The Structural Crisis of Capital Accumulation in the USA and Its Causa Prima

©Alexander V. RYZHENKOV
Institute of Economics and Industrial Engineering
Siberian Branch of Russian Academy of Sciences
17 Lavrentiev Avenue Novosibirsk 630090 Russia
E-mail address: ryzhenko@ieie.nsc.ru

Abstract. *This paper re-defines three hypothetical laws of capital accumulation including endogenous rate of accumulation and capital-output ratio as state variables. An original non-linear relationship relates their growth rates. Other main state variables are output per worker, employment ratio and relative labour compensation. A comprehensive Phillips equation, governing real labour compensation, is an element of the initial hypothetical law (HL-1). HL-2 substitutes the former equation by a new one that reflects a long-term tendency of relative labour compensation to fall. A capital strive to maximal profit alters HL-2 in 2008. An alternative control law (HL-3) determines a growth rate of surplus value by a gap between target and current employment ratios while an integral absolute divergence of relative labour compensation from the average one for 1979–2008 is minimised.*

Based on the US macroeconomic data mainly for 1969–2008, computer simulation runs for a later period (through 2062) exhibit how an application of HL-3 in 2008 and afterwards could alleviate severity of the current crisis in the restructured US economy compared to evolution based on altered HL-2. The recovery from the present structural crisis of capital accumulation, worst after the World War II, will last until 2011–2013 when the pre-crisis maximum of net output is restored and until 2014–2017 when the pre-crisis maximum of employment is reached again.

1. Introduction

This research addresses long-term tendencies in the US economy such as the declining countervailing power of labour, falling labour share in GDP, lower industrial capacity utilisation and atrophy of net non-residential investment. It focuses on the courses of the present structural crisis of capital accumulation in the USA especially on record high unemployment as its specific manifestation.[1]

This paper continues a research thread of a class conflict theory of macropolicy based upon the Marxian concept of cycle. The key assumptions are: first, the contradiction between value and use-value of labour power (its ability to create surplus value) is a fundamental factor of capitalist development (including the present structural crisis); second, investment are the main trigger mechanism of industrial cycle, third, capital has been pursuing policies aimed at maximisation of profit that requires the industrial cycle, fourth, from capitalist point of view, "benefit" of a crisis is that it purges the excesses of the previous boom, leaving the economy in a healthier state.

In order to increase a stationary and average profit rate, capital accumulation tends to decrease a stationary and average relative labour compensation using mass unemployment as a forceful instrument. Thus the fast and sharp decline of output, employment and profit (observed in 2008–2009) is, mostly likely, the necessary consequence of such a profit-lead policy. The latter is perceptibly preferred

[1] The recent report points out: "The recession's impact on the labor market has been severe: employment in December 2009 was 7.2 million below its peak level two years earlier, and the unemployment rate was 10 percent. Moreover, although real GDP has begun to grow, employment losses are continuing" (Economic Report of the President 2010: 68).

by the dominant adverse societal culture exposed vividly in the recent influential book (Galbraith 2008).

This paper also emphasises that in the USA, as in Italy (Ryzhenkov 2008), the labourers, rather paradoxically, are more interested than capitalists in investing a higher profit share in the domestic economy; this issue is too important to be decided by capitalist only or by anonymous ‘market forces’. The history teaches stabilising policy that contradicts capital interests cannot be implemented without a prior pro-labour power shift.

The rest of this paper is organised in the following way.

Section 2 re-formulates two hypothetical laws of capital accumulation for the modern US economy (HL-1 and HL-2). They contain a new partial non-linear dynamic law for rate of accumulation that reflects a pro-cyclical character of this variable. Whereas HL-1 contains a comprehensive Phillips equation for the rate of change of real labour compensation, HL-2 subordinates that rate to the rate of growth of output per worker. Their intensive deterministic forms are composed of five non-linear ordinary differential equations with the following state variables: output per worker, rate of accumulation, capital-output ratio, relative labour compensation and employment ratio.

Section 3 explores a historical fit of HL-1 and HL-2 for the US Economy in 1969–1982 and 1983–2008 sub-periods and offers other behaviour reproduction tests for these laws. Their non-observable parameters are identified through application of a simplified version of the extended Kalman filtering (EKF) to macroeconomic data over the basal period 1969–2008 as a whole. The official US macroeconomic statistics serve thereby as an empirical base.

Section 4 elaborates control law of capital accumulation (HL-3) for the modern US economy that determines growth rate of surplus value by a gap between target and actual employment ratios. An intensive deterministic form of HL-3 contains the same state variables as HL-1 and HL-2, only differential equations for relative labour compensation are different in these three laws.

Section 5 investigates inertia scenario I based on unaltered HL-2 as well as two profit enhancing scenarios II and III maintained by parametrically altered HL-2 and HL-3, respectively. The latter two differ in chosen policy optimisation: whereas scenario II maximises total profit over forty years, scenario III minimises for twenty years an absolute total divergence of relative labour compensation from its average magnitude for 1979–2008.

Table 1 lists the state and other variables of all three hypothetic laws. Time is viewed as a continuous variable. So the appropriate measure for the rate of change of a variable x is the derivative of x with respect to time ($\dot{x} = dx/dt$), while its growth rate is logarithmic derivative $\hat{x} = \ln(x)' = \dot{x}/x = dx/(xdt)$. The same convention is appropriate for all variables. The main variables with their units of measurement follow: a [millions of 2005 dollars per worker per year], k, u, v [dimensionless], s [years]. Calculations of u and s are done with the nominators and denominators measured in current prices. The employment ratio v is for the civil labour force (without accounting the latent and stagnant unemployment). The net fixed capital (K) is a sum of private and governmental produced non-residential fixed assets.

The presented models consider relations between classes of capitalists and workers at rather high level of abstraction. The commodity market is *not* cleared á la vulgar Say’s Law because of fundamental contradiction between *value* and *use-value* of commodity. Still an explicit treatment of disequilibria on good market is left for future research. Capitalist class owns means of production and circulation; workers own their labour power that they sell to capitalists for a restricted period of time. Only one good is produced as net output in macro-economic setting. These models abstract from differences between product real labour compensation and purchasing power real labour compensation arising due to differences between price index of net output and that of workers’ consumption bundle.

Strictly speaking prices exists in these models only for two commodities: labour power and workers’ consumption good whereas there is no interest rate and no price of capital good, which is in entire possession of the collective capitalist. The collective capitalist does not sell surplus product on the good market explicitly. Therefore surplus product is not a visible commodity and has neither perceptible labour value nor observable price. It is assumed for simplicity that abstract labour embodied in surplus product does represent surplus value and that net output unit price is identically one whereas profit equals surplus product.

Table 1. The main variables of HL-1, Hl-2 and HL-3

Variable	Notation
Real net output	P
Nominal net output	$P*1 = P$
Employment	L
Labour force	N
Output per worker	$a = P/L$
Employment ratio	$v = L/N$
Fixed capital (net)	K
Worker's real labour compensation	w
Unit value of labour power (relative labour compensation)	u
Capital-output ratio	$s = K/P$
Surplus product	$M = (1 - u)P$
Profit	$(P - wL)*1 = P - wL$
Surplus value	$S = (1 - u)L$
Rate of capital accumulation	k
Net accumulation of fixed capital	$\dot{K} = kM = k(1 - u)P$
Capital intensity	K/L
Profit rate (profitability)	$M/K = (1 - u)/s$
Rate of surplus value	$S/(L–S) = (1 - u)/u$

The inverse of output per worker ($1/a$) represents a total labour input embodied in a unit of net output, so it approximates a magnitude of labour value of this unit.[2] The value of a unit labour power is $u = w/a$, unit surplus value is $1 - u$; total surplus value is the labour value of surplus product, measured by surplus labour, $S = (1 - u)L$.

Total profit $M = Sa$ is the money form of surplus product. In hypothetical laws, net output unit price (1) is omitted below for simplicity. A target employment ratio in HL-3 only is denoted as $X = const$.

2. Two Hypothetic Laws of Capital Accumulation in the US Economy

The advanced capital does not include variable capital since workers are paid at the end of each completed circulation process. Capital of circulation, natural capital and resource rent are not taken into explicit account; therefore magnitudes of general profit rate are biased. International relations are not presented explicitly.

Net national product (NNP) represents net output. As nowadays the US income receipts from the rest of the world exceed income payments to the rest of the world (including interest payments), NNP is bigger than net domestic product. Still a far greater part of surplus product is domestically produced.

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[2] Let Q is the total product, A is the direct material input per unit of total output, $l = L/Q$ is the direct labour input per unit of total output; $P = (1 - A)Q$ is the net output, while $Q = (1 - A)^{-1}P$. Then $L = lQ = l[(1 - A)^{-1}P] = P/a$ is the total labour input, and $1/a = l(1 - A)^{-1}$. The labour value of an output unit is approximated by the total labour embodied in this unit: $\omega = \omega A + l = l(1 - A)^{-1} = 1/a$.

National income equals NNP less statistical discrepancy in the US national accounts statistics used in this paper (BEA 2009, Economic Report of the President 2010).

Marx' notion of capitalist surplus product is the base for all three following definitions of (total) profit. They use BEA national income and product accounts.

The *first* definition grasps profit as a residual: NNP (gross national product less consumption of fixed capital) minus total labour compensation measured as pre-tax compensation of employees (including supplements) and minus imputed (by the author) labour compensation of self-employed persons as a part of proprietors' income.

In the *second* equivalent definition, profit consists of net domestic operating surplus of private enterprises, current surplus of government enterprises, less imputed (by the author) labour compensation of self-employed persons as a part of proprietors' income, plus taxes on production and import less subsidies, plus statistical discrepancy, plus income receipts from the rest of the world, less income payments to the rest of the world.

The *third* definition results from the second after adding details: total profit consists of remaining part of proprietors' income with inventory valuation and capital consumption adjustments, rental income of persons with capital consumption adjustments, corporate profits with inventory valuation and capital consumption adjustments, net interest and miscellaneous payments, taxes on production and imports less subsidies, business current net transfer payments, current surplus of government enterprises and statistical discrepancy (that is not included in national income but included in NNP).

Below profit is considered only as aggregate. Therefore the *first* definition is mostly relevant.

2.1. An Extensive Deterministic Form of HL-1

If $t < T_n$, a deterministic model consists of the following equations:

$$P = K/s; \tag{1}$$

$$L = P/a; \tag{2}$$

$$u = w/a,\ 0 < u < 1; \tag{3}$$

$$\hat{a} = m_1 + m_2 \hat{K/L} + m_3 \psi_1(\hat{v}), \tag{4}$$

$$\psi_1(\hat{v}) = sgn(\hat{v})|\hat{v}|^j,\ m_1 > 0,\ 1 > m_2 > 0,\ m_3 > 0,\ 1 > j > 0;$$

$$\hat{K/L} = n_1 + n_2 u + n_3(v - v_c), \tag{5}$$

$$n_2 > 0,\ n_3 > 0,\ 1 > v_c > 0;$$

$$v = L/N,\ 1 > v > 0; \tag{6}$$

$$n = n_a + p_1 e_1^{-M_1|K/L - K_c/L_c|^{i_1}} \tag{7a}$$

for $0 < K/L < K_c/L_c,\ M_1 = 1,\ p_1 > 0$;

$$n = n_a + p_1 e_2^{-M_2(K/L - K_c/L_c)^{i_2}} \tag{7b}$$

for $K/L \geq K_c/L_c,\ M_2 = 1,\ p_1 > 0$;

$$\hat{w} = -g + rv + b\hat{K/L},\ g > 0,\ r > 0; \tag{8a}$$

$$P = wL + M = Q + \dot{K} = wL + (1 - k)M + \dot{K}; \tag{9}$$

$$\dot{K} = k(1 - u)P = kM,\ 0 \leq k \leq 1; \tag{10}$$

$$\hat{k} = c_1 \psi_2(\hat{s}),\ c_1 < 0,$$

$$\psi_2(\hat{s}) = sgn(\hat{s})|\hat{s}|^{j_2},\ 1 \geq j_2 > 0. \tag{11}$$

Equation (1) postulates a technical-economic relation connecting the net fixed capital (K), net output (P) and capital-output ratio (s). Equation (2) relates output per worker (a), net output (P) and labour input, or employment (L). Equation (3) describes the relative labour compensation (u), or unit labour value, as the ratio of real labour compensation (w) to output per worker (a).[3]

Equation (4) is an extended technical progress function. It includes: the rate of change of capital intensity, K/L, and direct positive scale effect, $m_3\psi_1(\hat{v})$; $|x| \geq 0$ is an absolute value of x; $sgn(x) = -1$ for $x < 0$, $sgn(x) = 1$ for $x \geq 0$.

