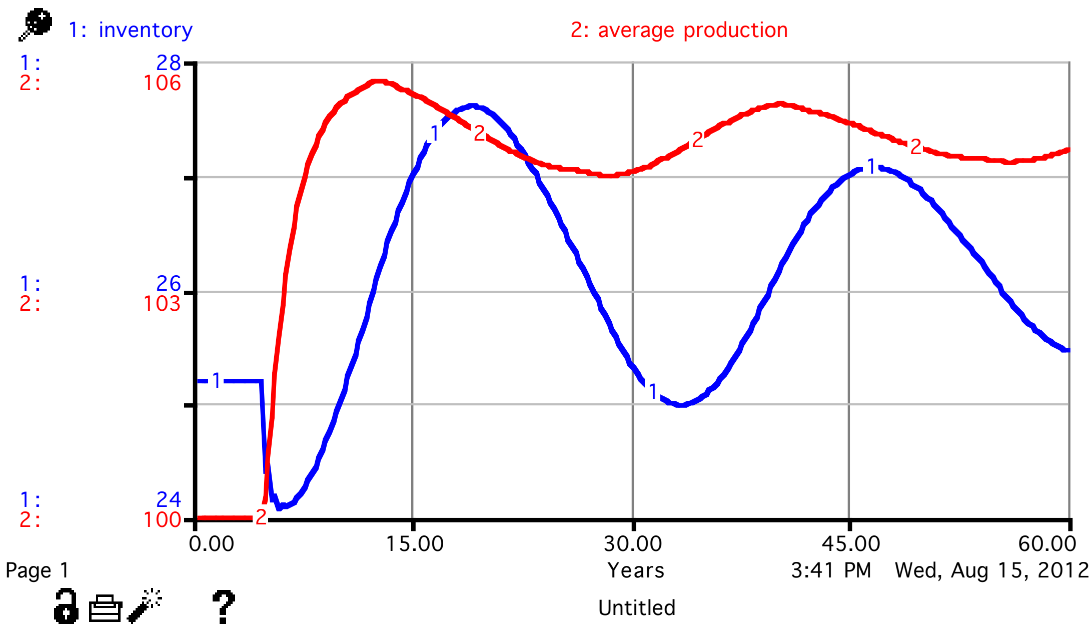

Figure 9: **A cyclical trend of about 25 year periodicity arising from interaction of capital investment decisions with Aggregate demand and supply sectors.**

A simple model of investment goods capital plant management process leading to Kondratieff cycle or long wave

A periodicity of 50-70 years observed by Nikolas Kondratieff is explained by Forrester as a function of the interaction between consumption goods and investment good production sectors. When consumption goods production sector wants to create additional plant and equipment, it places orders for these on the investment goods production sector. When investment goods production sector has orders beyond its capacity to deliver, it must expand its own capacity before filling capital orders of the consumption goods sector. Thus it places additional orders to produce investment goods on itself that Forrester termed self-ordering.

The stock and flow structure of the fixed consumption goods capital sector is the same as in Figure 7 accept that capital formation rate is constrained by the production of capital goods by the investment goods production sector in both consumption and investment goods sectors. The structure of the fixed investment goods capital sector is shown in Figure 10, that has a supply chain comparable to that of Figure 9 with some differences. First, the investment goods capital orders depend in equilibrium directly on its rate of retirement instead of an average of the past capital formation/hiring rates rate used in the models of respectively Kuznets and Business cycles, which are indirectly linked to the rate of capital retirement. In fact, basing these on average capital formation rate creates a powerful positive feedback that creates explosive growth not observed in reality. This also implies a less competitive and somewhat inward perspective that is typical of a rather specialized investment goods industry. Second, since, its specialized production is driven by orders and it often does not maintain an inventory, the investment goods on order must be seen as the investment goods sector's order backlog rather than a part of its capital supply line. Hence, it would need to produce more in response to capital on order rather than scaling down its orders as in the consumption goods sector whose capital on order is a part of its capital supply line. Last, its orders are driven both by the needs of the investment of the consumption goods sector as well as its own needs creates self-ordering as elaborated in Figure 11. The desired capital in the investment goods sector depends on desired investment goods production that is a function of the summation of both investment goods on order for consumption goods production and those for investment goods production.

