The Effect of Numbers and Aggressiveness of Bidders on Market Performance: A Behavioral Simulation of Sealed Bid, First Price, Private Value Auctions

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ABSTRACT:
Economists have a well-documented analytical approach to determining optimal pricing strategies and market efficiencies for a host of market types, including the spectrum of auction markets. While their analysis is impressive and certainly yields "optimal" bidding strategies (for the given set of assumptions), the mathematics underlying these analyses do not necessarily reflect how actual participants in such markets make bidding decisions. This paper presents the results of a simulated sealed bid, first price, private value multi-round auction. Auction participants follow a relatively simple algorithm for determining their bid. The number of bidders and their determination to earn the item up for auction (their "aggressiveness") is varied in order to explore the impact these parameters have on auction efficiency. The results show that both the number of bidders and their aggressiveness have dramatic impacts on both price and auction efficiency. The number of bidders is a factor in most analytical solutions. Aggressiveness, like other behavioral factors, is less common. While traditional economic analysis informs auctioneers that having more bidders will generate higher prices and greater revenue, it does not typically discuss bidder aggressiveness.

KEYWORDS:
Economics, Auctions, Simulation, Market Efficiency

INTRODUCTION

Economists have a well-documented analytical approach to determining optimal pricing strategies and market efficiencies for a host of market types, including the spectrum of auction markets. While it is impressive and certainly yields "optimal" bidding strategies (for the given set of assumptions), the mathematics underlying these analyses do not necessarily reflect how actual participants in such markets make bidding decisions. This paper presents the results of a simulated sealed bid, first price, private value multi-round auction in which the participants adjust their bidding strategy based on whether they earn the item or not. Aggressive bidders adjust the strategy differently than passive bidders – raising their bids more sharply when they lose an auction, for example. As a result, market performance varies by aggressiveness of auction participants. Before exploring bidder behavior, following is a short primer on auctions for readers less familiar with the topic.

SEALED BID, FIRST PRICE, PRIVATE VALUE AUCTIONS

Auctions can be structured in countless ways. Three of the central features of an auction are how bids are processed, how the price is determined, and how individuals value the item being auctioned.

Bid Processing

Bids can be gathered in a variety of ways. The most common ways are orally, via sealed bids, or electronically.

The stereotypical oral auction has either a fast-talking auctioneer standing near livestock, or a precise-speaking person standing behind a podium at the front of a room. But any auction where the bids (or asks – the seller's equivalent of a bid) are taken verbally constitutes an oral auction. So, for example, the "pits" of any major trading exchange are a type of oral auction. Oral auctions allow participants to make multiple bids – if someone bids more than you, then you are allowed, indeed encouraged, to raise your bid. There is no deadline in an oral auction; the auction continues until no further bids are made.

Bids can also be gathered on a one-time basis during a defined period leading up to the auction. Participants submit their bids to the auctioneer, historically, written and placed in sealed envelopes (hence the term for this sort of process: "Sealed Bid"), prior to a stated deadline. In most cases, only one bid per participant is allowed. At the appointed time, bids are opened to determine which bid is the highest.

Increasingly, with the rise of the Internet and especially eBay, bids are gathered electronically. Strictly speaking, electronic auctions are not exactly an alternative to oral and sealed-bid auctions – an electronic auction can be structured to mimic closely the characteristics of either of the two older style auctions – however they often have characteristics that differentiate them from their low-tech predecessors. EBay's auctions, for example, have many of the characteristics of an oral auction, but also a few elements of a sealed bid auction (most notably, a predetermined and public end). The unique structure of an eBay auction has invited the invention of "sniping" – making a last minute bid with the intent of preventing anyone from raising the bid. In fact, applications have been developed to facilitate sniping. But sniping does not make sense in oral and sealed bid auctions, so electronic auctions merit their own category.

Determining Price

Traditionally the price paid by the high bidder is the amount of his or her bid. Such auctions are called "First Price" auctions constitute the vast majority of real world auctions. Indeed, some people find it hard to fathom any other mechanism for establishing the price in an auction.

