A DYNAMIC MODEL OF TWO COMPETING CITIES: THE EFFECTS OF COMPETITION ON TOLLS AND LAND USE

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Abstract

In this paper we model the impacts of competition between cities when considering demand management strategies on both the optimal tolls and residential location choices. The work builds on earlier work which studied competition in a small network using a static equilibrium approach. That work showed that while both cities have an incentive to charge alone, once they begin, they are likely to fall into a Nash trap or prisoner's dilemma where both cities are worse off. Our research extends this by setting up a system dynamics model which includes all modes and longer term location responses. An isolated city is studied first and a simplified welfare function is used to determine the optimal toll around the central area and its impacts on location decisions and other transport indicators. A twin city is then added. Traffic from the neighbouring city may be charged and the revenue retained - a form of tax exporting behaviour which should increase the welfare of the city. We study the impact on the optimal tolls set by the cities and how the game develops between cities of equal size and amenity. The impact on location decisions and other transport indicators are presented along-side the implications for regulation and the development of cities within regional partnerships.

Key Words : Transport policy, road pricing, competition, Nash trap, land-use.

Introduction

This paper sets out to model the interaction between two competing cities who are considering toll cordons as a means of demand management. It is part of a larger research project which aims to answer the following policy questions :

- In what ways do and could cities compete using fiscal demand management policies?
- How should cities design their policies to achieve individual and collective 'best' outcomes?
- Should cities consider sharing revenue streams – should they compete or co-operate?
- How significant are these policies to the redistribution of business and residents between cities?

Firstly, cities do compete with each other. For more than fifty years, Public Choice Theory has explored the notion that cities compete to attract and retain residents and businesses (Tiebout, 1956; Basolo, 2000). Likewise, the Public Finance & Tax Competition literature identifies competition between cities on tax-and-spend policies (Wilson, 1999; Brueckner, 2001).

Research in the transport literature, on the other hand, has focused predominantly on intra-city issues. The strong focus in recent years has been on road user charging, economic theory suggesting benefits will accrue to a city from a combination of congestion relief and recycling of revenues within the city (Walters, 1961). Beyond the theoretical benchmark of full marginal cost pricing, the design of practical charging schemes, such as those adopted by local authorities in recent Transport Innovation Funds (TIF) bids, have generally focused on pricing cordons around single, mono-centric cities (Shepherd et al, 2008). As research has demonstrated, it is possible in such cases to design the location and level of charges for a cordon so as to systematically maximise the potential welfare gain to the city (Shepherd and Sumalee, 2004; Sumalee et al, 2005), yet there is an implicit premise here that the city acts *in isolation*.

Our research extends the work on optimal toll cordons by considering multiple cities and consists of two main strands, the first a set of structured interviews with cities and local authorities to determine city motivations when considering competition; the second a series of modelling exercises which aim to capture the behaviour within static and dynamic models.

Work on the first task by Marsden and Mullen (2012) has looked at the motivations of decision-makers in local government in different towns and cities of four major city regions in England. It showed that towns and cities both compete and collaborate to maximise their own competitive position. The major cities are seen as the main powerhouses of growth, with other towns and cities trading on particular distinctive skills sets or tourist offers and spill over effects from the major cities. Working together they can act as a more powerful voice to argue for investment from central government.

The aim of this paper is to model the impacts of competition between cities when considering demand management strategies on both the optimal tolls and upon longer-term decisions such as business and residential location choices. The research uses a dynamic land use transport interaction model of two neighbouring cities to analyse the impacts by setting up a game between the two cities who are assumed to maximise the welfare of their residents. The work builds on our earlier work by Koh et al (2012) who studied competition in a small network using a static equilibrium approach for private car traffic alone. Our research extends this by setting up a dynamic model which includes all modes and longer term location responses.

The dynamic model (based on MARS described below) is used first to study an isolated city (representative of Leeds) and a simplified welfare function is used to determine the optimal toll around the central area and its impacts on location decisions and other transport indicators. A twin city is then added to the model thus introducing traffic between the cities. This traffic may be charged to enter the central area along with own residents, however the revenue may be retained by the city - a form of tax exporting behaviour which would in theory increase the welfare of the city. However as shown by Koh et al (2012), both cities will have an incentive to charge and a game may evolve where in the long run both cities may be worse off in terms of welfare than if none had charged in the first place.

With this simple model set up we study the impact on the optimal tolls set by the cities and how the game develops between cities of equal size and amenity. The impact on location decisions and other transport indicators are presented along-side the implications for regulation and the development of cities within regional partnerships. Finally we compare the results with those which may be obtained by a dynamic game between two players.

Prior Research

Koh et al (2012) investigated a simple static network assignment problem under elastic demand where two adjacent authorities could toll road users to enter their central zones (see figure 1). Each city was assumed to maximise the welfare of their own residents, taking into account consumer surpluses and recycling of any toll revenues without lump sum re-distribution.