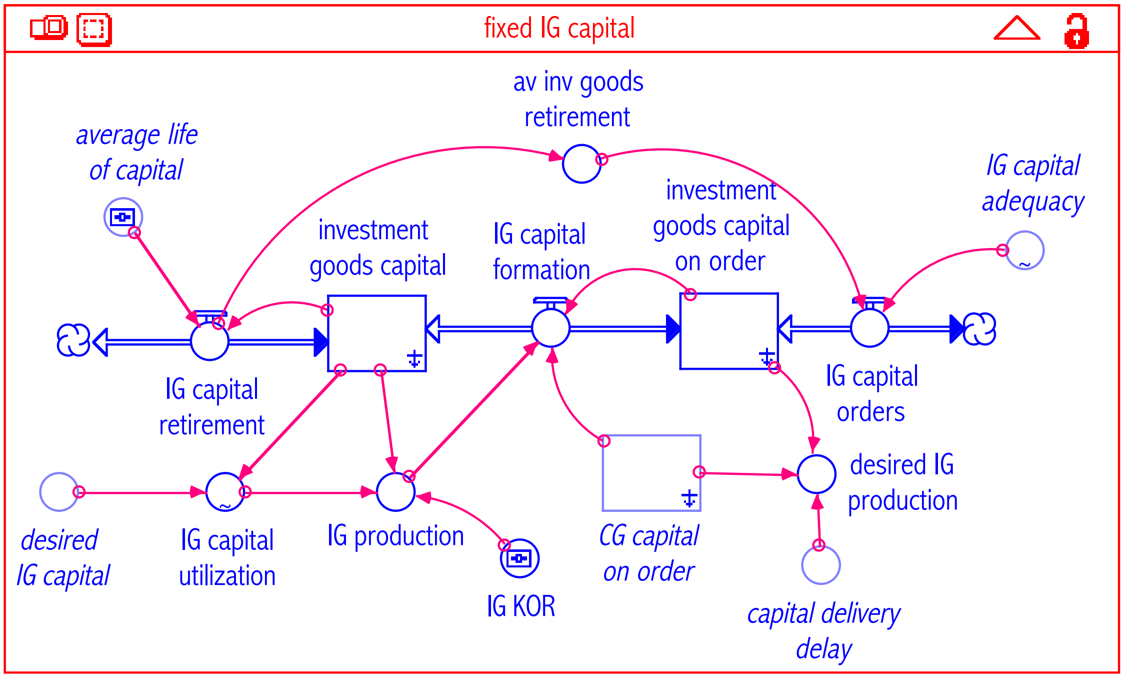

Figure 10 Creation of fixed investment goods capital

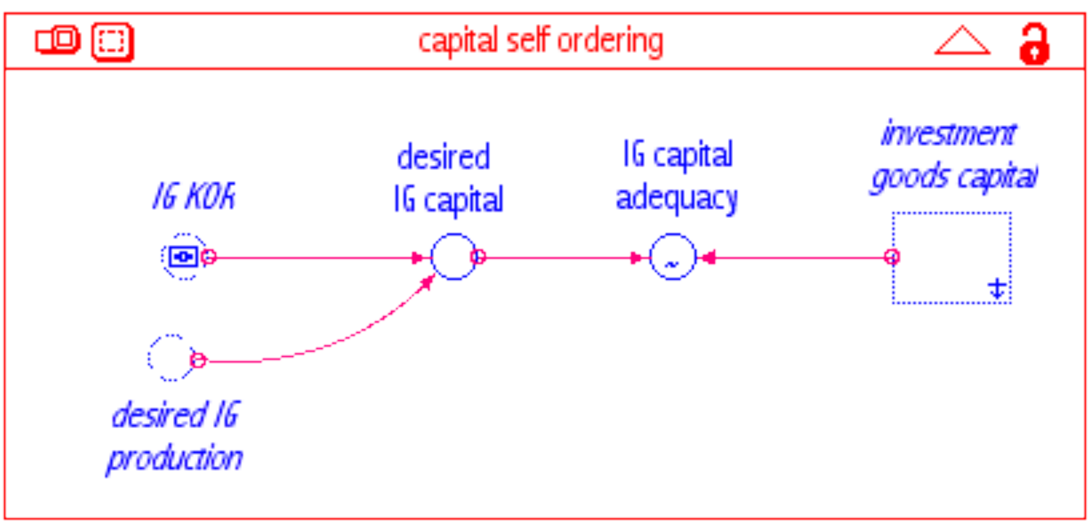

Figure 11 Capital self-ordering process

The interaction of the consumption and investment goods sectors and the self-ordering practice create key feedbacks shown in Figure 12. An increase in consumption goods capital orders requires first expanding the capacity of the investment goods production sector before it can meet additional capital goods demand. By the time this demand is met, the investment goods production has over-expanded and needs to scale down its production. This process creates two negative feedback loops that have a long time constant and a positive feedback loop that exacerbated the overshoot over the adjustment process. Note that aggregate demand, aggregate supply and workforce sectors are excluded from this slice of the system, as they do not directly contribute to the long term dynamic addressed in this model. External disturbance to this system will be created by autonomously stepping up desired consumption goods capital rather than orders for consumer goods.

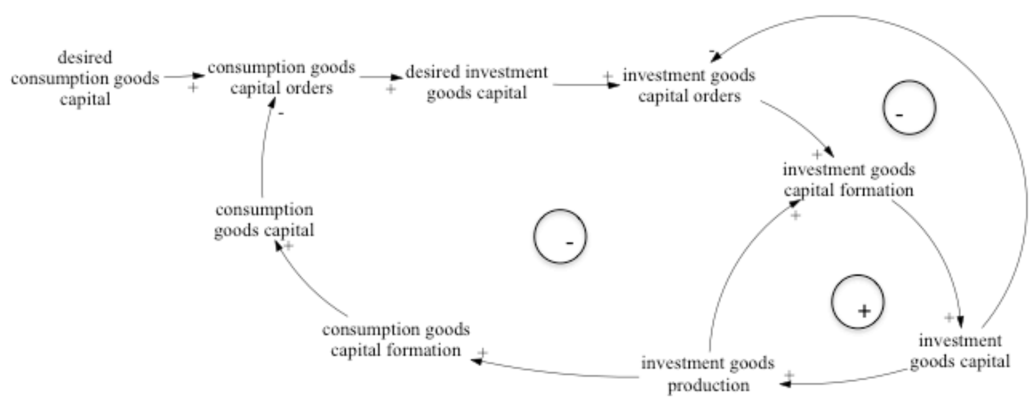

Figure 12 Feedback loops in Forrester's long wave model creating overshoot in the capital sector

Figure 13 shows the behavior of total capital in the system (both for consumption goods production and capital goods production) in this model. Two cases are shown: Graph one assumes a capital life of 15 years, which generates a cycle with about 55 years periodicity. Graph 2 assumes a capital life of 12 years and generates a cycle with 70 years periodicity.

The delays in delivery create over-ordering in the consumption goods sector. The self-ordering process of the capital goods sector, which defers deliveries to the consumption goods sector until the desired production capacity for capital goods has been achieved, further increases this over-ordering. Once the capital goods sector has achieved this capacity, its self-orders vanish even though its production capacity is enough to cater for both its self-orders and those from the consumption goods sector. Hence some of its capacity must lie idle. This also leads to speeding up clearance of the backlog of orders from the consumption goods sector, which in turn scales down its orders, which leads to further reduction of capacity utilization in the capital goods production sector. This spirals into creating a sustained down turn. The delivery lead times in this system are greatly affected by the durability of the capital since both replacement capital ordering and self-ordering are affected by the durability. It should be noted that this model excludes endogenous growth and decline processes that subsume the impact of worker lay offs from reduced capacity utilization on household income and demand.