However, economists have theorized certain advantages arise from a mechanism where the high bidder pays an amount equal to the second-highest bid. An auction employing such a mechanism is called a "Second Price" auction.

Individual Valuation of the Item Being Auctioned

There are traditionally two ways to categorize auctions according to how individuals value the item being sold. In "Common Value" auctions, the item being sold has a value that is common, but uncertain, to all potential buyers. The classic example of a Common Value auction is an oil lease. The amount of oil that the lease will yield is (effectively) the same for all potential bidders, but each bidder has a different estimate of what that amount is.

Items with a well-known common value are not effectively traded using an auction. Bidding would quickly settle on or near the known common value.

In "Private Value" auctions, each potential buyer places a unique value on the item being sold and knows exactly what that value is. An example of a Private Value structure is a fine art auction. Bidders will place their own unique value on the object up for auction – the value that *they* place on adding the piece to their collection. Others will value it differently.

Not all Private Value auctions are as esoteric as art auctions. When an apartment building goes up for auction, much of the value of the building may be common across all bidders. What makes it a Private Value auction is that a well-run management company or an insightful developer may value the property more than others, and therefore have a unique valuation on the building.

Structure of the Simulated Auction

The simulation described here is the first of a series of explorations into auction markets from a behavioral perspective. Each type of auction will require a unique model. While there could be some debate about which type of auction is *the* most important, Sealed Bid, First Price, Private Value, auctions are a good place to begin. It is relatively common, easy to explain and easy to model. So, specifically, the simulation reported here represents an auction that is:

- Sealed Bid – Participants make a single bid.
- First Price – The high bidder earns the item and pays the amount of that high bid.
- Private Value – Each bidder has a unique value for the item, known only to that bidder.

PRIVATE VALUES AND BIDDING STRATEGIES

Each participant receives a normalized private value. Values are assigned to participants based on a random normal distribution with a mean of 50 and a standard deviation of 15. More than 99% of the values generated by this distribution will be between 0 and 100, and values are explicitly bounded to be within this range. Nearly 2/3 of the values will be within one standard deviation of the mean: between 35 and 65.

The auction is repeated sequentially 100 times, and each round is independent from the last – that is, the private value that a participant receives in a given round is unrelated to the value they received in previous rounds.

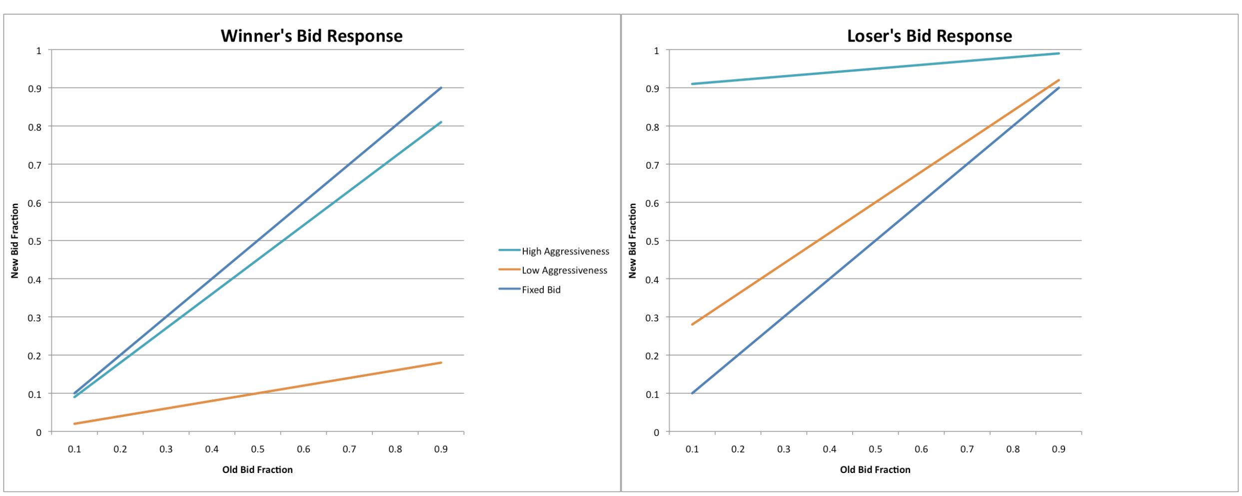
Participants multiply their private valuation by a "Bid Fraction" to determine their bid. For example, a private valuation of 70 and a bid fraction of 0.8 yield a bid of 56. The participant that submits the highest bid "earns" that round's item and pays a price equal to their bid. Auction winners earn a "profit" equal to the difference between their valuation and the price paid. All other participants in the auction earn 0.