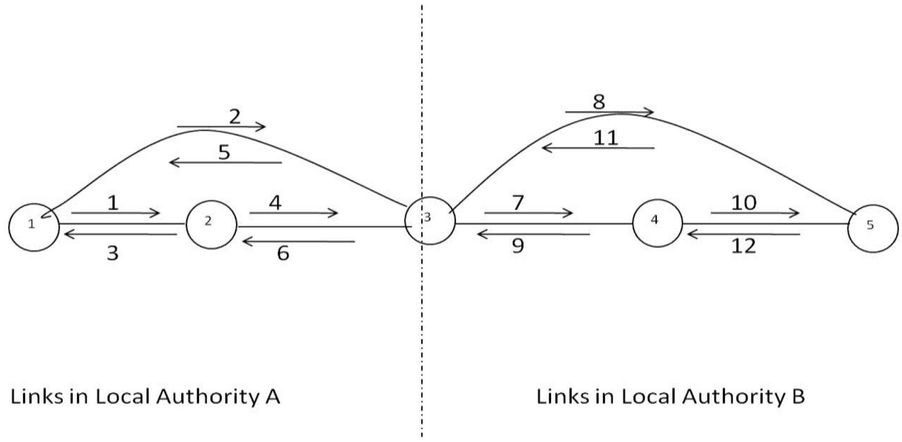

Figure 1 : Simple network used by Koh et al (2012)

As route choice is included, the problem is more complex than it first appears. We have two actors at the upper level who control the tolls within each city, while at the lower level the users respond to these tolls and seek efficient routes to satisfy their travel needs. The inter-play of the two authorities in each aiming to maximize its own welfare by setting a toll, conditional on the other authority's toll, while anticipating the impact on the travellers, leads us to an example of a so-called Equilibrium Problem with Equilibrium Constraints (EPEC) (Mordukhovich, 2005). The natural game between the two authorities can be seen as an example of a Nash game whereby the cities play against each other and are able to extract revenues from non-residents.

Using the response surfaces (see figure 2) Koh et al (2012) showed that there existed four local Nash Equilibria and that the Nash outcome was a classic prisoner's dilemma whereby once the game starts, both players end up in a position where they are both worse off than if they had not started the game.

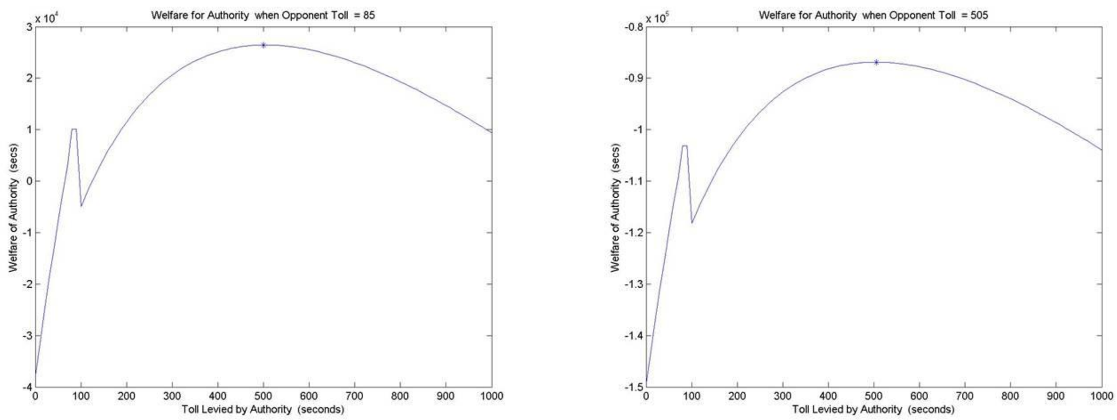

Figure 2 : (Left Pane) Welfare Plot for Authority A Showing Optimum at around 505 when Authority B levies a toll of 85; (Right Pane) Welfare Plot for Authority A Showing Optimum at around 505 when Authority B levies a toll of 505.

Koh et al (2012) also investigated the implications of having a higher level regulator in place, the possible gains to be had from signalling or collusion and the implications of the same game where one city is stronger than the other.

In this paper we build on this work by looking at the same game set up but with a dynamic model (MARS) which includes all modes of transport and longer term reactions such as relocation of residents and businesses. The aim is to see whether the same general results are to be found and whether local authorities would actually set tolls according to the Nash solution and hence fall into the "Nash Trap" where both cities are worse off in terms of welfare.

