A Soft Landing Model and a Mass-Spring Damper Based Control Heuristic[1]

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Abstract
This paper presents a soft landing model and a related control heuristic. The aim of this modeling effort is to represent the process of landing a spacecraft on the surface of a celestial body. This problem is known as the soft landing problem because crashing the spacecraft to the surface should be avoided. At the same time, long landing period necessitates extensive use of fuel, which should also be avoided. Consequently, the main goal in soft landing problem is to land the spacecraft as gently and as fast as possible. We adapted a control heuristic from the mass-spring damper model. According to the initial simulation runs, the adapted heuristic is successful in landing the spacecraft.

Keywords: soft landing; spacecraft; control heuristic; control law; mass-spring damper.

1. Introduction

Soft landing is an interesting and challenging problem in space exploration. The landing process should be controlled so as to land a spacecraft undamaged. By controlling the process, one tries to achieve a fast and safe landing on the surface of a celestial body. The velocity at the instant of landing should be minimal to prevent a crash. When landing on celestial bodies with no atmosphere (e.g. the moon), deceleration strategies that rely on the drag force (e.g. a parachute) do not work due to the absence of atmospheric molecules. Therefore, a reverse force thruster, which will decelerate the vehicle, is needed (see Figure 1). At the instant of landing, an impact force is generated which may be harmful to the vehicle. For a successful landing, this impact force must be under a certain limit and ideally it should be as low as possible. Another goal in landing is to decrease the time to land in order to decrease fuel consumption (Liu, Duan, and Teo, 2008; Zhou et al., 2009).

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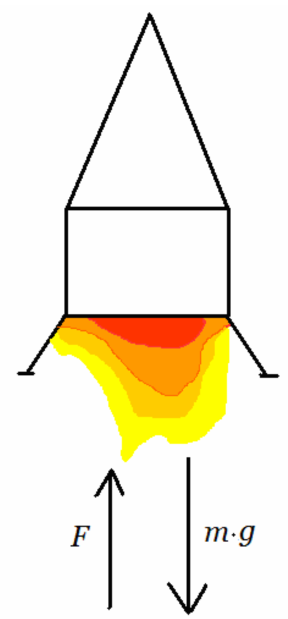

Figure 1: Free body diagram of the vehicle with a control force (F) generated by the reverse force thruster and the gravitational force ($m \cdot g$)

In this paper, we first constructed a stock-flow model of this problem. Later, we adapted a control heuristic from the mass-spring damper model. This control heuristic (i.e. control law) guarantees safe landing conditions for the spacecraft according to the initial simulation runs that we obtained. The total duration of landing seems plausible. The model and the heuristic is presented in the following sections.

2. The Model Structure and Equations

The stock-flow diagram of our soft landing model is given in Figure 2. This diagram represents only the physical structure of the described problem. *Height* (i.e. the vertical distance between the spacecraft and landing surface) and *Velocity* (i.e. the vertical velocity) are the two stock variables (accumulations) in the model. Note that our model ignores horizontal movement. *Velocity* is at the same time the one and only flow of *Height*. *Velocity* has a single flow too; *Acceleration*. *Net Force* and *Mass* determine *Acceleration*. In our model, *Mass* is a constant because we assumed that the change in the mass due to fuel consumption is negligible. *Height* is controlled via *Velocity*, *Velocity* via *Acceleration*, *Acceleration* via *Net Force*, and *Net Force* via *Control Force* (see equations 1-6).