THE BEHAVIORAL RESPONSE FUNCTION

Participants adjust their ***Bid Fraction*** from one round to the next, based on whether they earn the item or not and on how aggressive they are.

Participants that earn the item in one round will lower their ***Bid Fraction*** in the following round. This is consistent with two common behavioral responses to winning an auction. First, winners might feel sated and therefore a lessened sense of urgency to win the subsequent auction. Second, winners might wonder, "Did I pay too much?" If so, the behavioral response is also a lower ***Bid Fraction*** in the following round. Participants that fail to earn the item will raise their ***Bid Fraction*** in the following round. This represents the idea that "if I'm going to earn an item, I had better raise my bids."

How much a participant adjusts their ***Bid Fraction*** is a function of how determined they are to win – their aggressiveness. This response function is asymmetrical with respect to winning and losing. Regardless of aggressiveness, auction winners lower their ***Bid Fraction*** and auction losers raise theirs. However, an aggressive bidder will raise their ***Bid Fraction*** sharply upon a loss, but will lower their ***Bid Fraction*** only slightly upon a loss (owing to their deep desire to win). Less aggressive bidders are less troubled by losing an auction and will raise their bids relatively slowly. But when they win an auction, they will sharply cut their ***Bid Fraction***.



High aggressiveness is, in part, a function of the personality of the individual participant – some people are simply more competitive and determined to win than others. It is also a function of the nature of the item being auctioned – some items (e.g. Beatles paraphernalia) generating a more emotional response than others (e.g., rare stamps, perhaps).

MEASURES OF MARKET EFFICIENCY

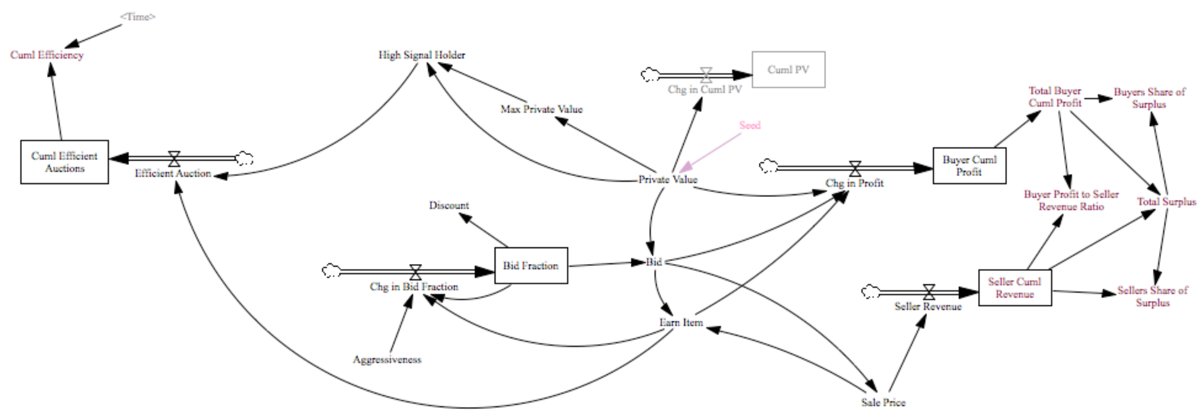
Markets often do a terrific job at allocating goods to the people who value them most. But despite what some pundits may suggest, they are not perfect mechanisms. So it's useful to explore how well a market performs this allocation.