The MARS model

MARS is a dynamic Land Use and Transport Integrated model. The basic underlying hypothesis of MARS is that settlements and activities within them are self organising systems. MARS is based on the principles of systems dynamics (Sterman 2000) and synergetics (Haken 1983). The present version of MARS is implemented in Vensim®, a System Dynamics programming environment. MARS is capable of analysing policy combinations at the city/regional level and assessing their impacts over a 30 year planning period. Figures 3 and 4 show examples of the causal loop diagrams for the main responses included within MARS for commute trips by car and for development and relocation of residences respectively.

Figure 3 shows the CLD for the factors which affect the number of commute trips taken by car from one zone to another. Starting with the balancing feedback loop B1, commute trips by car increase as the attractiveness by car increases which in turn increases the search time for a parking space which then decreases the attractiveness of car use – hence the balancing nature of the loop. Loop B2 represents the effect of congestion – as trips by car increase speeds decrease, times increase and so attractiveness is decreased. Loop B3 show the impact on fuel costs, in our urban case as speeds increase fuel consumption is decreased – again we have a balancing feedback.

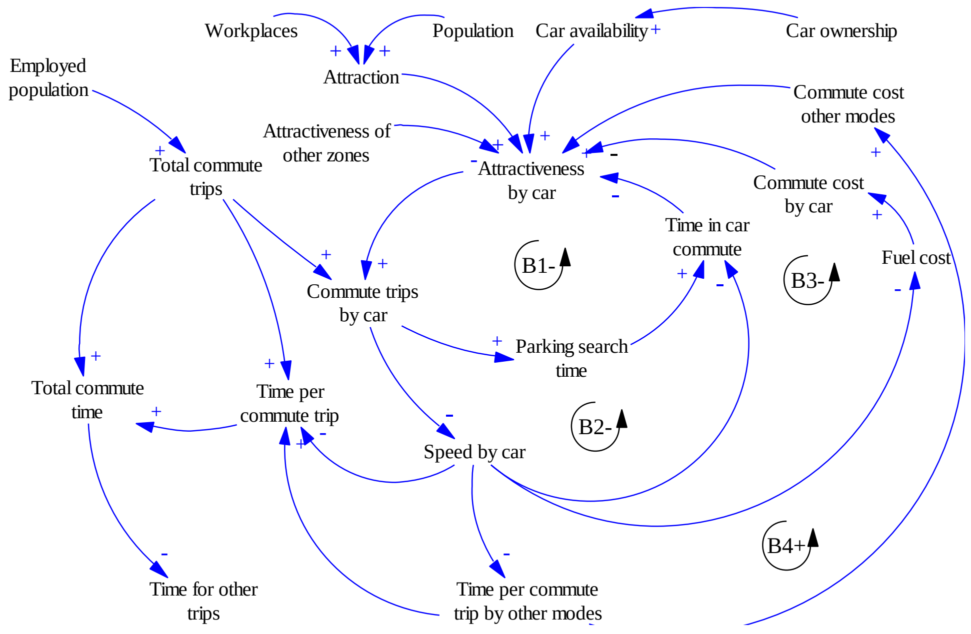

Figure 3: CLD for the transport model – commute trips by car in MARS

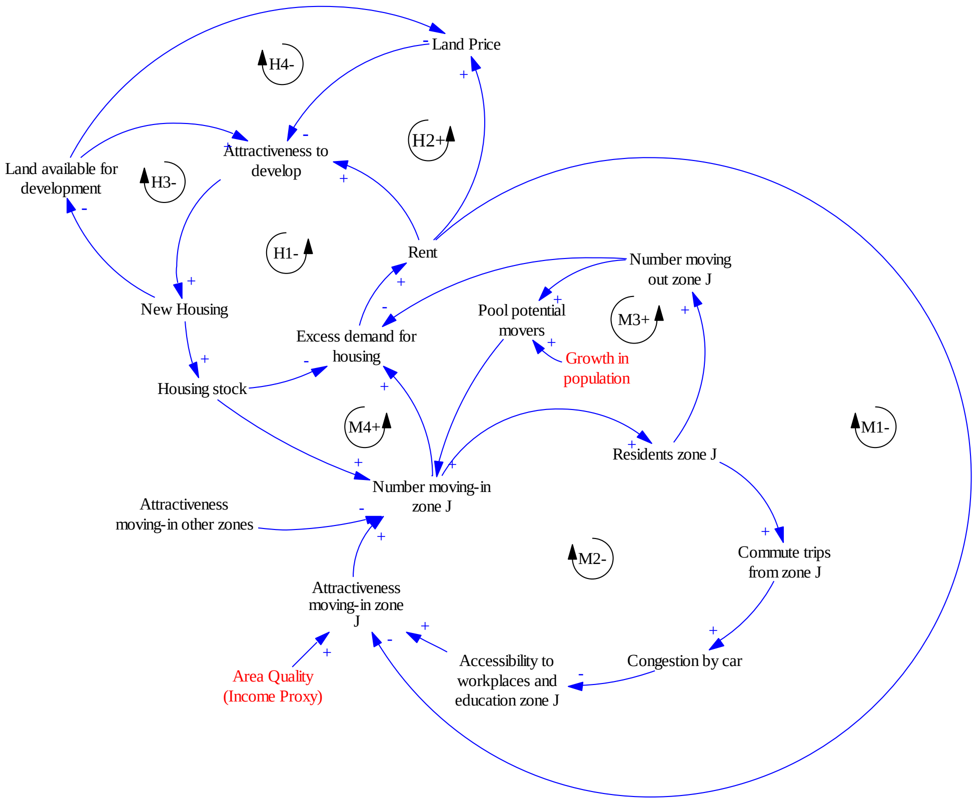

Figure 4: CLD for development of housing in MARS

Figure 4 shows the CLD for the development of housing and the interaction with location choice of residents in MARS. Starting with the development of housing, loop H1 is a balancing feedback loop which shows that the attractiveness to the developer to develop in a given zone is determined by the rent which can be achieved. The level of the rent is driven by the excess demand for housing which in turn is related to the housing stock and new housing developments. As new houses are developed the stock is increased which reduces the excess demand which then reduces the rent achievable which reduces the attractiveness to develop – resulting in a balancing loop. Loop H2 is a reinforcing loop as new housing reduces the excess demand which reduces rent and hence land price which in turn makes development more attractive all other things being equal. Loop H3 represents the restriction of land available for development; as land available is reduced then the attractiveness to develop is reduced. Loop H4 extends H3 to represent the effect of land availability on land price.