The non-linear continuous function $\psi_1(\hat{v})$ is analytical except at singular points with $\hat{v} = 0$ where its positive first derivative ($\psi_1'(\hat{v}) = j|\hat{v}|^{j-1} > 0$) becomes infinite. The derivatives of the function $\psi_1(\hat{v})$ of higher orders go to plus or minus infinity at the vicinity of $\hat{v} = 0$. This substantial singularity explains why the growth rate of output per worker changes stepwise at local maximums and minimums of the employment ratio.

Equation (6) outlines the rate of employment (v) as a result of the buying and selling of labour–power. The variable v plays decisive role in determination of the rate of change of the real labour compensation (w). In the comprehensive Phillips equation (8a) for $t < T_n$, the rate of change of the real labour compensation (w) depends on the employment rate (v), as in the usual Phillips relation, and on the rate of change of capital intensity (K/L) additionally. Capital intensity (K/L) is a proxy for qualification. Mechanisation (automation) manifests itself in growing capital intensity.

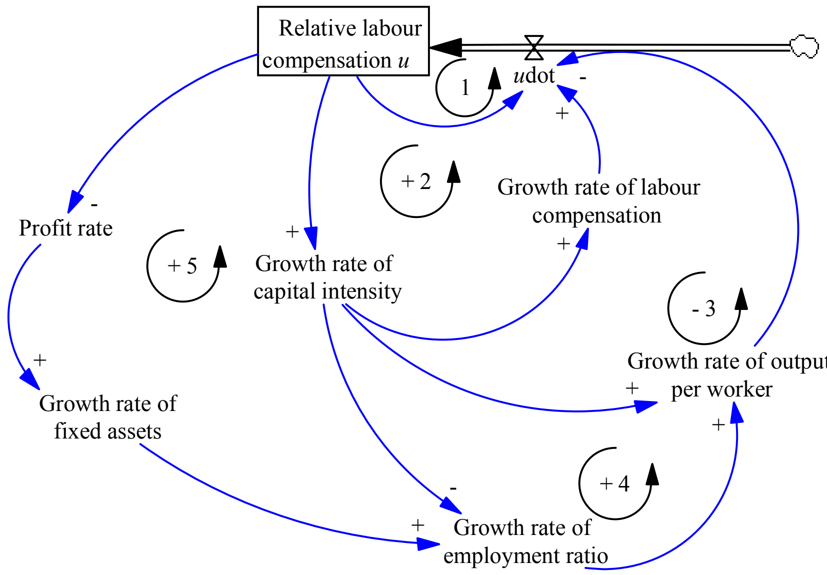

Figure 1. Five 1st order feedback loops of relative labour compensation (u) in HL-1

Figure 1 presents all 1st order feedback loops of relative labour compensation (three positive, one negative and one of changing polarity) leaving loops of higher orders aside. Consider two of them (numbered 2 and 3). In infinitesimal time interval, an increment of relative labour compensation fosters increases in the growth rate of capital intensity that, on the one hand, facilitates growth rate of labour

[3] The equity $u = 1$ is not compatible with capitalist production relations as the use value of labour power ceases to exist for capitalists when they get no surplus value at all. The equity $u = 0$ would exclude the specific premise of capitalist production relations, namely, market supply of labour force. Therefore $0 < u < 1$. The necessity of unemployment for capital accumulation requires $0 < v < 1$.

compensation and promotes the increment of relative labour compensation further; on the other hand, these increases in the growth rate of capital intensity uphold growth rate of output per worker that push relative labour compensation in the opposite direction. If parameters of the equations (4) and (8a) are such that $b < m_2$, the loop 3 dominates over loop 2.

A high relative labour compensation and high employment ratio promote mechanization (automation) that shapes the labour supply. The rate of change of capital intensity (K/L) in the equation (5) is a function of the relative labour compensation (u), difference between the real employment ratio (v) and some base magnitude (v_c).

Following reasoning stays behind a hypothetical partial law for the labour supply. Before reaching a critical magnitude, mechanisation (automation) pushes new demographic groups (children, women, aged, immigrants from less developed countries) into a labouring population (as far as qualification really or potentially satisfies technological requirements) thus chiefly accelerating the growth of supply of labour force. Afterwards mechanisation (automation) becomes mainly a decelerating factor for the growth of supply of labour force because a substantial part of working-age population does not possess adequate qualification for being hired or self-employed.

Accordingly, the equations (7a) and (7b) determine the growth rate of supply of labour force (N) as a non-linear continuous function of capital intensity alone. Capital intensity, in turn, is a product of capital-output ratio and output per worker $(K/L = sa)$, it is implicitly applied in the equation (14) below where $n = n(sa)$.

The growth rate of supply of labour force is monotonically increasing for $K/L \le K_c/L_c$, reaching an absolute maximum $n_{\max} = n_a + p_1$ at the point $K/L = K_c/L_c$; this rate is monotonically decreasing for $K/L \ge K_c/L_c$. Time evolution of supply of labour force (N) is typically S-shaped. A magnitude of the constant n_a is not determined a priory.

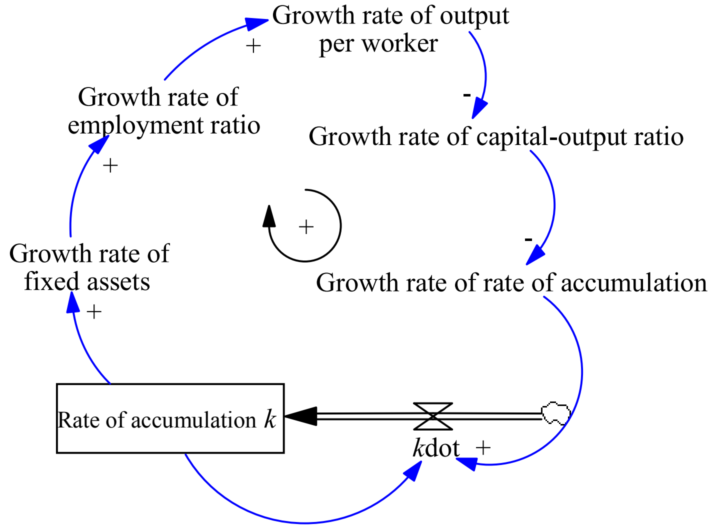

Figure 2. Endogenous rate of accumulation k reinforcing economy of scale in HL-1, HL-2 and HL-3

Consider the equation (9). Net national output produced (P) is the sum of labour compensation (wL) and profit (M). $\dot{K}$ denotes net formation of fixed capital; Q sums net export of goods and services E_1, net income receipts from the rest of the world E_2, net residential investment $\dot{R}$, net increment of inventories $\dot{I}$, final private C and public consumption expenditures G. In their turn, private consumption, net residential investment and public consumption consist of workers' and capitalists' parts (respectively, $C = C_w + C_c$, $\dot{R} = \dot{R}_w + \dot{R}_c$ and $G = G_w + G_c$). Notice that the equation (9) satisfies requirement that produced net domestic product ($P - E_2$) equals net domestic product finally used ($\dot{K} + Q - E_2$). These details help clarify the common boundary of the hypothetic laws (HLs) in section 2.1.

Net non-residential investment, being a priority fraction of surplus product (kM), covers net formation of fixed capital in the equation (10) abstracting from delays. The equation (11) defines a derivative control over rate of capital accumulation, (k), whereby its growth rate depends strongly negatively (for $c_1 < 0$) and non-linearly (for $1 > j_2 > 0$) on a growth rate of capital-output ratio. For the chosen new non-linear functional form (11) explicit analytical integration is not possible.

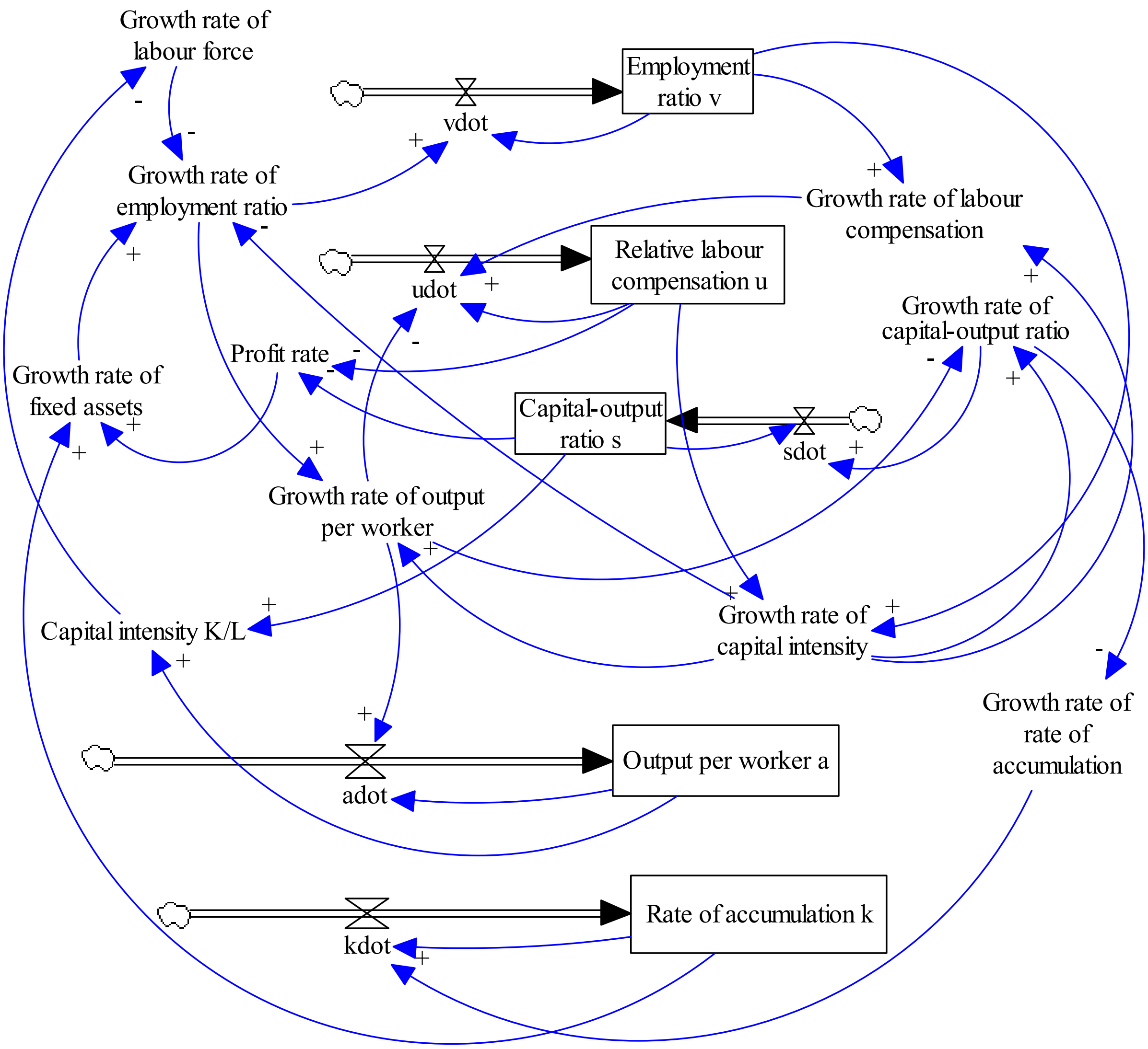

Figure 3a. A condensed causal loop diagram of HL-1 for $t < T_n$ (for the equation (7b))

Following considerations support logically a working hypothesis on a pro-cyclical nature of rate of accumulation. In the economic literature, output-capital ratio ($1/s$) represents typically a proxy of utilization of the productive capacity. The mathematical properties of function $\psi_2(\hat{s})$ in the equation (11) in respect to the argument $\hat{s}$ are the same as the above properties of function $\psi_1(\hat{v})$ in the equation (5) in respect to the argument $\hat{v}$, although measurement units of these functions and of related parameters c_1 and m_3 differ. The chosen functional form (11) allows not only modelling abrupt and vigorous changes of rate of capital accumulation (k) near turning points of industrial cycles but its long term declining trend as well. This variable substantially neutralises (for $c_1 < 0$) the secular tendency of profit rate to fall.