$$Height_0 = 1000 \quad [m] \tag{1}$$

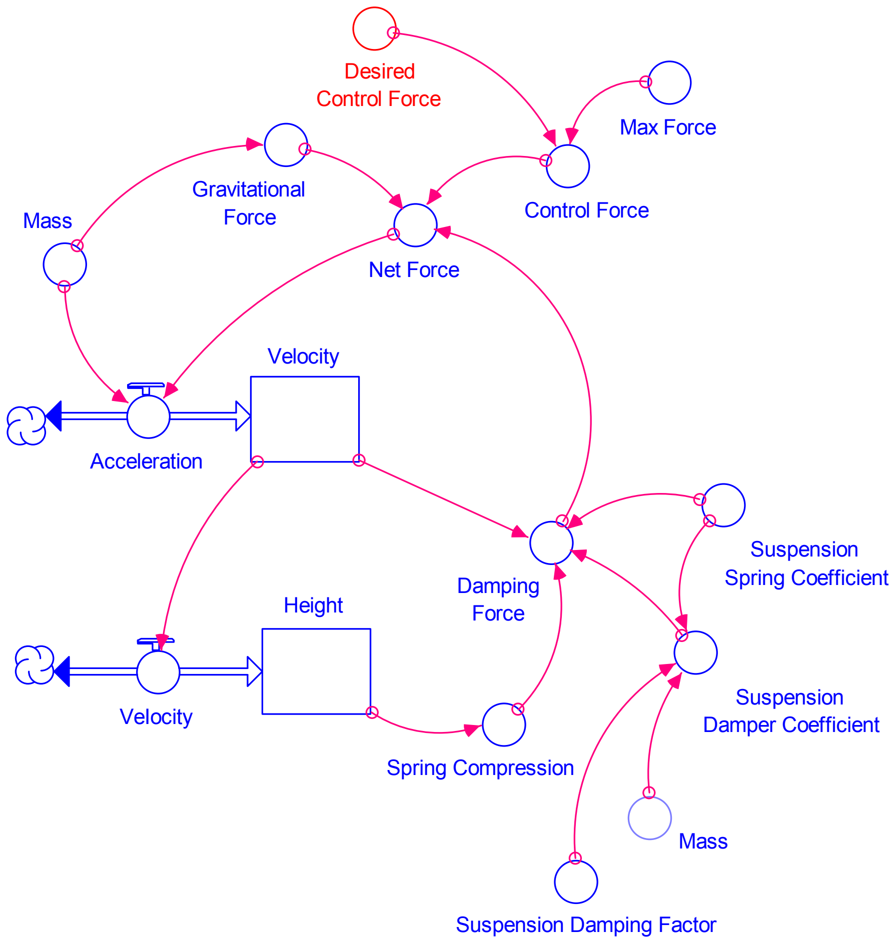

Figure 2: Stock-flow diagram of the model

$$Height_{t+DT} = Height_t + Velocity_t \cdot DT \quad [m] \tag{2}$$

$$Velocity_0 = -10 \quad [m/s] \tag{3}$$

$$Velocity_{t+DT} = Velocity_t + Acceleration \cdot DT \quad [m/s] \tag{4}$$

$$Acceleration = Net\ Force / Mass \quad [m/s^2] \tag{5}$$

Positive *Height*, *Velocity*, *Acceleration*, and force directions are upward from the surface. *Height* equals zero means that the vehicle touches the ground, but the springs of the landing gear are at rest, thus they bear no force at *Height* equals zero.

$$Net\ Force = Control\ Force + Damping\ Force + Gravitational\ Force \quad [N] \tag{6}$$

$$\textit{Gravitational Force} = 9810\,[N] \tag{7}$$

Gravitational Force, *Control Force*, and *Damping Force* add up to the *Net Force* acting on the vehicle (Equation 6). *Gravitational Force* acts on the vehicle due to mass and gravity and is assumed to be constant during landing; it does not change with the distance to the surface.

$$\textit{Damping Force} = \left\{\begin{array}{l} \text{IF } \{\textit{Spring Compression}\} = 0 \text{ THEN } 0 \\ \text{ELSE} \begin{pmatrix} \textit{Suspension} \\ \textit{Spring} \\ \textit{Coefficient} \end{pmatrix} \cdot \begin{pmatrix} \textit{Spring} \\ \textit{Compression} \end{pmatrix} - \begin{pmatrix} \textit{Suspension} \\ \textit{Damper} \\ \textit{Coefficient} \end{pmatrix} \cdot \textit{Velocity} \end{array}\right\} \; [N] \tag{8}$$

$$\textit{Spring Compression} = \text{MAX } (-\textit{Height}, 0) \quad [m] \tag{9}$$

$$\textit{Suspension Damper Coefficient} = \left\{ \textit{Suspension Damping Factor} \cdot \sqrt{\begin{pmatrix} \textit{Suspension} \\ \textit{Spring} \\ \textit{Coefficient} \end{pmatrix} \cdot \textit{Mass}} \right\} \; \left[\frac{N \cdot s}{m}\right] \tag{10}$$

$$\textit{Suspension Damping Factor} = 2 \quad [\text{dimensionless}] \tag{11}$$

$$\textit{Suspension Spring Coefficient} = 19{,}620 \quad [N/m] \tag{12}$$

The landing gear of the spacecraft is comprised of dampers and springs. *Damping Force*, which is a result of the compression of the landing gear, is generated after the spacecraft contacts the landing surface. Note that the variable *Spring Compression* represents the amount of compression. *Suspension Spring Coefficient* and *Suspension Damper Coefficient* together determine the damping behavior; subcritical, critical, or supercritical. The values of these two coefficients are selected such that critically damped behavior is obtained after the touch down. See equations 8-12.