One important measure is how often the high value holder in the market actually earns the item being auctioned. The simulation affords us complete information about participants' private values, their bids, and whether they earn the item or not. So an explicit calculation of the rate at which the high value holder wins the auction is easily implemented and is reported in the Simulation Results section below.

Auction houses are usually less concerned about whether the high value holder earns the item than they are about the amount of revenue generated. Their clients hire them to get the most money possible for the items put up for sale. So total revenue generated is very important to sellers and auctioneers alike. Since values are generated randomly, direct comparison of total revenue across runs is problematic. However, the percent of total economic surplus that is captured by the seller can be compared across runs and is reported in the Simulation Results section.

SIMULATION AND RESULTS

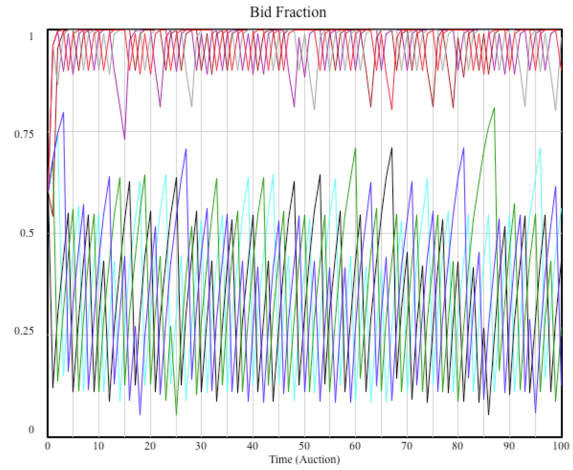
The preliminary model is built in Vensim™ (Ventana Systems) and involves 19 equations, including 5 stocks. Since all participants in the auction share a common structure (in terms of the process of receiving a valuation, calculating a bid fraction, and so forth), the model has a single array: Participants. The preliminary model is arrayed to represent four (4) participants. This allows for testing of the bid fraction response function with varying levels of aggressiveness. A diagram appears here; the equations are listed below.



The full model is under development and is being built using Forio Simulate™ (Forio Online Simulations). The Simulate platform and modeling language makes it easier to develop a user interface, share the simulation over the Internet, and develop related auction models later.

Typical Runs from the Preliminary Model

The graph on the right shows the ***Sale Price*** from two typical runs from the preliminary model. The upper line (in red) shows the price across 100 auctions resulting from a run with highly aggressive bidders. The lower line (in blue) shows the price pattern where the bidders have relatively low aggressiveness. It is important to note that the sequence of private valuations is *the same* for both runs. Clearly, aggressive bidders lead to higher realized prices, other things equal.



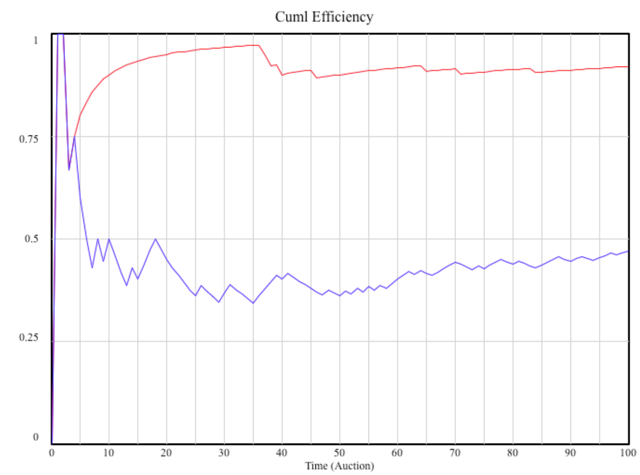
The difference is ***Sale Price*** even though the underlying valuations are the same can be directly traced to the different ***Bid Fractions*** that result from the different levels of aggressiveness.