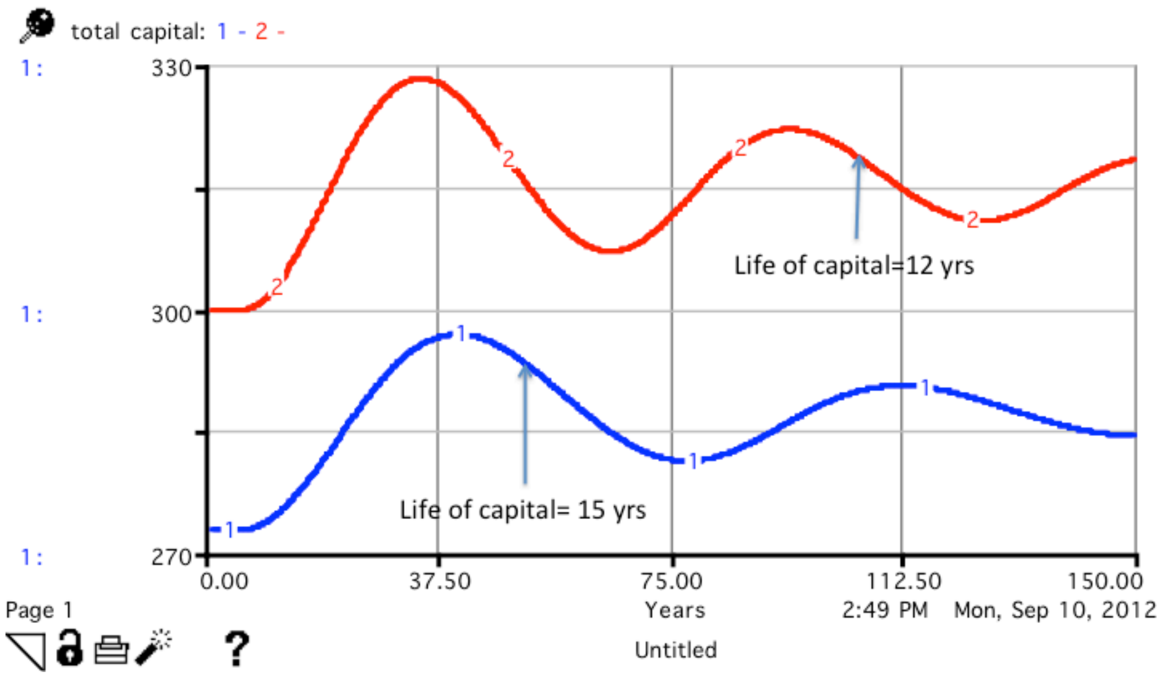

Figure 13 **A cycle of 50-70 years (depending on durability of equipment) generated by overgrowth in capital stock through self-ordering process**

Conclusion

Forrester's behavioral theory differs from the traditional economics explanation of business cycle in two ways, 1) it focuses on real and identifiable managerial interactions rather than on an invisible hand driving abstract processes of price, demand and supply interactions. 2) It explains periodicity of the cycle in terms of real time constants existing in the system. The simple slices of the theory I have attempted to construct in this paper do not includes the endogenous growth processes and the changes in household income that fuel demand, changes in labor market conditions and the functional income distribution that can affect both demand and investment behavior, which are important parts of the mainstream macro-economic theory. They only explain the long-term economic patterns as manifestations of every day managerial actions. This an important thread to pursue to create a disruptive change in economic theory.

Many years ago, I attempted to construct a simple model of the long wave subsuming endogenous growth and endogenous processes driving functional income distribution in addition to self-ordering of capital as posited by Forrester. I constructed this model as a part of the a paper for a seminar on Long wave Professor Forrester taught at MIT back in the fall of 1977. In this model, I especially also focused on value change as a contributing factor and even tried to address the phase relationship between the political value expression cycles reported by Zvi Namenworth (1970, 1973) and the long economic cycle. This paper appears in MIT system dynamics D-memo archive as D-2765-1 (Saeed 1978), although it has never been referenced in the later memoranda on long wave. I have since revisited this work and subsumed in it also the endogenous changes in household income and labor market conditions and how they might affect the long wave together with changes in functional income distribution, which I hope to report in the near future.