$$\textit{Control Force} = \left\{\begin{array}{l} \text{IF } \{\textit{Height}\} \le 0 \text{ THEN } 0 \\ \text{ELSE MIN } (\textit{Desired Control Force}, \textit{Max Force}) \end{array}\right\} \; [N] \tag{13}$$

$$\textit{Max Force} = 29430\,[N] \tag{14}$$

Desired Control Force determined by the control heuristic, which is explained in the next section, is an input to the *Control Force* of the reverse force thruster. *Control Force* cannot be more than the maximum force applicable by the thruster. Therefore, in the case that the *Desired Control Force* is more than *Max Force*, *Max Force* is applied.

The other important model assumptions are:

- There is no atmosphere in the landing area, thus no air friction exists that would cause a drag force on the vehicle.
- Upon touching the ground, the thruster is off and is not switched on again. The simplified model diagram in Figure 2 and Equation 13 do not reflect this assumption. By giving the simplified version of the model, we aim to improve the readability of the manuscript and prevent digression.

3. Dynamic Behavior of Landing

As mentioned before, the aim of soft landing is to land the spacecraft as gently and as fast as possible. This task requires simultaneous control of *Velocity* and *Height*, which –due to the physical structure of the problem– can only be indirectly affected by the reverse force thruster (see Figures 1 and 2 and equations 1-6). Therefore, the task of soft landing is a challenging one. The dynamic behavior of *Height* is given in Figure 3. Initially, the change in *Height* (i.e. *Velocity*) is relatively fast and, as the spacecraft approaches to the surface, the change in *Height* slows down. Hence, the behavior obtained by the control heuristic, which will be explained in the next section, is a reasonable one; by a fast initial decline, the heuristic tries to decrease the time to land; by a slow final approach, it keeps the impact force well below harmful values. At the instant of touchdown, the value of *Velocity* is -0.05 m/s creating a maximum impact force of 11,231.29 N, which is 1.14 times the weight of the space craft.

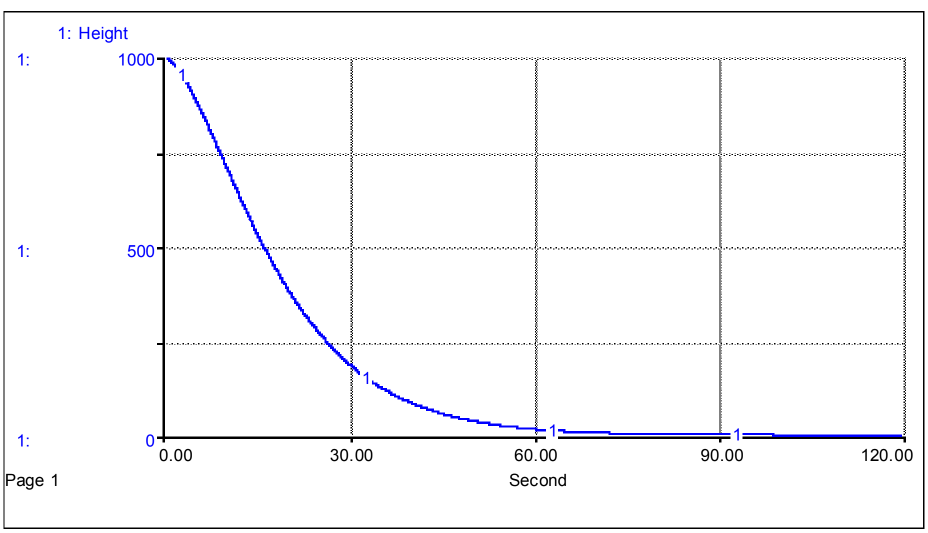

Figure 3: Dynamic behavior of *Height*

The dynamic behavior of *Velocity* and *Net Force* acting on the vehicle during landing are given in figures 4 and 5, which further explain the dynamic behavior obtained by the control heuristic. At first, the heuristic allows the spacecraft to accelerate in the negative direction towards the landing surface (see Figure 4, approximately within the time range of 0-10 seconds) by keeping *Net Force* negative (i.e. *Control Force* less than *Gravitational Force,* see figures 5 and 6). Aiming to decrease the duration of landing, *Velocity* continues to increase during this initial period. After this initial phase, *Velocity* decreases until the vehicle touches the surface (see Figure 4, approximately within the time range of 10-100 seconds). In this later phase, the heuristic produces more *Control Force* than *Gravitational Force* (Figure 6) resulting in a positive *Net Force* (Figure 5). At the moment of landing, *Control Force* is turned off and *Damping Force*, which is zero throughout the simulation up to this point, takes over and stops the vehicle (see figures 5 and 6, approximately around 100 seconds).