The diagram on the left shows the deviation of ***Bid Fractions*** between the two runs. Clustered near the top, in red and purple hues, are the ever-changing ***Bid Fractions*** associated with the aggressive bidders. Being aggressive bidders, their bid fraction never stays far from "1," choosing earning the item over earning a "profit."

Towards the lower-center of the diagram, in green and blue hues, are the ***Bid Fractions*** of the less aggressive bidders. They discount their private valuations much more steeply than the aggressive do, leading to significantly lower prices.

Market Efficiency

The steeper discounts of less aggressive bidders leave more room for someone other than the high private valuation holder to earn the item. In other words, the steeper discounts lead to lower efficiency. And the graph to the right shows exactly that. In the first few rounds, the efficiencies are the same. In round 5, the efficiencies diverge, and within 10 rounds the ***Cumulative Efficiencies*** are at their long-term levels.



In a market with four (4) aggressive bidders, the high value holder earns the item roughly 90% of the time. The high value holder earns the item less than half the time if the bidders are not very aggressive.

Completion of the full model in Forio Simulate will enable easy cross-tabulation of market efficiency and other performance metrics, by number of participants and aggressiveness. An illustrative example of this analysis appears on the right.

The result will be a kind of "contour map" that shows how efficiency is affected by market size (number of bidders) and the aggressiveness of those in the market.

The development of the full model and the completion of this analysis are on schedule to be completed well before the conference convenes.

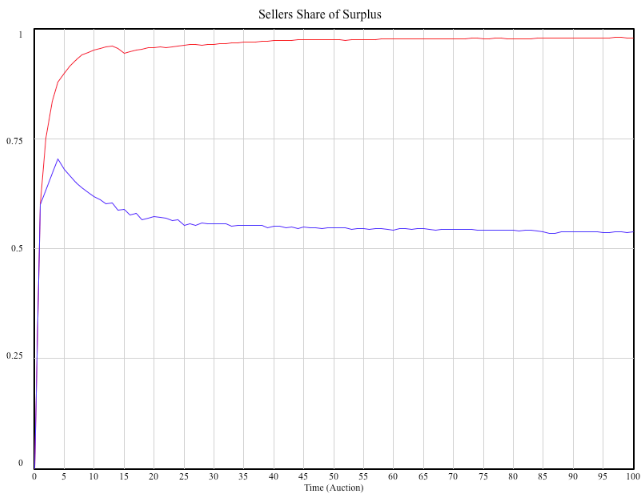
Illustrative

EFFICIENCY

(Percent of Auctions Won By High Value Holder)

		Aggressiveness								
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
# of Bidders	**2**	1	2	2	3	3	4	5	6	6
	3	1	2	3	4	5	6	7	8	8
	4	2	3	4	6	6	7	9	11	11
	5	2	3	5	7	8	10	11	13	14
	6	2	5	6	7	9	11	13	14	17
	7	2	5	7	9	11	13	16	17	19
	8	2	6	8	10	13	15	18	20	22
	9	3	6	8	11	14	17	19	22	25
	10	4	6	10	12	15	18	21	25	27
	11	4	7	11	14	17	20	23	27	30
	12	4	8	12	15	18	22	25	30	33
	13	4	8	13	16	20	24	27	32	35
	14	4	9	13	18	21	26	30	34	38
	15	5	10	14	19	23	28	32	37	41
	16	6	10	15	20	24	29	34	38	44
	17	5	11	15	21	26	31	36	42	46
	18	6	12	17	22	27	33	38	44	49
	19	6	12	17	23	29	35	41	46	52
	20	6	13	18	25	30	37	43	48	55
	21	7	13	19	25	32	38	45	51	57
	22	8	14	21	27	34	40	46	53	60
	23	8	14	22	28	35	42	49	56	63
	24	7	14	22	30	37	44	50	58	65
	25	8	15	23	30	38	46	53	60	68
	26	9	16	24	31	40	48	55	63	70
	27	9	17	25	33	41	49	57	65	74
	28	9	17	26	34	42	51	59	68	77
	29	9	18	27	35	44	53	61	70	79
	30	10	18	27	37	45	54	63	72	81

Seller Capture of Economic Surplus



As described earlier, another important market performance metric is the fraction of total economic surplus captured by the sellers and their agents. The graph on the left reports the results from the preliminary model. It clearly shows the advantage conveyed to sellers by the aggressive bidders.