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Annex
Model equations

1. Business cycle model

aggregate demand
average_shipments(t) = average_shipments(t - dt) +
(change_in_av_shipments) * dt
INIT average_shipments = orders
INFLOWS:
change_in_av_shipments = (shipments-
average_shipments)/shipment__averaging_time
backlog(t) = backlog(t - dt) + (orders - orders_filled) * dt
INIT backlog = desired_backlog
INFLOWS:
orders = 100+STEP(5,4)
OUTFLOWS:
orders_filled = shipments
average_desired_production =
SMTH1(desired_production,desired_production_averaging_time)
desired_inventory = average_shipments*inventory_coverage
desired_production = average_shipments+(desired_inventory-
inventory+backlog-desired_backlog)/time_to_correct_inventory_or_backlog
desired_production_averaging_time = .5
inventory_coverage = .5
shipment__averaging_time = .25
time_to_correct_inventory_or_backlog = .5

aggregate sypply
average_production(t) = average_production(t - dt) +
(change_in_av_production) * dt
INIT average_production = orders
INFLOWS:
change_in_av_production = (production-
average_production)/production_averaging_time
inventory(t) = inventory(t - dt) + (production - shipments) * dt
INIT inventory = desired_inventory
INFLOWS:
production = workforce*overtime_factor*labor_productivity
OUTFLOWS:
shipments = average_production*backlog_pressure*inventory_constraint
backlog_coverage = .5
desired_backlog = average_production*backlog_coverage
desired_labor_for_production = desired_production/labor_productivity
labor_productivity = 1
production_averaging_time = 1
backlog_pressure = GRAPH(backlog/desired_backlog)

(0.00, 0.00), (0.2, 0.06), (0.4, 0.14), (0.6, 0.3), (0.8, 0.59), (1.00, 1.00), (1.20, 1.46), (1.40, 1.74), (1.60, 1.87), (1.80, 1.95), (2.00, 2.00)
inventory_constraint = GRAPH(inventory/desired_inventory)
(0.00, 0.00), (0.2, 0.426), (0.4, 0.648), (0.6, 0.798), (0.8, 0.918), (1.00, 1.00), (1.20, 1.07), (1.40, 1.12), (1.60, 1.15), (1.80, 1.19), (2.00, 1.20)
overtime_factor = GRAPH(desired_labor_for_production/workforce)
(0.00, 0.00), (0.2, 0.348), (0.4, 0.588), (0.6, 0.768), (0.8, 0.906), (1.00, 1.00), (1.20, 1.08), (1.40, 1.14), (1.60, 1.18), (1.80, 1.19), (2.00, 1.20)

labor
hires_in_process(t) = hires_in_process(t - dt) + (hiring_starts - worker_hiring) * dt
INIT hires_in_process = worker_attrition*hiring_delay
INFLOWS:
hiring_starts = average_hiring_rate+(desired_labor_for_production+desired_hires_in_process-workforce-hires_in_process)/time_to_correct_workforce
OUTFLOWS:
worker_hiring = hires_in_process/hiring_delay
workforce(t) = workforce(t - dt) + (worker_hiring - worker_attrition) * dt
INIT workforce = (orders/labor_productivity)
INFLOWS:
worker_hiring = hires_in_process/hiring_delay
OUTFLOWS:
worker_attrition = workforce/average_life_of_employment
average_hiring_rate = SMTH1(worker_hiring,hiring__averaging_time)
average_life_of_employment = 2.5
desired_hires_in_process = average_hiring_rate*hiring_delay
hiring_delay = .5
hiring__averaging_time = .5
time_to_correct_workforce = 1