The variable k represents the capitalists' propensity to reinvest surplus value in an Eagly model (Eagly 1972). That model postulates that there exists some minimum acceptable profit rate which the capitalists regard as inadequate to justify further capital accumulation. When this minimum profit rate is reached, capitalists stop net capital accumulation; correspondingly, k can be either one or zero depending on relation of profit rate with its threshold. This presentation is not used in the present paper as too abstract and not empirically relevant. Other substantial drawback of that model is abstracting from relevant positive feedback loops arising mainly owing to the positive direct scale effect included in the extended technical progress function (4).

In an infinitesimal time interval, an increment in rate of accumulation facilitates growth of fixed capital and of employment ratio that, due to direct positive scale effect, fosters decline in capital-output ratio. The latter is, in turn, favourable for further extension of rate of accumulation (Figure 2). This positive feedback loop is an element of the greater structures of HL-1 (Figure 3a), of HL-2 (Figure 3b) and of HL-3 (Figure 3c).

Looking at the HLs boundary and beyond

A boundary of these HLs focused on the domestic economy (in a context of the world economy) is not shown explicitly yet. On this stage of the research, a short characteristic of a specific approach to external socio-economic relations may suffice.

The starting point is the equation (9) for relations of components of NNP produced with those of NNP used. Introduce a total of net export and net income receipts from the rest of the world (E):

$$E = E_1 + E_2, \qquad (9a)$$

where (for the US economy in the mean time) $E_1 < E < 0$ while $E_2 > 0$.

Assume that workers' labour compensation (before taxes!) equals their private and public consumption plus net residential investment

$$wL = uP = C_w + G_w + \dot{R}_w. \qquad (9b)$$

Then according to the equations (9), (9a) and (9b)

$$P = wL + M = \dot{K} + Q = \dot{K} + C_w + C_c + \dot{R}_w + \dot{R}_c + \dot{I} + G_w + G_c + E_1 + E_2. \qquad (9c)$$

Re-grouping of terms in the equation (9c) leads to

$$\dot{K} + C_c + G_c + \dot{R}_c + \dot{I} = M - E_1 - E_2. \qquad (9d)$$

In the equation (9d), a sum (on the left) of domestic non-residential investment, capitalists' private and public consumption, their net residential investment, net increment of inventories equals profit (before taxes!) plus net import ($-E_1$) and net income payments to the rest of the world ($-E_2$). The uses (on the left) *in toto* exceed surplus product ($M - E_2$) domestically produced by quantity of net import ($-E_1$), whereas domestic surplus product is typically much higher than net increment of fixed capital ($M - E_2 >> \dot{K}$). The net foreign expenses ($-E > 0$) are covered by net foreign borrowing (not explicit in HLs).

Notice that in a special abstract (limit) case without net accumulation of fixed capital ($k = 0$) and without a change of inventory ($\dot{I} = 0$), the equation (9d) is simplified to $C_c + G_c + \dot{R}_c = M - E_1 - E_2$. A sum of capitalists' private and public (including military) consumption, net residential investment exceeds domestic profit by net export. The net foreign expenses are covered by net foreign borrowing again.

Although in our time the US income receipts from the rest of the world exceed income payments to the rest of the world, current account is negative due (arithmetically!) to, first, negative net export and, second, positive net current taxes and transfer payments to the rest of the world (given in foreign transaction current account). Negative current account less minor net capital account transaction equals net negative lending (given in foreign transaction capital account). Lavishness and military expenditures may foster accumulation of foreign debt especially during the protracted wars.

According to domestic capital account, a sum of positive net investment, minor net capital account transactions and negative net lending equals a sum of negative net national (private and government) saving and statistical discrepancy. This is a concretization for the USA of well-known identity: net domestic investment (including net change of inventories) ≡ net national saving + net foreign borrowing + statistical discrepancy.

A wide-spread fallacy is a superfluous interpretation of this identity (tautology) as a principal causal relationship: "The supply of [fixed] capital is determined by national saving and capital flows from abroad" (CBO Memorandum: 5). Its implicit, yet absurd, conclusion is that these capital flows became the main direct factor of US net fixed capital formation in 2003–2008 (when foreign borrowing was higher than net national saving, according to Economic Report of the President 2010: Table B-32).

An initial unproven belief of the Council of Economic Advisers (CEA) follows from an equivalent identity (current account reduced to national saving minus investment plus some measurement error): "U.S. saving was very low [in relation to investment], which led to substantial borrowing from the rest of the world" (Economic Report of the President 2010: 108).

CEA demolishes this unproven belief without notice by admitting later: "This accounting definition provides a description but not an explanation of the drivers of the current account. One important driver is the business cycle" (ibid, 131). A critical mind will appreciate this refinement as a flash that even lightens an interesting empirical regularity – apparent positive correlation between the US current account (as per cent of GDP) and rate of unemployment for 1980–2009 (ibid, 130). Thereby CEA vaguely characterises unemployment rate as driven by the business cycle without offering a plausible model.

It is easy to see that the above non-accurate interpretation of accounting identity disregards the fundamental laws of capital motion and especially laws of motion of fictitious capital explored by K. Marx in the three volumes of "Capital" (the second and third published by F. Engels after his friend and the main author of these volumes passed away).[4] This paper explores and validates HLs that generate circular trends and industrial cycles and, particularly, fluctuations in the rate of unemployment being in congruence with Marx' theory and mostly supported by statistical data. These HLs imply that, first, net fixed capital formation is determined in the US economy by mostly domestic and partially foreign surplus labour embodied in surplus product and, second, that surplus labour and surplus product, in their turn, depend on net domestic fixed capital formation.[5]

Foreign states and private investors, often seeking out safety, accumulate fictitious capital as claims for a part of surplus value (flow) created by American labourers. Net additional claims are reflected as a financial account excess (flow) that equals a current account deficit (flow) with its sign reversed if capital account and statistical discrepancy are left aside. Negative net lending (positive net borrowing) as a flow facilitates foreign indebtedness (a stock) and thus it promotes income payments to the rest of the world (a flow); in turn, net increment of foreign indebtedness (a flow) lessens net US-owned assets abroad (a stock) and worsens the US net international investment position (a stock) although assets revaluation may have an opposite effect on this position.[6]

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[4] "The formation of a fictitious capital is called capitalisation. Every periodic income is capitalised by calculating it on the basis of the average rate of interest, as an income which would be realised by a capital loaned at this rate of interest" (http://www.marxists.org/archive/marx/works/1894-c3/ch29.htm).

[5] In 2008, net income receipts from the rest of the world amounted to 141.9 billion dollars or 1.1 per cent of NNP and 3.5 per cent of surplus product (Economic Report of the President 2010: Table B-26, author's calculations).

[6] International transaction accounts (ITAs) and international investment position accounts (IIPAs) reflect these processes statistically (BEA 2010). Changes attributable to valuation adjustments in IIPAs

2.2. An Intensive Deterministic Form of HL-1

An intensive deterministic form of HL-1, derived from the equations (1)–(7), (8a), (9) – (11), consists of five non-linear ordinary differential equations (11), (12) – (14) and (15a): if $t < T_n$,

$$\dot{a} = \{m_1 + m_2\,[n_1 + n_2 u + n_3(v - v_c)] + m_3\,\psi_1(\hat{v})\}a, \tag{12}$$

$$\dot{s} = \{-m_1 + (1 - m_2)[n_1 + n_2 u + n_3(v - v_c)] - m_3\,\psi_1(\hat{v})\}s, \tag{13}$$

$$\dot{v} = \left[k\frac{1-u}{s} - n_1 - n_2 u - n_3(v - v_c) - n\right]v\,, \tag{14}$$

$$\dot{u} = \{-g + rv - m_1 + (b - m_2)[n_1 + n_2 u + n_3(v - v_c)] - m_3\,\psi_1(\hat{v})\}u. \tag{15a}$$

Analysing HL-1 with a help of the Lie derivative

Formally, properties of a system of ordinary non-linear differential equations can be examined with the help of the Lie derivative or divergence defined in the present case for the vector-function $f\,(a, k, s, v, u)$ as

$$\mathrm{div}(f) = \frac{\partial \dot{a}}{\partial a} + \frac{\partial \dot{k}}{\partial k} + \frac{\partial \dot{s}}{\partial s} + \frac{\partial \dot{v}}{\partial v} + \frac{\partial \dot{u}}{\partial u}\,. \tag{16}$$

For the HL-1 intensive form (11) – (15a), where $\hat{a} + \hat{s} + \hat{v} = \frac{k(1-u)}{s} - n$, the Lie derivative is calculated as follows: if $t < T_n$,

$$\mathrm{div}(f) = \frac{k(1-u)}{s} - n + \hat{u} - n_3 v + (b - m_2)n_2 u$$
$$+ m_3\psi_1{}'(\hat{v})\left(\frac{k}{s} + n_2 u\right) + c_1\psi_2(\hat{s}) - c_1\psi_2{}'(\hat{s})m_3\psi_1{}'(\hat{v})\frac{k(1-u)}{s}\,. \tag{17a}$$

In vicinity of critical (singular) points, including a non-trivial stationary state, where $\psi_1{}'(\hat{v}) \to +\infty$ for $\hat{v} \to 0$ and $\psi_2{}'(\hat{s}) \to +\infty$ for $\hat{s} \to 0$, the Lie derivative (17a) moves for $k > 0$ to positive infinity since the compound element $m_3\left[\frac{k}{s} + n_2 u - c_1\psi_2{}'(\hat{s})\frac{k(1-u)}{s}\right]\psi_1{}'(\hat{v})$ goes to positive infinity as $c_1 < 0$, $m_3 > 0$ and $\frac{k(1-u)}{s} > 0$. So induced technical progress, economy of scale and procyclical character of rate of accumulation are at least locally destabilising in vicinity of such critical points in the initial model.

A non-trivial stationary state in HL-1

For finding a non-trivial stationary state of a system of ordinary differential equations, it is necessary to equate each of the expressions on the right to zero. As $\dot{a} = 0$ is not true for a non-trivial stationary state, this system does not possess a non-trivial stationary state. A slightly changed system has it if equations (7b′) and (7c) substitute the equation (7b)

$$n = n_a + p_1 e_2^{-M_2(K/L - K_c/L_c)^{i2}} \quad \text{for } K_m/L_m > K/L \ge K_c/L_c\,, \tag{7b′}$$

$$n = n_a \quad \text{for } K/L \ge K_m/L_m \tag{7c}$$

(the partial derivatives $\partial n/\partial s = 0$ and $\partial n/\partial a = 0$ for the latter equation).

---

are connected with changes of stock market and real estate prices, changes in exchange rates, etc. ITAs abstract from them.

This redefinition of the partial dynamic law of labour supply enables to have solutions with a steadily growing $(n_a > 0)$, declining $(n_a < 0)$ or constant labour force $(n_a = 0)$. Defining n by the equations (7a), (7b′) and (7c) allows also dropping equation (12) from the system thus reducing the number of remaining differential equations in it.