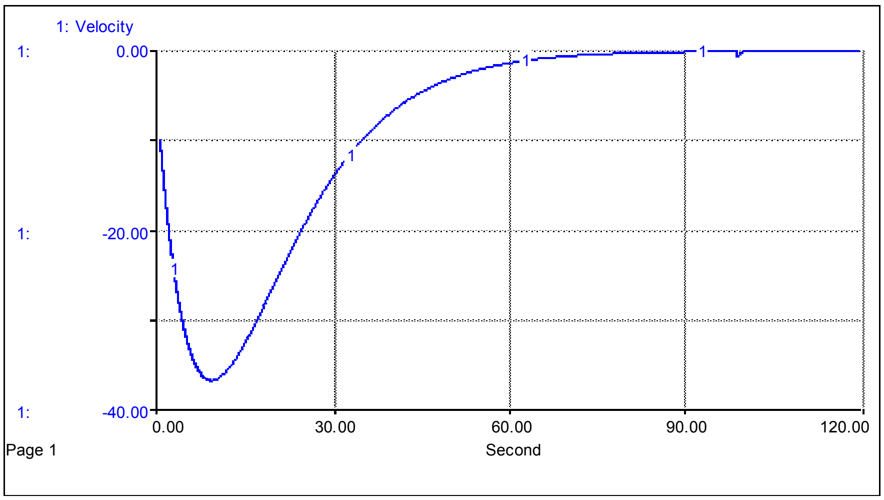

Figure 4: Dynamic behavior of *Velocity*

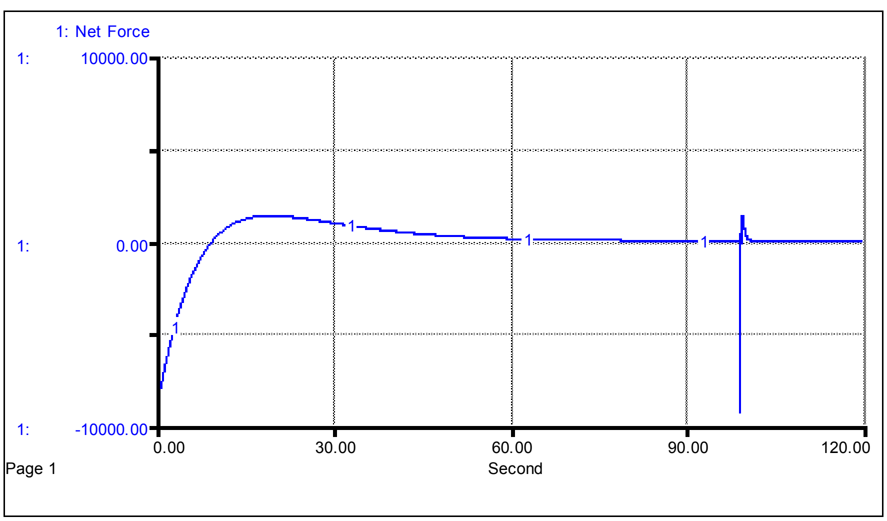

Figure 5: Net force acting on the vehicle during landing

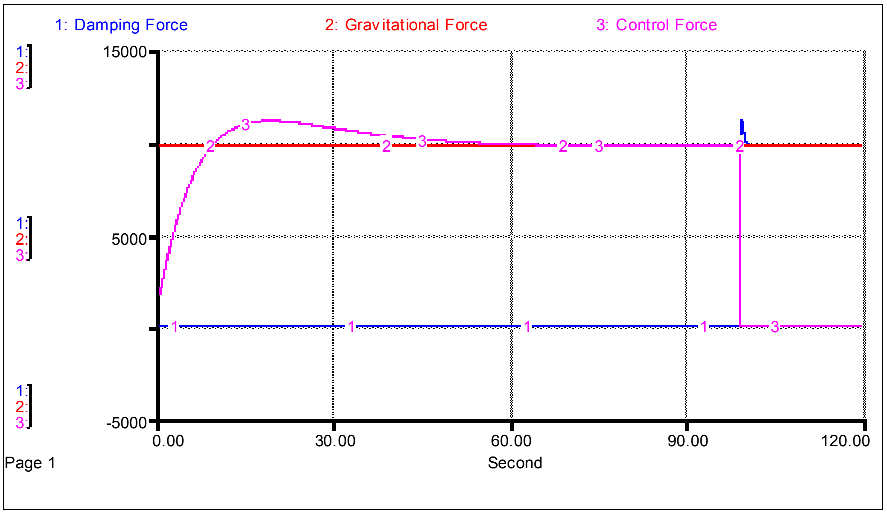

Figure 6: Absolute values of the forces acting on the vehicle during landing

4. A Mass-Spring Damper Based Control Heuristic

The control heuristic that we used is adapted from the mass-spring damper model. Therefore, we first present the mass-spring damper model and its equations, and later the adapted heuristic and its relation to the mass-spring damper model.

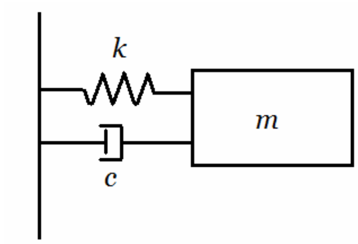

Figure 7: Mass-spring damper model

The mass-spring damper model given in Figure 7 is a well known structure. The equations for a non-driven (no external forces acting on the body) mass-spring damper model with mass m, spring constant k, and damper coefficient c can be given as (equations 15-20):

$$F_{spring} = -k \cdot x \tag{15}$$

$$F_{damper} = -c \cdot \dot{x} \tag{16}$$

$$F_{total} = F_{damper} + F_{spring} = -c \cdot \dot{x} - k \cdot x \tag{17}$$

$$\ddot{x} = \frac{F_{total}}{m} \tag{18}$$

From equations 17 and 18, we obtain:

$$m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = 0 \tag{19}$$

The damping ratio ζ of the system is defined as:

$$\zeta = \frac{c}{2 \cdot \sqrt{m \cdot k}} \tag{20}$$

The system can be underdamped, overdamped, or critically damped depending on the value of the damping ratio ζ. For ζ values under 1, the system is underdamped and for ζ values over 1, the system is overdamped. The case that the damping ratio ζ is exactly 1 is called critically damped. When the system is underdamped, the spring dominates the movement and the body oscillates. In the critically damped case, the body approaches the rest condition without an overshoot. When the system is overdamped, the damper dominates the dynamics. However, this does not cause a qualitative change in the dynamic behavior compared to the behavior obtained in the critically damped case. The only difference is that the body approaches the rest condition slower in the overdamped case (Åström and Murray, 2008).