The housing development loops are linked to the residents' location choice. Firstly the main elements considered to influence the choice of location are rent, accessibility and area quality. As area quality is difficult to measure it is normal to take some kind of proxy for quality, in this case average income. The main loops in the residential choice are M1which is a balancing feedback loop – as more people move-in excess demand increases which increases rent which then reduces attractiveness to move in. M2 is also a balancing loop which shows that as the number of residents increases in a zone then congestion out of that zone increases which reduces accessibility to workplaces and so reduces attractiveness to move in.

Loop M3 is a positive feedback loop which simply shows that as the number of residents increases in a zone then the potential for moving out also increases (set as 10% of residents per year in the simplest case). This increases the pool of potential movers, which also includes growth in population (which could come from natural growth or in-migration and is taken from an exogenous forecast per annum). Loop M4 is a positive feedback loop which extends H1 – as more people move in this increases excess demand which increases rent and so increases attractiveness to develop which in turn increases the housing stock. Here it should be noted that housing stock available can limit the number of people allowed to move in to a zone as any excess demand is reallocated to other zones. This process reflects reality where excess demand must be taken up elsewhere if the capacity for residential occupation is reached in any one time period.

As MARS is not the subject of this paper readers are referred to Pfaffenbichler et al (2010) for a detailed description.

Case Study

In order to investigate the competition between cities and the impacts of competition on optimal tolls and other indicators, it was decided to use a simple hypothetical case study based on an aggregate representation of an existing more "spatially" complex model. This follows the discussion in Ghaffarzadegan et al (2011) who highlight the benefits of using small system dynamics models in addressing public policy issues.

Thus we first of all developed a 2 zone model of Leeds based on the full 33 zone model and then doubled this to form a twin city model with 4 zones based on the idea of two identical cities within reach of one another (see figure 5).

In the single city model the population of Leeds is split between the inner zone 1 and outer zone 2 with more growth predicted in the outer zones by 2030. We call this City A and the additional hypothetical neighbouring city we will call City B.

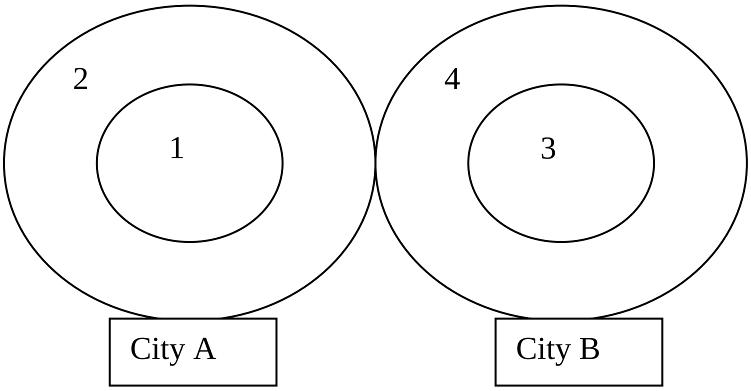

Figure 5 : The Twin City zones

The forecast population in the 2-zone model of Leeds was validated against that of the full sized Leeds model with 33 zones. This full sized Leeds model in turn was calibrated to UK TEMPRO forecasts of population, jobs and workers (DfT, 2010). Table 1 shows the population in the inner and outer areas (zone 1 and 2 in the 2 zone model) for the 2-zone and 33-zone model of Leeds. This validates the 2 zone model and note that both models exhibit more growth in the outer zones i.e. urban sprawl continues in Leeds as forecast by TEMPRO.