The lower order system of the equations (11), (13) – (15a) has a continuum of non-trivial stationary states defined independently of the parameters c_1 and m_3. Whereas stationary employment ratio v_a and stationary relative labour compensation u_a are determined distinctively for all non-trivial stationary states, stationary capital-output ratio s_a and stationary rate of accumulation k_a are not given unambiguously, these two are connected by a linear relationship as the definition (18) shows.

Define a particular non-trivial stationary state for a stationary rate of accumulation $1 \geq k_a = k_0 \geq 0$

$$E_a = (k_a, s_a, v_a, u_a), \tag{18}$$

where $s_a = k_0 \dfrac{1-u_a}{i}$, $v_a = \dfrac{g+(1-b)(i-n_a)}{r}$, $u_a = \dfrac{i-n_a-n_1-n_3(v_a-v_c)}{n_2}$. The stationary growth rate of real labour compensation, output per worker and capital intensity is $\hat{w}_a = \hat{a}_a = K_a \hat{/} L_a = m_1/(1-m_2)$; the stationary growth rate of net fixed capital and net output is $\hat{K}_a = \hat{P}_a = i = n_a + m_1/(1-m_2)$. At this stationary state, the growth rate of the labour value of net fixed capital, employment and labour force is $K_a \hat{/} a_a = \hat{L}_a = n_a$. The stationary profit rate is $(1-u_a)/s_a$. It could be easily shown, that exogenous infinitesimal increases in a stationary growth rate of output per worker raise a stationary employment ratio but diminish stationary relative labour compensation.

Whereas the social factors do influence on the long-run stationary ratio of profit to labour compensation (rate of surplus value) in HL-1, in the neoclassical case the profit-labour compensation ratio is entirely determined by parameters of a production function quite independently of other substantial socio-economic parameters.

The system (11), (13)–(15a) cannot be linearised at a stationary state E_a. This stationary state E_a is not asymptotically stable as explained in the above remarks on the Lie derivative. Computer simulations (skipped) show that it, being locally unstable in the sense of Liapunov too, repels trajectories to an attracting limit cycle (owing to singularity of functions $\psi_1(\hat{v})$ and $\psi_2(\hat{s})$ for zero arguments) with a period of about 11 years (for $v_c \approx 0.925$) that does not result from the Andronov – Hopf bifurcation.

The existence of limit cycle is not yet proven analytically. Still multiple computer simulations with different integration techniques demonstrate that transient to very close vicinity of limit cycle endures centuries and millenniums. Although full transition to limit cycle and limit cycle itself cannot be simulated precisely, simulations depict them with sufficient accuracy. Different evidences support this conclusion. First, adjacent cyclical motions are very similar to each other. Second, there is proximity of average magnitudes of variables v and u to their stationary magnitudes for limit cycles approximations in simulations (Tables 5a and 5b).[7]

2.3. An Extensive and Intensive Deterministic Forms of HL-2

Reasons of the first restructuring of HL-1 into HL-2 are explained in Section 3.2. If $t \geq T_n = 1983$, a new extensive deterministic model involves the equations (1)–(7), (9)–(11) and equation (8b) for the growth rate of labour compensation that substitutes equation (8a) and relates to a threshold employment ratio (constant) V:

[7] Runge – Kutta integration with automatically adjusted step size is used (RK4 auto) in scenarios I–III.

$$\hat{w} = \hat{a} - d, \tag{8b}$$

where an auxiliary discrete variable $d = d_1 > 0$ if $0 < v < V < 1$, or $d = d_2 < 0$ if $1 > v \geq V$.

The left and centre panels of Figure 4 confront new and former immediate causes of the growth rate of labour compensation before and after the first restructuring. The right panel is characterised later.

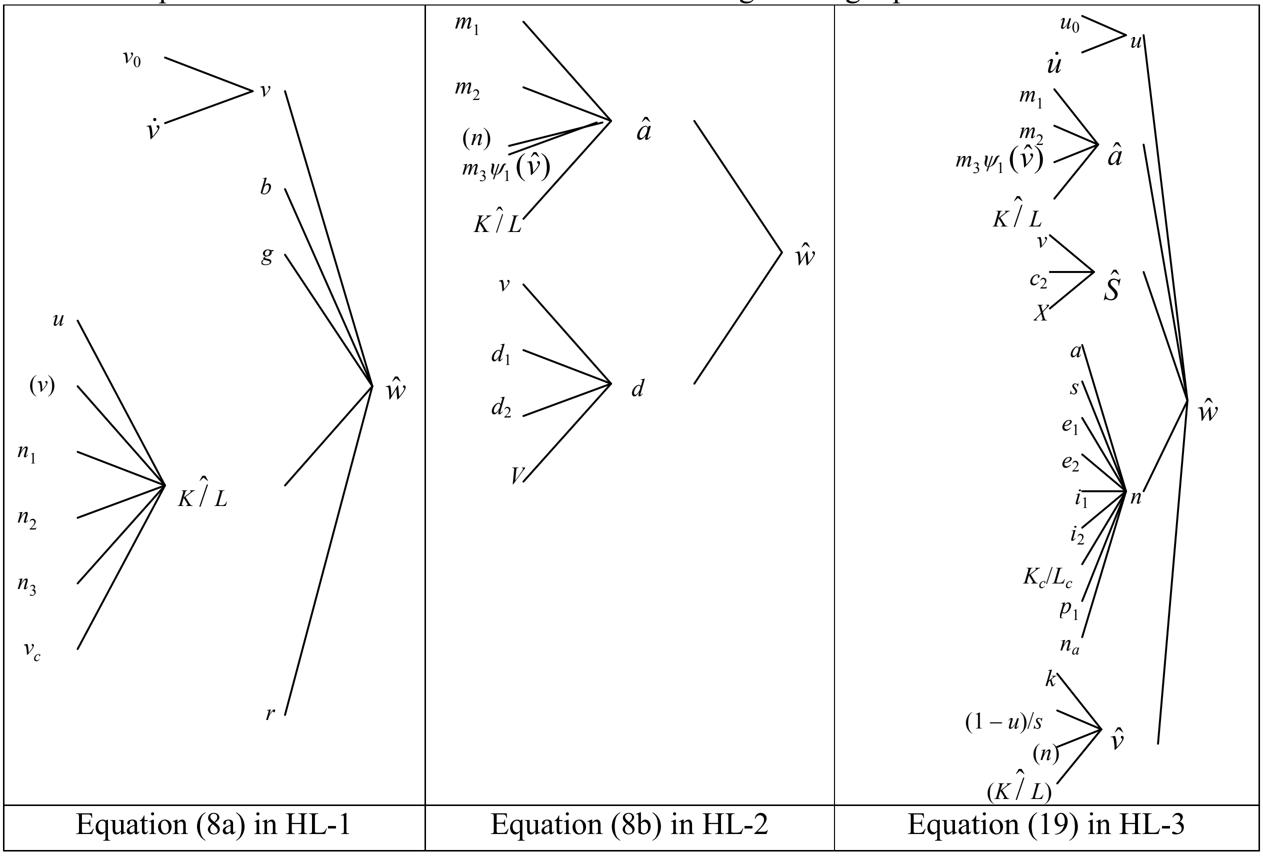

Figure 4. Causes trees of depth 2 for growth rate of labour compensation $\hat{w}$ for HL-1, HL-2 and HL-3

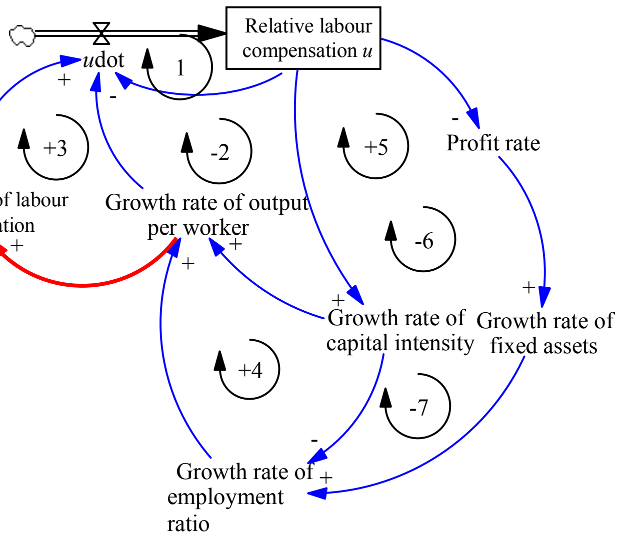

Figure 5. The all 1st order feedback loops of relative labour compensation u in HL-2

Figure 5, like Figure 1, presents again all 1st order feedback loops of relative labour compensation (three positive, three negative and one of changing polarity) leaving loops of higher orders aside. Consider two of them (numbered 2 and 3). In both, in an infinitesimal time interval, an increment of relative labour compensation promotes increases in the growth rate of capital intensity that facilitates growth rate of output per worker, this either diminish the initial increment of relative labour compensation (loop 2) or facilitates growth rate of labour compensation that is favourable for further increment of relative labour compensation (loop 3). If $d > 0$ in the equation (8b), the loop 2 dominates over loop 3, and vice versa (if $d < 0$). More detailed Figure 3b displays the encompassing HL-2 structure.

An intensive deterministic form of HL-2, derived from the equations (1)–(7), (8b), (9) – (11) that involve its extensive deterministic form, includes five non-linear ordinary differential equations (11), (12) – (14) and (15b). The latter substitutes equation (15a): if $t \geq T_n$

$$\dot{u} = -du, \qquad (15b)$$

where $d = d_1 > 0$ if $v < V$, or $d = d_2 < 0$ if $v \geq V$.

The trajectory of $u(t)$ consists of growing and declining exponential parts connected in piece-wise manner. Local maximums and minimums of u correspond to occurrences of $v = V$ when the variable d changes abruptly.

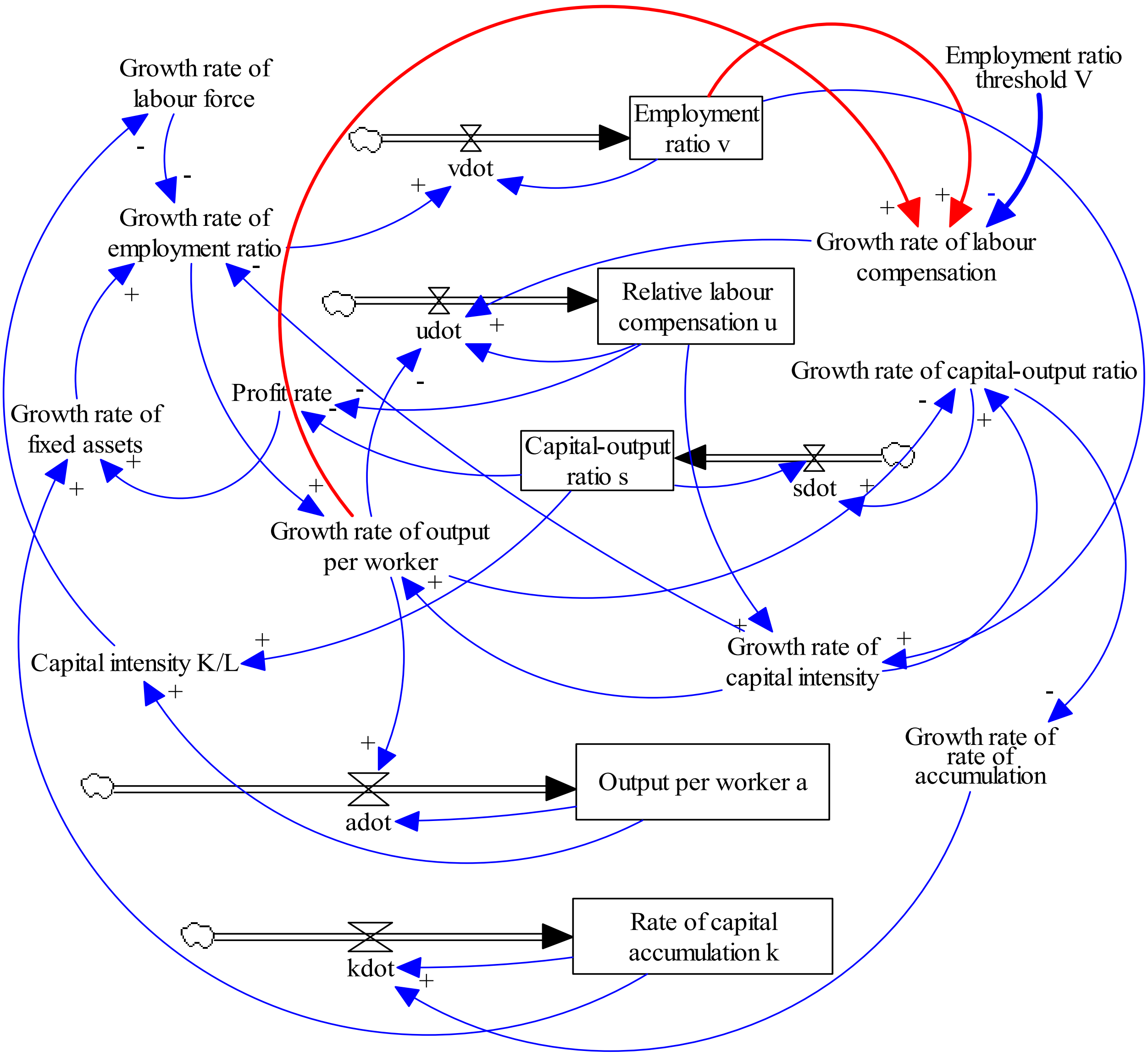

Figure 3b. A condensed causal loop diagram of HL-2 (for the equation (7b))