The equations of the adapted heuristic are given below (equations 21-24):

$$\mathit{Desired\ Control\ Force} = \mathit{Desired\ Net\ Force} + \mathit{Gravitational\ Force} \quad [N] \tag{21}$$

Desired Control Force, which is the input variable in red in Figure 2, is the output of the heuristic.

$$\begin{aligned} &\mathit{Desired\ Net\ Force} = \\ &\quad -\mathit{Velocity\ Coefficient} \cdot \mathit{Velocity} - \mathit{Height\ Coefficient} \cdot \mathit{Height} \quad [N] \end{aligned} \tag{22}$$

$$\mathit{Height\ Coefficient} = 10 \quad [N/m] \tag{23}$$

$$\mathit{Velocity\ Coefficient} = 200 \quad [N \cdot s/m] \tag{24}$$

Although the soft landing model presented in this paper has more variables, it can be reduced to a second order differential equation similar to Equation 19. The formulation for *Desired Net Force* is obtained by changing the variable names in Equation 17.

5. Conclusions and Future Research

In this paper, we presented a soft landing model. The heuristic used in controlling the landing process is obtained from the mass-spring damper model. The heuristic produces plausible behavior.

In the continuation of this study, we plan to extend our model by adding an action formation delay, which is assumed to be caused by an actuator, and a measurement/report formation delay, which is assumed to be caused by a sensor. The addition of these delays to our model will make it more realistic because the actuators and sensors present in a soft landing system contribute to the dynamic complexity of that system as they are sources of delays. We anticipate that the addition of these delays will cause deterioration in the dynamic behavior to a great extent. In order to overcome the problematic behavior, we plan to adapt and use the heuristics developed by Yasarcan and Barlas (2005) and Yasarcan (2011), which are specifically suitable for this kind of control problems.

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