In the four-person market populated by aggressive bidders, the sellers capture nearly 100% of the economic surplus that is available in the market. If low aggressive bidders populate the market, the surplus is split roughly evenly.

IMPLICATIONS

As economic analytical theory suggests, the number of bidders has a direct impact on market efficiency and revenue generation – the more participants in an auction, the higher the efficiency and the greater the revenue generated.

Bidder aggressiveness has a very strong influence on market efficiency and revenue generation. Even markets with relatively large numbers of bidders can have surprisingly poor efficiency if their aggressiveness is low.

Sellers and their agents (e.g., auction houses) should be particularly interested in selling items that have an emotional aspect to them because of the aggressiveness that is likely to accompany such items. And in general, aggressive bidders are very valuable to sellers and their agents.

NEXT STEPS

This is just the first step along a path with a lot of interesting possibilities. First, the results will be thoroughly compared to the latest literature in economics. Second, the marginal impact of aggressiveness will be explored. For example, what is the impact on market efficiency if a single aggressive bidder joins a market of, say, 20 otherwise fairly passive bidders? Third, a classroom application for conducting online, interactive experiments will be developed. Last, this entire set of activities will be expanded to include other types of auctions.

EQUATIONS

Total Surplus = Seller Cuml Revenue + Total Buyer Cuml Profit

Sellers Share of Surplus = Seller Cuml Revenue/Total Surplus

Buyers Share of Surplus = Total Buyer Cuml Profit / Total Surplus

Discount[Players] = 1 - Bid Fraction[Players]

Total Buyer Cuml Profit = SUM(Buyer Cuml Profit[Players!])

*Efficient Auction = SUM(High Signal Holder[Players!]*Earn Item[Players!])*

High Signal Holder[Players] = IF THEN ELSE(Private Value[Players]=Max Private Value, 1 , 0)

Buyer Profit to Seller Revenue Ratio = Total Buyer Cuml Profit / Seller Cuml Revenue

Max Private Value = VMAX(Private Value[Players!])

Cuml Efficiency = Cuml Efficient Auctions / Time

Cuml Efficient Auctions = INTEG (Efficient Auction, 0)

Earn Item[Players] = IF THEN ELSE(Bid[Players]=Sale Price,1 , 0)

*Bid[Players] = Private Value[Players] * Bid Fraction[Players]*

Chg in Cuml PV[Players] = Private Value[Players]

*Chg in Bid Fraction[Players] = (Earn Item[Players] * (Aggressiveness[Players] - 1)*
** Bid Fraction[Players]) + (1 - Earn Item[Players])*
** (1 - Bid Fraction[Players]) * Aggressiveness[Players]*

*Chg in Profit[Players] = Earn Item[Players] * (Private Value[Players] - Bid[Players])*

Cuml PV[Players] = INTEG (Chg in Cuml PV[Players], 0)

Bid Fraction[Players]= INTEG (Chg in Bid Fraction[Players], 0.6)

Seller Revenue = Sale Price

Players: P1,P2,P3,P4

Private Value[Players] = RANDOM NORMAL(0, 100, 50, 15, Seed)

Buyer Cuml Profit[Players]= INTEG (Chg in Profit[Players], 0)

Sale Price = VMAX(Bid[Players!])

Seed = 1234

Seller Cuml Revenue = INTEG (Seller Revenue, 0)

Aggressiveness[Players] = 0.1