2. Kuznets cycle model

aggregate demand
backlog(t) = backlog(t - dt) + (orders - orders_filled) * dt
INIT backlog = desired_backlog
INFLOWS:
orders = 100+1*STEP(5,4)
OUTFLOWS:
orders_filled = shipments
average_desired_production =
SMTH1(desired_production,desired_production_averaging_time)
desired_production = average_shipments+(desired_inventory-
inventory+backlog-desired_backlog)/time_to_correct_inventory_or_backlog
desired_production_averaging_time = 1
time_to_correct_inventory_or_backlog = .5

aggregate supply
average_production(t) = average_production(t - dt) +
(change_in_av_production) * dt
INIT average_production = orders
INFLOWS:
change_in_av_production = (production-
average_production)/production_averaging_time
average_shipments(t) = average_shipments(t - dt) +
(change_in_av_shipments) * dt
INIT average_shipments = orders
INFLOWS:
change_in_av_shipments = (shipments-
average_shipments)/shipment__averaging_time
inventory(t) = inventory(t - dt) + (production - shipments) * dt
INIT inventory = desired_inventory
INFLOWS:
production =
capital_for_cons_goods_production*capital_utilization_factor*capital_product
ivity
OUTFLOWS:
shipments = average_production*backlog_pressure*inventory_constraint
backlog_coverage = .5
capital_productivity = .5
desired_backlog = average_production*backlog_coverage
desired_capital_for_cons_goods_production =
desired_production/capital_productivity
desired_inventory = average_shipments*inventory_coverage
inventory_coverage = 0.25
production_averaging_time = 1
shipment__averaging_time = 1

backlog_pressure = GRAPH(backlog/desired_backlog)
(0.00, 0.00), (0.2, 0.06), (0.4, 0.14), (0.6, 0.3), (0.8, 0.52), (1.00, 1.00), (1.20, 1.40), (1.40, 1.61), (1.60, 1.76), (1.80, 1.90), (2.00, 1.99)
capital_utilization_factor = GRAPH(desired_capital_for_cons_goods_production/capital_for_cons_goods_production)
(0.00, 0.00), (0.2, 0.348), (0.4, 0.588), (0.6, 0.768), (0.8, 0.906), (1.00, 1.00), (1.20, 1.08), (1.40, 1.14), (1.60, 1.18), (1.80, 1.19), (2.00, 1.20)
inventory_constraint = GRAPH(inventory/desired_inventory)
(0.00, 0.00), (0.2, 0.426), (0.4, 0.648), (0.6, 0.798), (0.8, 0.906), (1.00, 1.00), (1.20, 1.08), (1.40, 1.13), (1.60, 1.18), (1.80, 1.19), (2.00, 1.20)

fixed capital
capital_for_cons_goods_production(t) = capital_for_cons_goods_production(t - dt) + (capital_formation - capital_retirement) * dt
INIT capital_for_cons_goods_production = (orders/capital_productivity)
INFLOWS:
capital_formation = capital_on_order/capital_delivery_delay
OUTFLOWS:
capital_retirement = capital_for_cons_goods_production/average_life_of_capital
capital_on_order(t) = capital_on_order(t - dt) + (capital_orders - capital_formation) * dt
INIT capital_on_order = capital_retirement*capital_delivery_delay
INFLOWS:
capital_orders = average_capital_formation_rate*capital_on__order_adequacy*capitl_adequacy
OUTFLOWS:
capital_formation = capital_on_order/capital_delivery_delay
average_capital_formation_rate = SMTH1(capital_formation,capital_formation_averaging_time)
average_life_of_capital = 20
capital_delivery_delay = 2
capital_formation_averaging_time = 1
desired_capital_on_order = average_capital_formation_rate*capital_delivery_delay
capital_on__order_adequacy = GRAPH(capital_on_order/desired_capital_on_order)
(0.00, 5.00), (0.2, 3.28), (0.4, 2.33), (0.6, 1.83), (0.8, 1.33), (1.00, 1.00), (1.20, 0.725), (1.40, 0.5), (1.60, 0.275), (1.80, 0.1), (2.00, 0.00)
capitl_adequacy = GRAPH(desired_capital_for_cons_goods_production/capital_for_cons_goods_production)
(0.00, 0.00), (0.2, 0.06), (0.4, 0.14), (0.6, 0.27), (0.8, 0.56), (1.00, 1.00), (1.20, 1.44), (1.40, 1.71), (1.60, 1.85), (1.80, 1.95), (2.00, 2.00)