Analysing HL-2 with a help of the Lie derivative

For the HL-2 intensive form (11), (12) – (14) and (15b), where $\hat{a}+\hat{s}+\hat{v}=\frac{k(1-u)}{s}-n$, the Lie derivative is calculated as follows:

$$\mathrm{div}(f) = \frac{k(1-u)}{s} - n \;-n_3 v + c_1\psi_2(\hat{s}) + m_3\psi_1'(\hat{v})\frac{k(1-u)}{s}[1-c_1\psi_2'(\hat{s})] - d. \quad (17b)$$

In vicinity of critical (singular) points where $\psi_1'(\hat{v}) \to +\infty$ for $\hat{v} \to 0$ and $\psi_2'(\hat{s}) \to +\infty$ for $\hat{s} \to 0$, the Lie derivative (17b) moves for $k > 0$ to positive infinity since the compound element $m_3\psi_1'(\hat{v})\frac{k(1-u)}{s}[1-c_1\psi_2'(\hat{s})]$ goes to positive infinity as $c_1 < 0$, $m_3 > 0$ and $\frac{k(1-u)}{s} > 0$. So induced technical progress, economy of scale and pro-cyclical character of profit investment share are at least locally destabilising in vicinity of such critical points in HL-2.

A non-trivial stationary state with positive relative labour compensation in HL-2 does not exist for $d \neq 0$ in the equation (8b). The existence of limit cycle is not yet proven analytically. Still multiple computer simulations with different integration techniques demonstrate that transient to very close vicinity of limit cycle endures centuries and millenniums. Although full transition to limit cycle and limit cycle itself cannot be simulated precisely, simulations depict them with sufficient accuracy. Tables 5a and 5b support this conclusion.

3. A Historical Fit of HL-1 and HL-2 for the US Economy in 1969–2008

3.1. Probabilistic Forms of HL-1 and HL-2

For estimating probable states of the economy and for identifying unobserved parameters in the basal period the deterministic models HL-1 and HL-2 have been transformed in two respective stochastic models, taking into account measurement errors and an impact of factors neglected in the model assumptions.[8] This makes implicit allowances for short-term economic fluctuations by specification of the random components. The latter models include state equations and measurement equations for discrete moments of time

$$\mathbf{x}(\tau) = \mathbf{f}_\iota[\mathbf{x}(\tau-1)] + \mathbf{w}(\tau),$$

$$\mathbf{z}(\tau) = \mathbf{H}\mathbf{x}(\tau) + \mathbf{v}(\tau),$$

where τ = 1970, 1971,…, 2008 is an index of data samples, $\mathbf{x}(1969)$ – a vector of an initial state of the system, $\mathbf{w}(\tau)$ – a vector of equations errors (driving noise), $\mathbf{v}(\tau)$ – a vector of measurement errors. The deterministic parts $\mathbf{x}(\tau) = \mathbf{f}_\iota[\mathbf{x}(\tau-1)]$, $\iota = 1, 2$ corresponds to the systems (11) – (15a) for $\iota = 1$ and $1969 \leq t < T_n = 1983$, (11) – (15b) for $\iota = 2$ and $2008 \geq t \geq T_n$. The symbol **H** is for a square matrix. The residuals are not due entirely, or largely, to pure random influences. On the contrary, these residuals contain highly systematic, non-random components.

A simplified version of an extended Kalman filtering (EKF), realised in the Vensim software developed by Ventana Systems, Inc., has been applied. This software enables to estimate the unobservable components of the both systems by a procedure of maximum likelihood.

[8] It is not possible to check whether the given deterministic model is able to replicate behaviour and create understanding of the observable economic behaviour without estimating parameters that usually requires construction of a stochastic model. A direct measurement of parameters' values, rarely achievable in macroeconomic modelling, is not for this particular study.

The value of one parameter was chosen a priory: $n_a = 0$. An application of the EKF to the US macroeconomic data for the basal period 1969–2008 has identified the other unobservable components of the above probabilistic forms of HL-1 and of HL-2: $b \approx 0.316$, $c_1 = -0.4$, $e_1 \approx 2.5$, $e_2 \approx 279.4$, $g \approx 0.042$, $i_1 \approx 0.2$, $i_2 \approx 0.520$, $j_1 \approx 0.476$, $j_2 = 0.05$, $K_c / L_c \approx 0.096$, $m_1 \approx 0.006$, $m_2 \approx 0.5$, $m_3 \approx 0.015$, $n_1 \approx -0.24$, $n_2 \approx 0.346$, $n_3 \approx 0.568$, $p_1 \approx 0.033$, $r \approx 0.059$, $v_c \approx 0.925$, $i \approx 0.011$; $1979 <= T_n = 1983 <= 1987$, $0.95 <= V = 0.955 <= 0.96$, $0.001 <= d_1 = 0.002 <= 0.002$, $-0.0082 <= d_2 = -0.003 <= -0.002$.

Parameters b, g and r from the comprehensive Phillips equation (8a) are not applicable for HL-2. In turn, parameters d_1, and d_2 from new partial dynamic law (8b) are not applicable for HL-1.

Table 2. Initial and average observable magnitudes for US economic development in 1969–2008

	Rate of accumulation (k)	Capital-output ratio (s)	Employment ratio (v)	Relative labour compensation (u)	Profit rate ($(1 - u)/s$)
Initial 1969	0.241	1.788	0.965	0.710	0.162
Average 1969–1982	0.213	2.018	0.936	0.714	0.142
Average 1983–2008	0.152	1.890	0.942	0.699	0.160

Simulation runs have used the observed magnitudes for the initial year (1969) posted in Table 2 (additionally $a_0 \approx 0.04521$ millions 2005 dollars per person a year, $N_0 \approx 80705.1$ thousands persons, $P_0 \approx 3520.7$ billions 2005 dollars). They calculated the most probable (still sub-optimal) magnitudes of state variables in the subsequent years.

3.2. Behaviour reproduction tests of HL-1 and HL-2

HL-1 and HL-2 probabilistic forms are to pass behaviour reproduction tests. In particular, the Theil inequality statistics (Table 3) are used for estimating historical fit (Theil 1966).

Rather small root-mean-square errors as the percentage of the means (RMSE as percentage of the mean) and prevailing non-systematic errors of incomplete co-variation (U^C) over bias (U^M) and over difference in variation (U^S) show that these probabilistic forms track observations of the major variables in the basal period agreeably (Table 3). Panels 1–6 on Figure 6, demonstrating a certain likeness between simulated and realised (observed) magnitudes in the basal period 1969–2008, support this conclusion.

Table 3. Decomposition of errors of the retrospective forecast for 1969–2008

Variable	$\sqrt{MSE}$ (units)	U^M	U^S	U^C	$\frac{\sqrt{MSE}}{mean}$,per cent
a	4.4E-05	0.002	0.091	0.906	0.070
s	0.004	0.003	0.061	0.936	0.21
v	0.002	0.194	0.195	0.611	0.22
u	0.009	0.167	0.034	0.799	1.282
k	0.024	0.202	0.065	0.732	13.96
$(1 - u)/s$	0.005	0.144	0.108	0.748	3.06

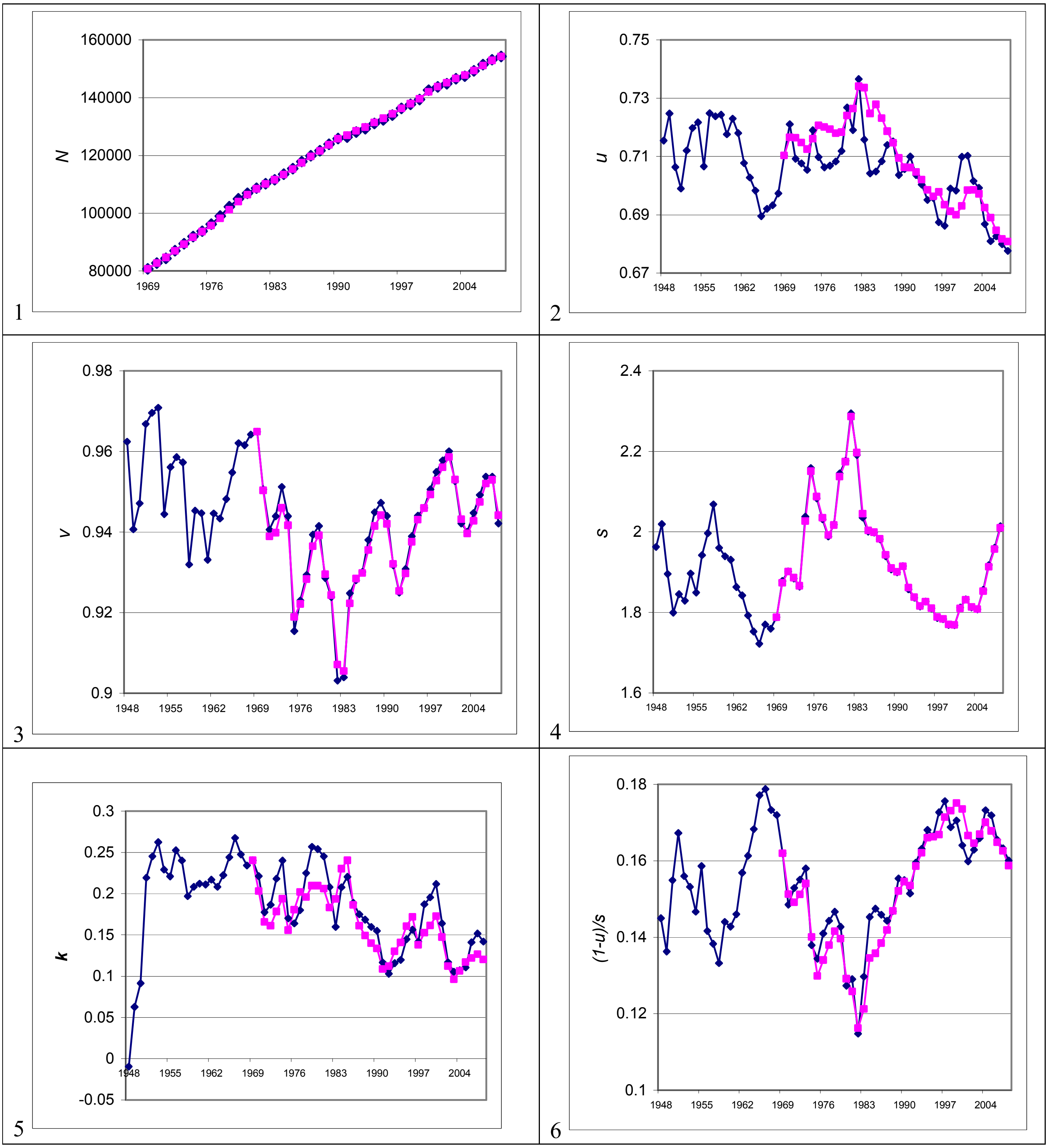

Figure 6. The observed (diamond) 1948–2008 and simulated (square) 1969–2008 magnitudes: 1 – civil labour force N (thousands of persons), 2 – relative labour compensation u, 3 – employment ratio v, 4 – capital-output ratio s, 5 – rate of accumulation k, 6 – profit rate $(1-u)/s$

Two highest magnitudes of the employment ratio, v, were observed and simulated in 1969 (best) and 2000 (second best), whereas its nadir was observed in 1982 and simulated in 1983 (Figure 6). Two highest magnitudes of the profit rate, $(1 - u)/s$, were observed in 1966 (best) and 1997 (second best), a trough – in 1982; the simulated highest magnitudes occurred in 1969 and 1999, simulated lowest one – in 1982 (Figure 6). The uncovered tendency of the profit rate to fall is unfavourable for the employment ratio in the long-term.