Table 1 Population of Leeds in Year 30

Region	2-zone model	33-zone model
Zone1 (1-13 of 33 zones)	342879	343384
Zone2 (14-33 of 33 zones)	621780	621801
Total	964659	965185

In what follows, each city may decide to charge car users to travel to the central area (zones 1 or 3) within the peak period. The charge will be applied to their own residents as well as those from the other city (where it exists).

Welfare measures

For a single city in isolation, the local authority is assumed to maximise the welfare of their citizens. The traditional form of welfare measure within the transport field is the Marshallian measure which sums consumer and producer surplus. The welfare measure includes the assumption that all revenues collected are recycled within the system i.e. shared back between the residents. For our case the measure may be estimated by the so called "rule of a half" (Williams, (1977)) and so the welfare measure is written as follows for the single city case :-

$$W = \sum_{i=1}^{n} \sum_{j=1}^{n} \left\{ -\frac{1}{2} \left[\left(t_{ij}^{1} - t_{ij}^{0} \right) \left(T_{ij}^{1} + T_{ij}^{0} \right) \right] - \frac{1}{2} \left[\tau \left(T_{ij}^{1} + T_{ij}^{0} \right) \right] + T_{ij}^{1} \tau \right\} \qquad (1)$$

where,

t_{ij}^{1} = travel time between each OD pair *ij* with road charge

t_{ij}^{0} = travel time between each OD pair *ij* without road charge

T_{ij}^1 = trips between each OD pair with road charge

T_{ij}^0 = trips between each OD pair without road charge

τ = toll charge to central zone

n = number of origins/destinations

It should be noted that for the isolated city, the local authority objective to maximise welfare will coincide with the objective of a higher level regulator e.g. the national government or some appointed regulator.

Once we move to the twin city case, we now have two authorities whose aim is to optimise the welfare of their residents – including their journeys to/from the other city region. The welfare measure for each city is similar to that used in the isolated city, but now we need to consider transfers of revenue from city A to city B and vice versa. The welfare for city A may be written as follows :-

$$W_A = W_{i \in A} + R_{B\ to\ A} - R_{A\ to\ B} \tag{2}$$

where,

$W_{i \in A}$ = welfare of residents from city A (origin i lies in city A), based on (1) above

$R_{B\ to\ A}$ = revenue collected by city A from residents of city B

$R_{A\ to\ B}$ = revenue paid by the residents of city A to city B

Now in the twin city case the higher level or global regulator would consider the total welfare of all residents i.e.$W_{total} = W_A + W_B$.

Examined Scenarios and Results

This section sets out the optimal tolls and welfare implications for a number of different cases, namely :-

- Isolated city (2 zone model)
- City A tolls alone (within the twin city set up i.e. 4 zone model)
- City A regulated (4 zone model)
- City A and City B – regulated (4 zone model)
- City A and City B Nash game (4 zone model)

In the first case, the city is considered in isolation and the local authority solution is to maximise the welfare of all residents. In this case there is no tax exporting behaviour and only one case to investigate. The next two cases consider City A tolling within the twin city set up so some tax exporting behaviour is now possible (City A tolls alone). The addition of the “City A regulated” test allows us to compare the solutions when a higher level regulator controls the toll in City A to maximise the welfare of residents from both cities. Finally there are two cases to consider where both cities are tolling users. The first is a regulated scenario where toll are set to maximise the total welfare of all residents, the second is the Nash game where cities are maximising their own residents’ welfare in a non-co-operative game. In this final scenario, tax exporting behaviour is assumed and revenues are not recycled between the cities.

Next, it should be noted that as the model predicts the impacts over a 30 year period we allow the tolls to vary over time. As the population is set to increase, the congestion is expected to increase so we may expect the tolls to increase over time. To avoid oscillations in the toll schemes, we limit the tolls to adjust linearly from the year of implementation (assumed to be year 5) until the final year (assumed to be year 30).

In order to find the optimal tolls levels, the VENSIM optimisation tool was used to maximise the appropriate welfare measure by varying the start and end values of the relevant tolls. Note that to calculate the Nash outcome, a diagonalisation approach was used whereby City A maximised Wa first with Tolls for City B held constant, then city B optimised with tolls for city A held at the previous iteration values. This was repeated until convergence which was within three rounds of the game.

Table 2 shows the resulting optimal tolls and changes in welfare in year 30 (taken as an example of welfare outcome).