3. long wave model

capitl self ordering
desired__IG_capital = desired_IG_production*IG_KOR
IG_capital_adequacy =
GRAPH((investment__goods_capital)/desired__IG_capital)
(0.00, 5.00), (0.2, 3.88), (0.4, 2.95), (0.6, 2.20), (0.8, 1.55), (1.00, 1.00), (1.20, 0.55), (1.40, 0.3), (1.60, 0.15), (1.80, 0.05), (2.00, 0.00)

fixed CG capital
CG_capital(t) = CG_capital(t - dt) + (CG_capital_formation - CG_capital_retirement) * dt
INIT CG_capital = desired_CG_capital
INFLOWS:
CG_capital_formation = IG_production-IG_capital__formation
OUTFLOWS:
CG_capital_retirement = CG_capital/average_life_of_capital
CG_capital_on_order(t) = CG_capital_on_order(t - dt) + (CG_capital_orders - CG_capital_formation) * dt
INIT CG_capital_on_order = CG_capital_retirement*capital_delivery_delay
INFLOWS:
CG_capital_orders =
average_CG_capital_formation_rate*CG_capital_adeuacy*CG_capital_on__order_adequacy
OUTFLOWS:
CG_capital_formation = IG_production-IG_capital__formation
average_CG_capital_formation_rate =
SMTH1(CG_capital_formation,CG_capital_formation_averaging_time)
average_life_of_capital = 15
capital_delivery_delay = 2
CG_capital_formation_averaging_time = 1
desired_CG_capital = 200+STEP(10,5)
desired_CG_capital_on_order =
average_CG_capital_formation_rate*capital_delivery_delay
CG_capital_adeuacy = GRAPH(CG_capital/desired_CG_capital)
(0.00, 5.00), (0.2, 3.88), (0.4, 2.78), (0.6, 2.08), (0.8, 1.40), (1.00, 1.00), (1.20, 0.675), (1.40, 0.4), (1.60, 0.25), (1.80, 0.1), (2.00, 0.00)
CG_capital_on__order_adequacy =
GRAPH(CG_capital_on_order/desired_CG_capital_on_order)
(0.00, 5.00), (0.2, 3.68), (0.4, 2.88), (0.6, 2.05), (0.8, 1.45), (1.00, 1.00), (1.20, 0.6), (1.40, 0.4), (1.60, 0.25), (1.80, 0.125), (2.00, 0.00)

fixed IG capital

investment_goods_capital_on_order(t) =
investment_goods_capital_on_order(t - dt) + (IG_capital_orders -
IG_capital__formation) * dt
INIT investment_goods_capital_on_order =
CG_capital_on_order*IG_KOR/(average_life_of_capital-IG_KOR)
INFLOWS:
IG_capital_orders = av_inv_goods_retirement*IG_capital_adequacy
OUTFLOWS:
IG_capital__formation =
IG_production*investment_goods_capital_on_order/(CG_capital_on_order+investment_goods_capital_on_order)
investment__goods_capital(t) = investment__goods_capital(t - dt) +
(IG_capital__formation - IG_capital_retirement) * dt
INIT investment__goods_capital = desired__IG_capital
INFLOWS:
IG_capital__formation =
IG_production*investment_goods_capital_on_order/(CG_capital_on_order+investment_goods_capital_on_order)
OUTFLOWS:
IG_capital_retirement = investment__goods_capital/average_life_of_capital
av_inv_goods_retirement = SMTH1(IG_capital_retirement,1)
desired_IG_production =
(CG_capital_on_order+investment_goods_capital_on_order)/capital_delivery_delay
IG_KOR = 4
IG_production = (investment__goods_capital/IG_KOR)*IG_capital_utilization
IG_capital_utilization =
GRAPH(desired__IG_capital/investment__goods_capital)
(0.00, 0.00), (0.2, 0.48), (0.4, 0.7), (0.6, 0.86), (0.8, 0.95), (1.00, 1.00), (1.20, 1.06), (1.40, 1.09), (1.60, 1.13), (1.80, 1.15), (2.00, 1.16)

Not in a sector
total_capital = CG_capital+investment__goods_capital