In the finished industrial cycle, the observed and simulated profit rate started to fall in 2005 due to increases in capital-output ratio despite diminishing relative labour compensation when relative over-accumulation of capital manifested itself. The observed and simulated employment ratio started to decline in 2007–2008.

The surmised restructuring of hypothetical laws of capital accumulation in basal period (transformation of HL-1 into HL-2 in 1983 – roughly the borderline for the new so-called neoliberal era) has found an additional support in a computer supported mental experiment. Based on initial HL-1, simulated data have been produced with a help of Kalman filtering with observations up to 1982.

Figure 7 presents unsatisfactory for capital actual, simulated and anticipated dynamics of profit rate and profit that required restructuring of this law. It was transformed in HL-2 that, probably, governed capital accumulation after 1982. A swollen unemployment of 1982–1983 facilitated this pro-capital transformation. As a recent paper demonstrates, the neoliberal era produced three relatively long expansions: 1982–1990, 1991–2000, and 2001–07 (Kotz 2009).

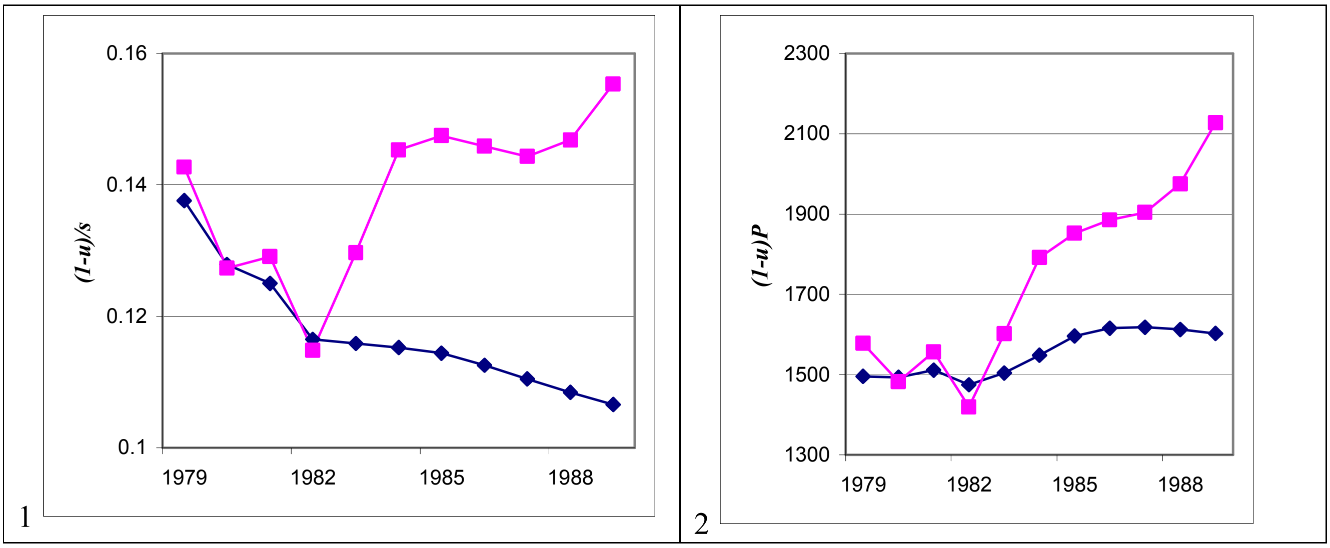

Figure 7. Profit rate (panel 1) and profit (panel 2): simulated (diamond), observed (square), 1979–1989

4. Supposing Control Law of Capital Accumulation for the Modern US Economy

Feed-forward control, as known, changes variables according to expected future states of the economy. It has been assumed that the decision-makers (the state officials, owners of capital, managers and, less likely, trade union leaders) set a desirable growth rate of total surplus value depending on a difference between a target (X) and current (v) employment ratios. An indicated growth rate of surplus value is

$$\hat{S} = c_2(X - v), \qquad (19)$$

where $v < X < V$ is typical for recessions and depressions. When $c_2 < 0$, surplus value vanishes and v sharply falls. The case $c_2 = 0$ would represent a tendency to equity in income distribution not observed in the studied historical period. So it is assumed realistically that the parameter c_2 is positive.

A new equation for relative labour compensation follows from the equation (19)

$$\dot{u} = (\hat{L} - \hat{S})(1-u) = [\hat{v} + n + c_2(v - X)](1-u). \quad (20)$$

A new equation for a growth rate of labour compensation follows from the equations (3) and (20):

$$\hat{w} = \hat{a} + \hat{u} = \hat{a} + [\hat{v} + n + c_2(v - X)]\frac{1-u}{u}. \quad (21)$$

As both $\frac{\partial \hat{w}}{\partial \hat{v}} > 0$ and $\frac{\partial \hat{w}}{\partial n} > 0$, and declining growth rates of employment ratio and of labour supply are detrimental for growth rate of real labour compensation if the all other conditions remain the same. The growth rate of real labour compensation continues to depend positively on the employment ratio (v).

It is easy to notice that the equation (19) is structurally different from the equations (8a) and (8b). The structure of HL-3 is different from the HL-1 and structure HL-2 only in this part (cf. Figures 3a, 3b and 3c) although the other parts are also affected.

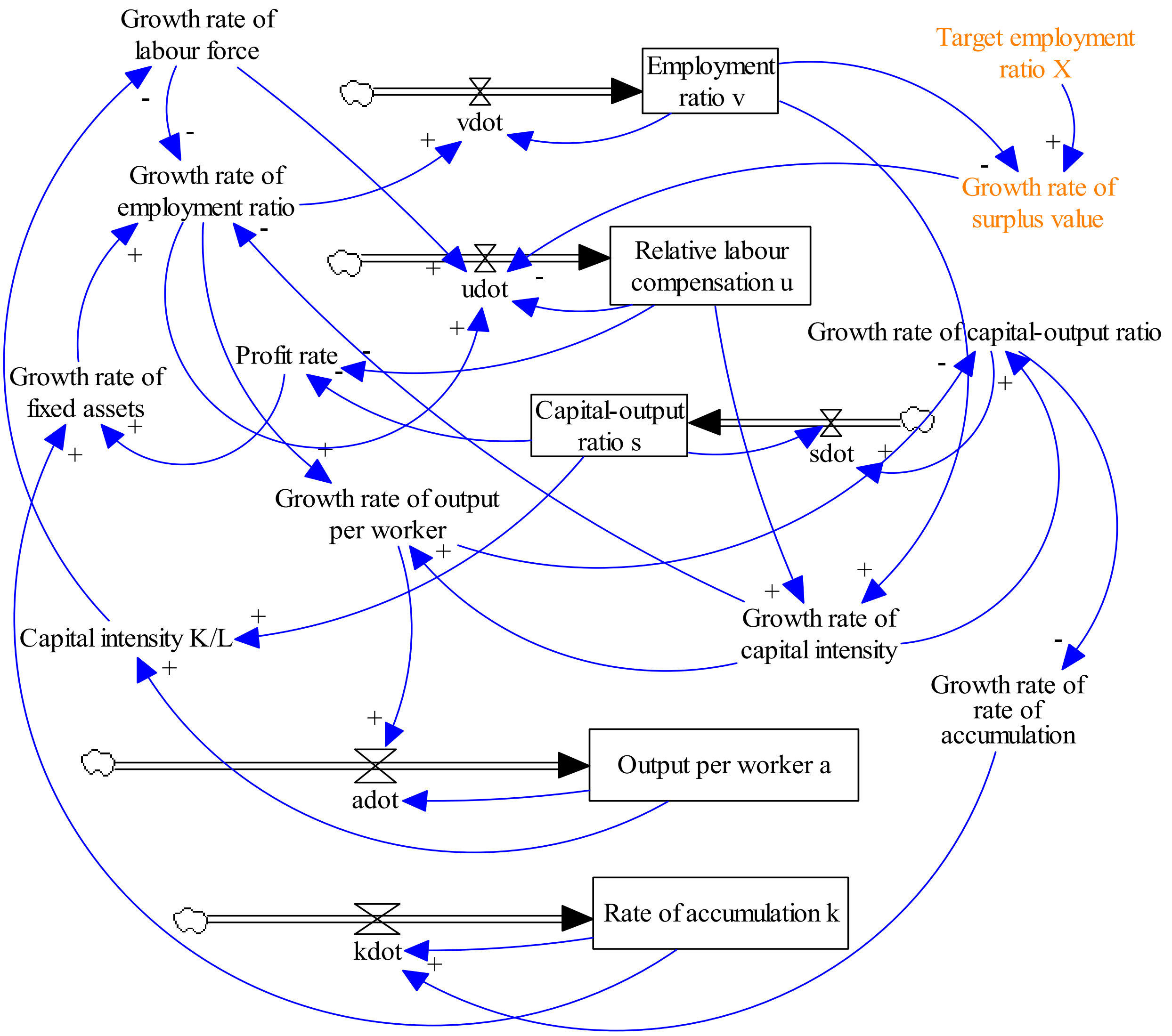

Figure 3c. A condensed causal loop diagram of HL-3 (for the equation (7b))

The impact of the growth rate of output per worker $(\hat{a})$ on $\hat{w}$ is unmitigated as in HL-2 (Figure 4 on the right). Besides that, the new compound non-linear term $[\hat{v}+n+c_2(v-X)]\frac{1-u}{u}$ substitutes the term d that is re-switching depending only on v. Now two 1st order feedback loops of relative labour compensation are negative and the shortest one has changing polarity. The restructuring of HL-2 into HL-3 eliminates all 1st order positive feedback loops of relative labour compensation altogether.

A comparison of the equation (19) with the equation (8a) reveals important differences too (cf. right and left columns on Figure 4). The former constant g has been transformed into a product of the two new constants (c_2, X) and of rate of surplus value $\left(\frac{1-u}{u}\right)$; non-linear positive dependence of $\hat{w}$ on the rate of change of the employment ratio ($\hat{v}$) multiplied by rate of surplus value has substituted its former positive linear dependence on the rate of change of capital intensity ($K\hat{/}L$); the former constant r has been transformed into a product of the new constant (c_2) and of rate of surplus value.