Table 2 : Optimal tolls and changes in welfare in year 30

Scenario	Tolls year 5 €	Tolls year 30 €	Change in Welfare City A € year 30	Change in Welfare City B € year 30	Change in Welfare Total € year 30
City A 2-zone only	1.81	1.88	54356		
Single City A	4.9	5.92	217714	-241630	-23915
Regulator Single City A	1.4	1.79	127362	-67271	60091
Regulator 2-city	1.88	2.23	74651	74651	149302
Nash Game	5.67	6.6	-1103	-1103	-2206

Table 2 exposes a number of observations. Firstly, if a city is assumed to be an isolated city then the optimal tolls are lower than when the same city is able to extract toll revenue from a neighbouring city. The welfare for city A is also significantly higher when toll exporting behaviour is allowed, at the expense of those in city B and the change in total welfare is negative. Regulation of this single city case results in lower tolls, with lower welfare for city A, a lower decrease in welfare for city B and a move from negative to positive overall welfare (as expected due to regulation which considers all residents). However residents from city B are still worse off and the regulator would wish to consider some form of compensating tax or redistribution of revenues.

Due to symmetry, the tolls under the two city regulated case are equal and are between the single city toll levels and those resulting from the Nash game. The regulated case returns the highest total welfare as would be expected, but the welfare to each city is much lower than could be achieved in an un-regulated single city case. The most interesting case is the Nash game where both cities impose the highest tolls and both cities are worse off than if they had not tolled in the first place. This is the classic prisoner's dilemma which appeared in the work of Koh et al (2012) who used a static assignment model. In order to understand this response we take a view of the benefits per city during the game (assuming full information by both players). Figure 6 shows how the welfare for City A and City B varies over time as the toll rises from the optimal values of 4.9 to 5.92€ assuming city A moves first. It is obvious that City A has an incentive to toll and derive benefits for its residents at the expense of those from City B. As the cities are assumed to be identical in this case study, then City B would have the same incentive and so would respond.

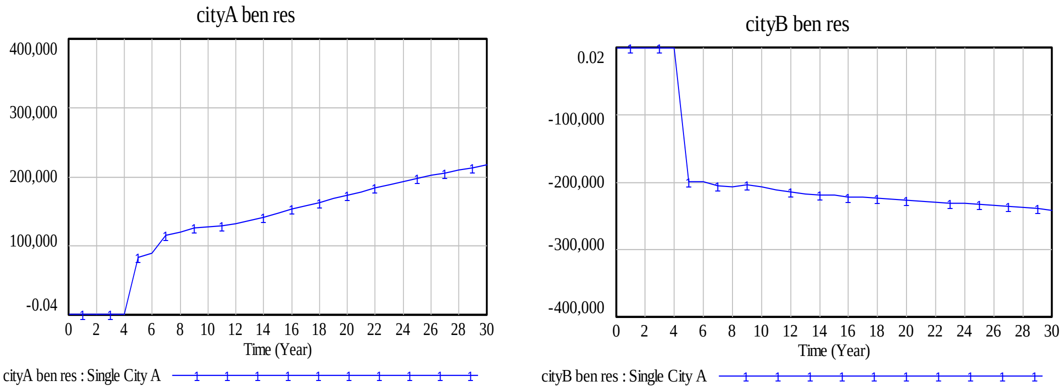

Figure 6 : Change in welfare for city A and city B residents with City A tolling alone

Figure 7 shows the resulting change in welfare for city A at the Nash outcome (it will be the same for city B due to symmetry). Note that the welfare profile begins with a significant drop in welfare and then begins to rise (lower decrease in welfare) as time progresses.

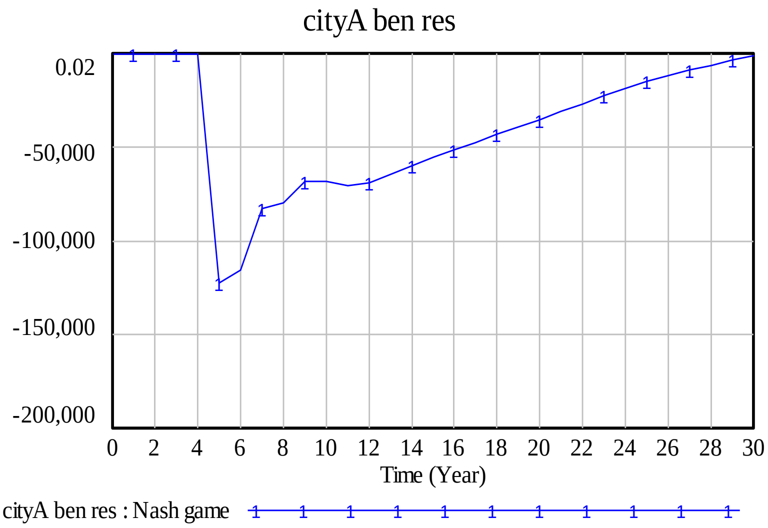

Figure 7 : City A welfare at the Nash outcome

Welfare loss decreases over time as the demand rises and congestion builds which then gives rise to greater time savings to both cities. This suggests that for more congested networks the Nash trap may not exist and that there may come a time when charging is beneficial to both cities even under the Nash outcome.