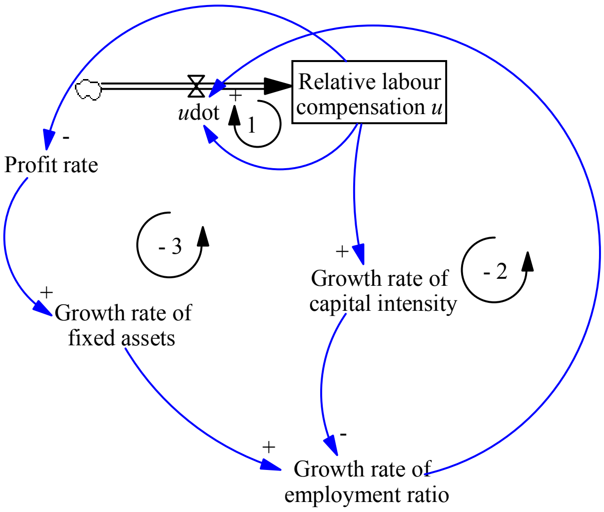

Figure 8. The all 1st order feedback loops of relative labour compensation u in HL-3

Analysing HL-3 with a help of the Lie derivative

For HL-3 defined by the equations (11) – (14) and (20), the Lie derivative is given by:

$$\mathrm{div}(f) = \frac{\partial \dot{a}}{\partial a}+\frac{\partial \dot{k}}{\partial k}+\frac{\partial \dot{s}}{\partial s}+\frac{\partial \dot{v}}{\partial v}+\frac{\partial \dot{u}}{\partial u}$$

$$= -n - n_3 v + m_3\psi'(\hat{v})\frac{k(1-u)}{s} - n_2(1-u) - \frac{\dot{u}}{1-u} + c_1\psi_2(\hat{s}) - c_1\psi_2'(\hat{s})m_3\psi_1'(\hat{v})\frac{k(1-u)}{s}. \quad (17c)$$

In vicinity of critical (singular) points, including a non-trivial stationary state, where $\hat{v}\to 0$ and/or $\hat{s}\to 0$, $\psi_1'(\hat{v})\to +\infty$ and $\psi_2'(\hat{s})\to +\infty$, respectively. Then Lie derivative (17c) moves to positive infinity for $k > 0$ since $m_3\frac{k(1-u)}{s} > 0$ and $c_1 < 0$. Thus in this case economy of scale and pro-cyclical character of rate of accumulation are at least locally destabilising in vicinity of such critical points.

A non-trivial stationary state

The initial equations (11) – (14) and the new equation (20) that substitutes the initial equation (15a) or (15b) embrace the intensive deterministic form of HL-3. If the equations (7a), (7b′) and (7c) for the growth rate of labour force are applied again, then the lower order system of the equations (11), (13),

(14) and (20) has a continuum of non-trivial stationary states defined independently of the parameters c_1 and m_3. Whereas v_b and u_b are determined distinctively, s_b and k_b are connected to each other by a linear relationship.

Define a particular non-trivial stationary state for a stationary rate of accumulation $1 \geq k_b = k_0 \geq 0$

$$E_b = (k_b, s_b, v_b, u_b), \tag{22}$$

where $s_b = k_0 \dfrac{1-u_b}{i}$, $v_b = X - \dfrac{n_a}{c_2}$, $u_b = \dfrac{i - n_a - n_1 - n_3(v_b - v_c)}{n_2}$, $i = \dfrac{m_1}{1-m_2} + n_a$. At this stationary state, the rates of change for the value of net fixed capital, employment, labour force and surplus value are the same and equal $\hat{K_b / a_b} = \hat{L}_b = \hat{S}_b = n_a$. The stationary profit rate is $(1 - u_b)/s_b = i/k_b$. Table 5a contains the stationary magnitudes of the distinctively determined state variables (their listing does not include k_b and s_b).

For the stationary state E_b (22) for the identified parameters magnitudes, the following properties are satisfied

$$\frac{\partial u_b}{\partial v_c} = \frac{n_3}{n_2} > 0, \tag{23a}$$

$$\frac{\partial u_b}{\partial n_3} = -\frac{v_b - v_c}{n_2} < 0, \tag{23b}$$

$$\frac{\partial v_b}{\partial v_c} = 0, \tag{24a}$$

$$\frac{\partial v_b}{\partial n_3} = 0. \tag{24b}$$

The probable plummeting of the magnitude of the parameter v_c in 2008 brings about the drop of the stationary magnitudes of the relative labour compensation in profit enhancing scenario III. It could be shown similarly based on the definition (22) that confronted with exogenous increases in a stationary output per worker growth rate the stationary employment ratio remains the same whereas the stationary relative labour compensation increases. The second consequence weakens the capital interest in the HL-3 practical application.

A system (11), (13), (14) and (20) cannot be linearised at the stationary state E_b. This stationary state E_b, as E_a of HL-1, is not asymptotically stable as explained in the above comments on the Lie derivative. Computer simulations (skipped) show that it, being locally unstable in the sense of Liapunov too, repels trajectories to an attracting limit cycle with a period of about 9 years (for $v_c \approx 0.925$). This limit cycle does not result from the Andronov – Hopf bifurcation, it arises due to singularity of functions $\psi_1(\hat{v})$ and $\psi_2(\hat{s})$ at zero, as in HL-1. The periods of the limit cycles are not necessarily the same.

5. Prospective scenarios of US Economic Development

The scenarios I, II and III are based on the unaltered HL-2, parametrically altered HL-2 and HL-3, respectively. Parameters values are given above (Section 3.1). Table 4 contains magnitudes of main variables in three different scenarios of US economic development for the scenarios' initial year 2008.

In scenario I related to unaltered HL-2 the magnitude identified by EKF for the basal period as a whole (1969–2008) of the critical parameter $v_c \approx 0.925$ remains the same. Without a step-wise drop of this magnitude HL-2 and HL-3 do not generate steep decreases in the employment ratio and in net output observed in 2008–2009. The outlook of this paper: outside the discarded scenario I, recovery begins after achieving bottom line of net output (P) in the profit enhancing scenario II – in 2010, in the profit enhancing scenario III – in 2009. Tables 5a and 5b compare characteristics of simulation runs for two basal sub-periods (1969–1982, 1983–2008) and three prospective scenarios for 2008 and far beyond.

Table 4. Initial magnitudes of main variables in three different scenarios for the year 2008

Scenario	Rate of accumulation (k)	Capital-output ratio (s)	Employment ratio (v)	Relative labour compensation (u)	Profit rate ($(1-u)/s$)
I	0.141	2.013	0.943	0.679	0.159
II	0.141	2.013	0.943	0.679	0.159
III	0.141	2.013	0.943	0.679	0.159
Observation	0.142	2.014	0.942	0.678	0.160

Table 5a. Parameters in two basal sub-periods and in three prospective scenarios for 2008 and beyond as well as distinctively determined stationary magnitudes

Parameter	Sub-period I, 1969–1982, based on HL-1	Sub-period II, 1983–2008 and scenario I based on unaltered HL-2	Scenario II based on altered HL-2	Scenario III based on HL-3
b	0.3163	…	…	…
g	0.0421	…	…	…
r	0.0588	…	…	…
i	0.0112	0.0112	0.0112	0.0112
v_c	0.925	0.925	0.8	0.8
d_1	…	0.002	0.02	…
d_2	…	-0.0032	-0.0032	…
n_3	0.568	0.568	0.568	0.28
c_2	…	…	…	0.8833
X	…	…	…	0.95
V	…	0.955	0.955	…
v_a	0.847	…	…	…
v_b	…	…	…	0.95
u_a	0.8536	…	…	…
u_b	…	…	…	0.604

Table 5b. Average magnitudes for approximations of limit cycles generated by the hypothetic laws

Law	HL-1	HL-2	Altered HL-2	HL-3
Time segment of limit cycle approximation	2885–2896	2885–2894	2885–2894	2878–2890
Approximate period of limit cycle	11	9	9	12
v	0.847	0.954	0.962	0.950
u	0.855	0.681	0.458	0.604
s	2.354	2.403	1.658	2.211
k	0.191	0.090	0.033	0.067
$(1-u)/s$	0.062	0.133	0.327	0.179
S (thousand workers)	28441	67289	112315	84862

5.1. Inertia Scenario I

An extrapolation of the retrospective forecast for the year 2008 and beyond, based on the unaltered deterministic model HL-2 is called the inertia scenario I. Figure 9 visualises this and other two scenarios.

Computer simulations reveal that phase variables (k, s, v, u), profit rate, growth rates of output per worker and real labour compensation as well as some other variables fluctuate. These middle-term fluctuations are anharmonic. The first distinguished complete cycle of the profit rate encompasses 2004–2011. Profitability tends secularly downwards (Figure 9, panel 6).

Table 6. Projecting 1st match with 1995–2008 maximal economic indicators in three scenarios

Variable	Year of previous maximum	Year of the 1st exceeding previous maximum in scenario		
		I	II	III
Net output (P)	2008	2010	2013	2011
Profit ($(1 - u)P$)	2008	2010	2008	2008
Surplus value ($(1 - u)L$)	2008	2010	2008	2010
Rate of surplus value ($(1 - u)/u$)	2008	outside reach	2008	2008
Profit rate ($(1 - u)/s$)	1999	outside reach	2012	2012
Employment (L)	2007	2010	2017	2014
Employment ratio (v)	2000	2011	2026	2017
Unit labour compensation (w)	2008	2008	2038	2009, 2018
Total real labour compensation (wL)	2008	2008	2026	2016

Profit and net output in real terms, surplus value and employment diminish in 2009 and recover to pre-crisis maximum already in 2010 (Figure 9, Table 6). Real labour compensation per worker and total remuneration increase despite the shallow crisis. Employment ratio exceeds pre-crisis maximum in 2011. Still relative labour compensation does not return to 1948–2008 and to 1979–2008 average magnitudes (Figures 6 and 9).

Why capital rejected this scenario? First of all, capital accumulation already experienced relative over-accumulation of capital in 2005–2007 that portended to overgrow in absolute over-accumulation nationally and world-wide. Besides this, simulations reveal that capital was able to anticipate the approaching crisis in the growth cycle.

A computer-supported mental experiment roughly reproduces conditions with information up to 2007 available in 2008. The model based on probabilistic and deterministic forms of HL-2 was simulated with parameters values identified with Kalman filtering applying observations up 2007 and extrapolated further without Kalman filtering.

It was likely anticipated that the growth rate of NNP had to decline in 2008–2009, employment ratio was to fall in 2008–2010 (Figure 10). Employment ratio in each year 2012–2015, 2021–2024 expected to be annoyingly higher than threshold ($V = 0.955$).[9]

[9] As written in a influential paper, "[The] class instinct of [business leaders] tells them that lasting full employment is unsound from their point of view and that unemployment is an integral part of the "normal" capitalist system" (Kalecki 1943: 326).