Finally whilst the most desirable case (from a regional perspective) should be the 2 city regulated case, it is noticeable from figure 8 that the total change in welfare in year 5 (year of implementation) is negative. Figure 8 shows that as with most dynamic systems, things may become worse before they get better. The negative benefits arise because it takes time for the users to respond to the charge levels. This means that initially at least there may be an overshoot or lagged response from travellers and this “out of equilibrium” behaviour results in a temporary decrease in welfare. It is caused by the fact that time benefits are not enough to outweigh the disbenefits of being charged.

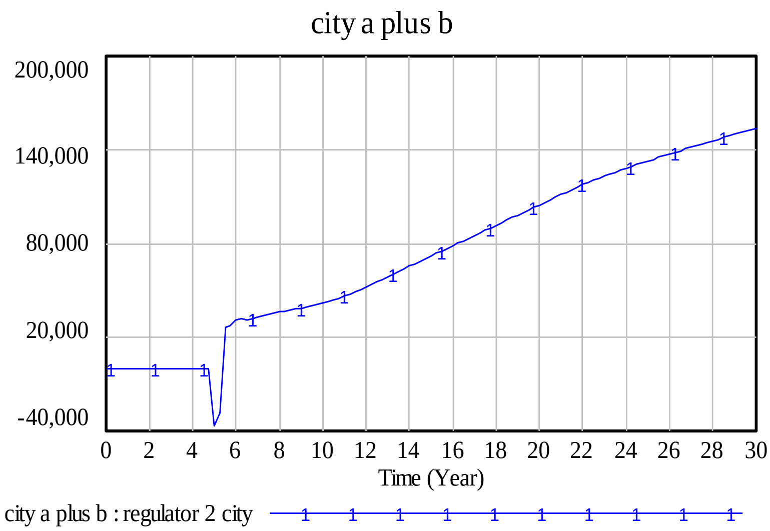

Figure 8 : Changes in Total welfare for the 2 city regulated case.

Impacts on residential/workplace location

Table 3 shows the change in residents in year 30 for each charging scenario. Firstly, the response in the longer term is that residents move into the charged zone. This is because the relative accessibility for those residing in the charged area improves compared to those residing in other areas. This is evident from all of the scenarios – be it for the 2-zone single city or 4-zone twin-city situation. With the 2 zone model, the movement of residents is within one city and so there is no gain in population. With the twin city model, the Single City A tolling alone shows the greatest response with the highest number moving to zone 1 due to the higher charges. Now with the twin city model, we can see that residents are attracted from all other zones and in effect City A becomes more attractive as a place to live and work – City B loses around 3800 residents to City A which whilst a relatively small number may be a cause for concern. Where both cities charge the same toll, the results are symmetrical as expected and mirror those of the 2 zone model with movement from outer to inner zones within each city. Similar effects were seen for workplaces, though the relative impact was much smaller with only around 400 jobs being transferred from City B to City A in the worst case scenario.

Table 3 Change in Residents in Year 30

Scenario	Change in residents Zone 1	Change in residents Zone 2	Change in residents Zone 3	Change in residents Zone 4
City A 2-zone only	2230#	-2230#	-	-
Single City A	7916	-4089	-585	-3242
Regulator Single City A	3495	-1898	-122	-1476
Regulator 2-city	3874	-3876	3881	-3879
Nash Game	6818	-6818	6818	-6818

based on a do-nothing scenario for City A 2 zone model

Dynamic game

The Nash solution above has been generated assuming that both cities have access to the same model of the future and that they play tolls in advance of actual time. It does not simulate the real time strategies of the cities. In order to do this we now set up a game within VENSIM whereby each player will act as a city authority and will have access to information on changes in current welfare for their city, revenues collected and tolls set by the opposing player.

Figure 9 shows the initial game set up in VENSIM where we allow users/players to set the tolls in 5 year steps and respond to the opposing player's toll. Whilst we are still to test this game with real stakeholders, our own response to the game situation suggests that it may be possible for the players to avoid the Nash trap. In figure 9 below, the initial tolls were set as low for City A and high for City B at year 5. Initially city B enjoys an increase in welfare and is able to extract revenue from city A. In response in year 10 city A increases the toll as does city B. This is in retaliation to the extraction of revenues. This continues until year 15. During the next 5 year period, both players see a negative welfare effect and then choose to reduce tolls. Further reductions bring further benefits. Whilst this is only one example of a possible response based on our own play of the game it demonstrates the possibility that decision-makers may be able to avoid the Nash trap.

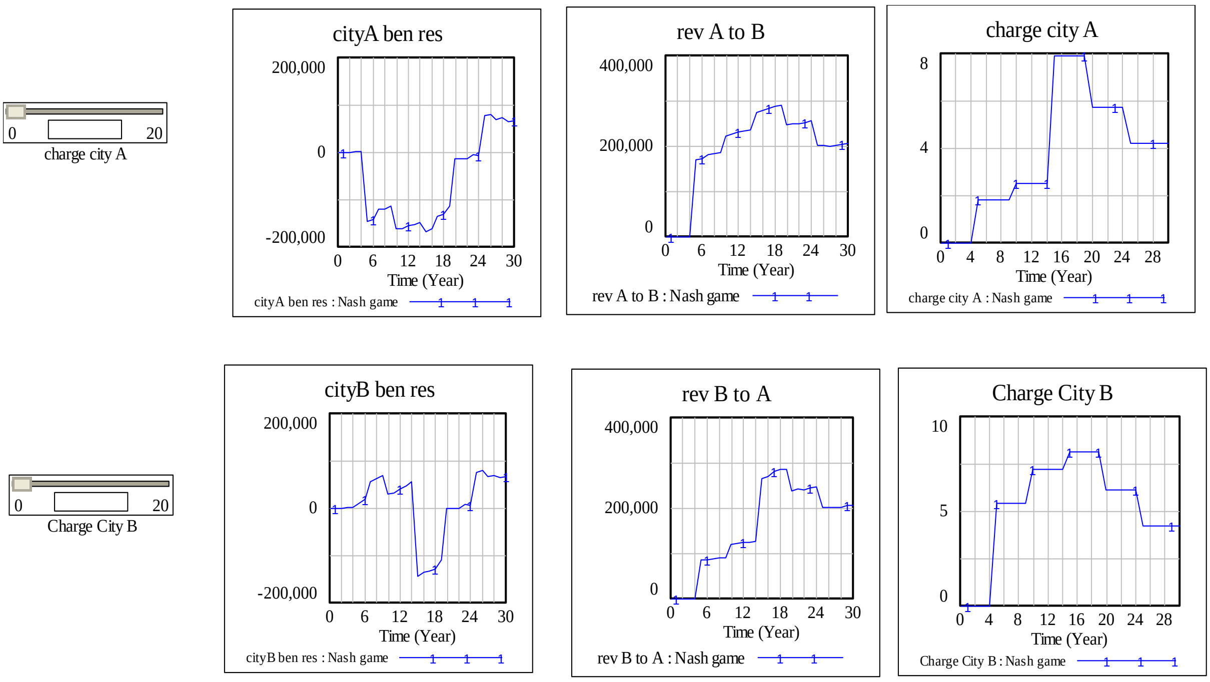

Figure 9: Initial attempt at the real dynamic game

Conclusions

This paper has reported on the development of a small model of an isolated city based on Leeds in the UK. We showed that adding a neighbouring city (assumed to be identical in this case), changes the optimal toll which City A may impose around its central area. The toll may be raised as City A may take advantage of those travelling from B to A. Regulation of a single city toll will reduce the optimal tolls and dampen the welfare effects, though those in City B are still worse off than in the no toll case. Where both cities toll, there is a possible win win outcome under a regulated scenario, however when cities only maximise their own residents' welfare a Nash game ensues and they may fall into a Nash trap where both cities are worse off and all users face higher tolls. These results are in line with the static game investigated by Koh et al (2012).

The longer term impact of tolls on residential location is to attract residents within the charged area. Whilst the impact is small, it was seen that with a twin city case where only one city tolls, then that city is likely to become more attractive as a place to live and so may become stronger at the expense of the other city. Whilst not reported here the same effect was also evident for workplaces/jobs though the relative impact was very small (around 400 jobs in total moved from City B to City A).

The above analysis which results in the Nash trap is based on the assumption that both cities make their decisions using a model which can forecast impacts over a thirty year period and that they (the decision makers) use the model to develop a toll strategy which responds to their competitor's strategy. So whilst we have implemented a dynamic model of the residents' response to tolls, the toll strategy itself is assumed to be played out between the cities in a quasi-static manner i.e. the game is played out over the full evaluation period and updated accordingly. In reality, transport planners are able to react and adjust a strategy in response to changes in observed data.

Given this, we implemented a dynamic version of the game within VENSIM and initial tests suggest that real time toll setting strategies may avoid the Nash outcome where both cities are worse off.

It is our intention to verify both how such decisions are made and the response to the game situation in the near future with local authority stakeholders. This research will look at which indicators are useful to the decision-makers and whether or not constraints on toll setting strategies such as only allowing positive changes in toll or full removal of tolls will affect the outcome of the game.

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