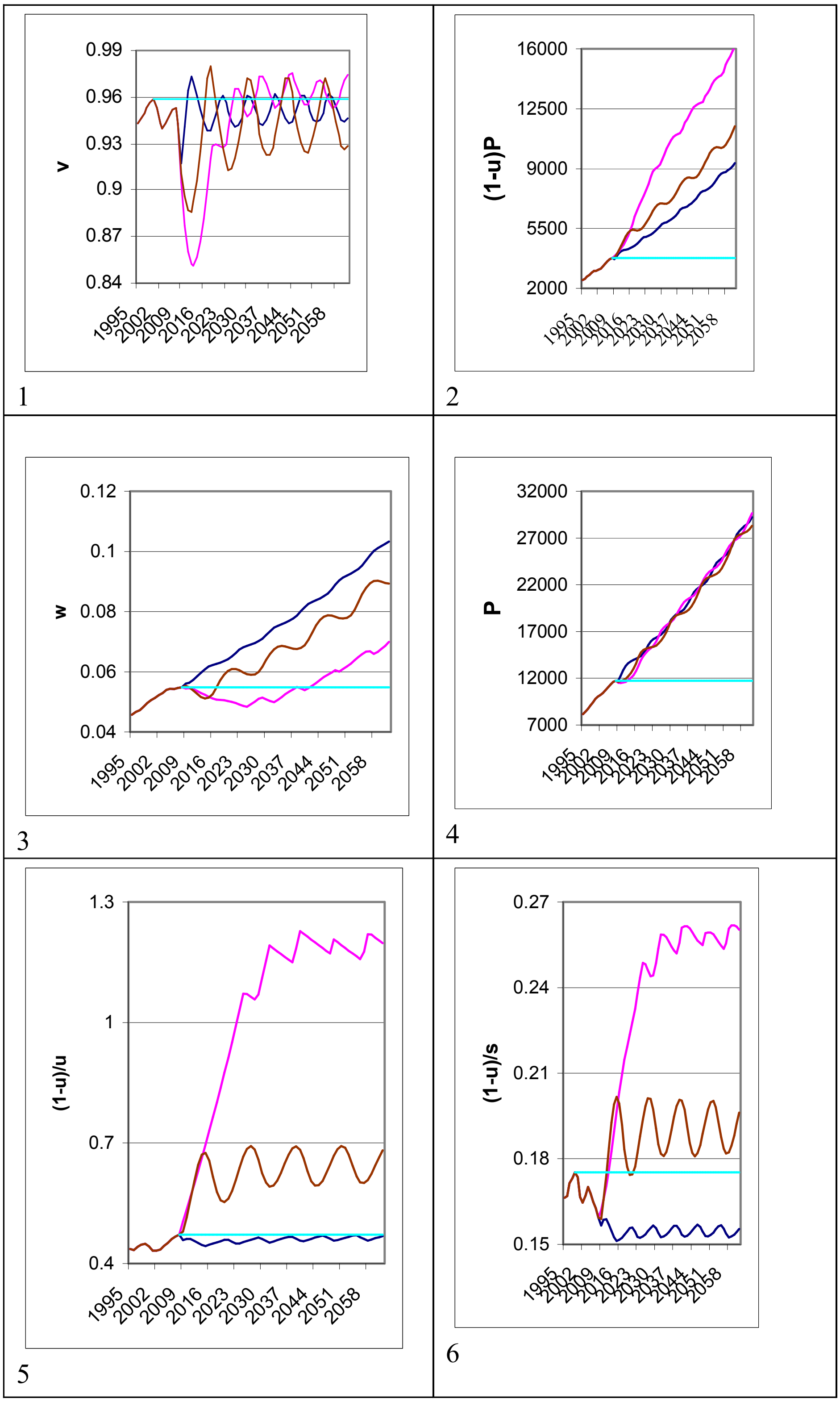

Figure 9. Evolution in three scenarios 1995–2062 (blue – I, violet – II, brown – III, aqua – frame matching maximum for 1995–2008; 1 – employment ratio, 2 – profit, 3 – labour compensation, 4 – net output, 5 – rate of surplus value, 6 – profit rate)

Excessive employment ratio was to determine worsening profitability: there will be no moving to the maximal post-war profitability observed in 1966, its next two local maximal levels (2012, 2021) are lower than even in 2004 (Figure 6, panel 6). Similarly, the rate of surplus value declines from 2012 to 2017, its next local maximum (2021) is a bit lower than the previous one of 2012 (Figure 10).

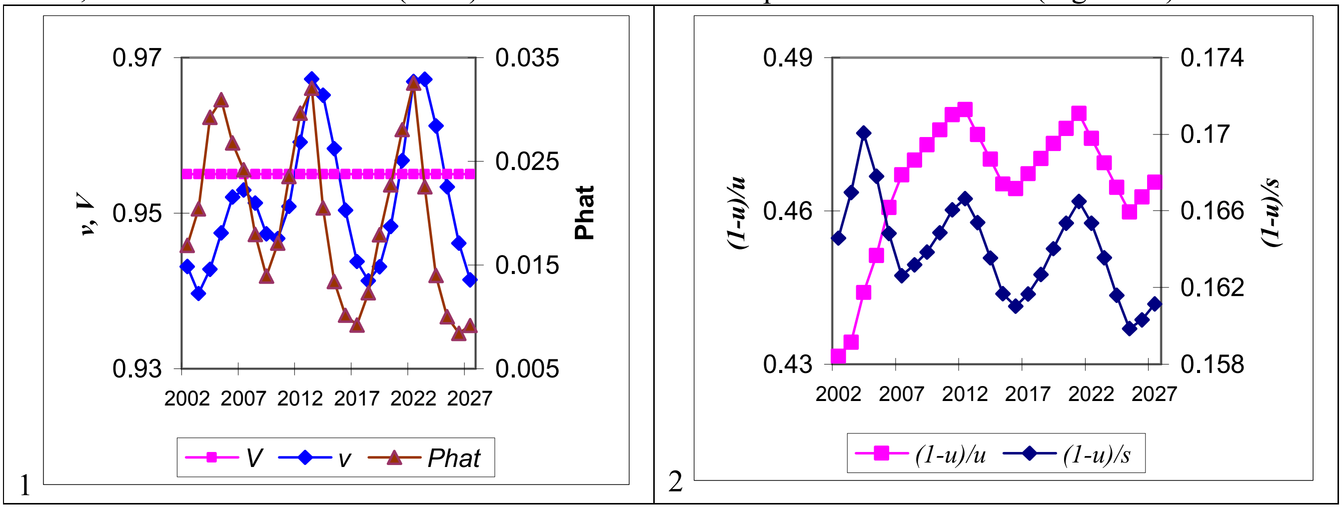

Figure 10. Panel 1 – simulated employment ratio (left scale, diamond for *v*, square for *V*) and simulated growth rate of NNP (triangle, right scale), 2002–2027; Panel 2 – simulated rate of surplus value (square, left scale,) and simulated profit rate (diamond, right scale), 2002–2027

Unsatisfactory for capital real and anticipated dynamics of profit and surplus value, rate of surplus value and profit rate in the inertia scenario I required at least parametrical alteration of HL-2. We will see that parametrical alteration of HL-2 in capital interests explains facts outlined in Introduction and enables a more realistic projecting of future developments than application of HL-2 in its previous shape. Mostly likely, intentional parametrical alteration of HL-2 (for improving long-term profitability and for elevating total profit) turned an approaching growth cycle recession into the immediate structural crisis.[10]

5.2. Two Profit Enhancing Scenarios

Scenario II

The integral profit 2008–2047 is maximised subject to the HL-2 equations (11), (12)–(14) and (15b) as well as to initial conditions of 2008. This payoff takes the magnitude of profit weighted by 1 (dimensionless). The focus of the current optimisation procedure is on three parameters that determine secular profitability trends and shape transients to limit cycles: parameter v_c from the mechanisation (automation) function (5) together with parameters d_1 and d_2 from the equation (8b) for growth rate of labour compensation.

We find optimal parameters for scenario II by maximising total profit for a selected time horizon under certain restrictions:

[10] Bob Herbert has written about "the bitter reality of the American present, a period in which big business has cemented an unholy alliance with big government against the interests of ordinary Americans, who, of course, are the great majority of Americans. The great majority of Americans no longer matter." See Herbert B. 2010 (May 21). More Than Just an Oil Spill / *The New York Times*, http://www.nytimes.com/2010/05/22/opinion/22herbert.html?_r=1 .

$$Maximise\left[\int_{2008}^{2047}(1-u)Pdt\right]$$

subject to +

$\dot{x} = f_{HL-2}[x(t), v_c, d_1, d_2]$,

$x_0 = [a_0, k_0, s_0, v_0, u_0]$, $0.8 \le v_c \le 0.94$, $0 \le d_1 \le 0.1$, $-0.01 \le d_2 \le -0.0032$.

The new magnitude of the critical parameter $v_c = 0.8$ (Table 5a) is lower than its former magnitude identified by EKF for the basal period as a whole (1969–2008). Without such or similar step-wise change an application of HL-2 (and of HL-3 below) does not generate steep decreases in the employment ratio and in net output observed in 2008–2009.

The magnitude of the vital parameter d_2 remains the same. This parameter matters only when the employment ratio equals or exceeds the threshold *V*. When the employment ratio is lower than threshold, a magnitude of the parameter d_1 is essential. It is quite reasonable that the lower magnitude of the parameter v_c, the worse is labour market for workers and consequently the lower is the magnitude of parameter d_1, as Table 5a shows.

Computer simulations reveal secular movements as well as middle-term fluctuations of phase variables (*k*, *s*, *v*, *u*), profit rate, growth rates of output per worker and real labour compensation with a period typical for industrial cycles in a range of (9–12 years). These fluctuations are anharmonic. After 2009, each of them represents growth cycle not proper industrial cycle as net output does not decrease (for results saved every time step of one year).

An amplitude of fluctuations over a certain period is measured as a difference between maximal and minimal magnitudes of the respective variable. In inertia scenario I, profitability experiences middle-term fluctuations with much smaller amplitude than in scenario II that promises drastic improvement of profitability surpassing the post-war maximum observed in1966 (0.179) in 2013.

The upward transient to regular cycle of profitability in scenario II endures up to 2026. The transient to regular cycle of employment ratio (rate of surplus value) in scenario II endures up to 2028 (2034). A plunge of employment ratio (to 0.845 in 2012) expresses labour destitution during the first cycle. The worker labour compensation (*w*) declines in scenario II in 2011–2026 (total labour compensation *wL* – in 2008–2014) to simulated level of 1998–1999.

Employment in absolute terms falls in scenario II until 2012 to the simulated level of 2000. In scenario II, the minimal net output level of 2010 will be at the level of 2007 (Figure 9, Table 6).

The analysis of scenario II gives support to the important conclusion made 35 years ago: "We conclude on the basis of an examination of the data that the political-economic function of macropolicy in the short-run is not to pursue sustained full employment nor a steady, relaxed economy with a stable reserve army. Rather its function is to ensure that the alternating pressures for expansion and contraction emanating from the private sector result in that cyclical pattern most conducive to long-run profit maximization. The goal of macropolicy is not to eliminate the cycle but to guide it in the interests of the capitalist class" (Boddy and Crotty 1975: 10).

Scenario III

In CBO's forecast (January 2010, p. 26), the persistently elevated level of unemployment depresses labor income in 2010. Beyond 2010, CBO expects labour income to grow more rapidly than GDP (as conditions in labour markets improve) and, by 2020, to approach the share of GDP that prevailed, on average, between 1979 and 2008.

CBO asserts (January 2010, p. 23): "The deep recession that began two years ago appears to have ended in mid-2009. Economic activity picked up during the second half of the year." CBO expects that the unemployment rate will average slightly above 10 percent in the first half of 2010 and then turn downward in the second half of the year. As the economy expands further, the rate of unemployment is projected to continue declining until, in 2016, it reaches 5 percent; that figure is equal to CBO's estimate of the natural rate of unemployment (which reflects, in part, the difficulty of making immediate matches between job seekers and jobs).[11]

Scenario III, based on HL-3, relates to the two targets similar to those stated be CBO: attaining relative labour compensation u_{mean} = 0.7023 observed over 1979–2008 for NNP substituting GDP in our models and achieving closeness to 5 percent rate of unemployment over 2008–2020.

A priory selection: v_c= 0.8 as in scenario II (instead of 0.925 in scenario I), X = 0.95 (corresponding the CBO estimation of the natural rate of unemployment). Parameter n_3 is selected in optimisation for balancing negative effects of step-wise drop of parameter v_c on relative labour compensation, its upper bound is lower than its identified magnitude for 1969–2008 for faster solving the next constrained optimisation task.

$$Maximise\left[-\int_{2008}^{2020}\left|u-u_{mean}\right|dt\right]$$

subject to

$$\dot{x}=f_{HL-3}[x(t),c_2,n_3],\ 0\le c_2\le 1.6,\ 0.28\le n_3\le 0.35, u_{mean}=0.7023,\ x_0=[a_0,s_0,v_0,u_0].$$

Midway scenario III uses the same initial magnitudes of state variables for 2008 (Table 4) and the same relevant parameters' values identified for the probabilistic forms of HL-1 and HL-2 by EKF for 1969–2008 except the updated (yielded by optimisation) magnitudes n_3= 0.28 in the mechanisation function (5) and $c_2 \approx$ 0.883 in the equation (19). Table 5a and 5b present relations of some important parameters with the stationary or average magnitudes of the main variables.

The Outlooks Through 2020

The unemployment rate increases to 12.5 per cent by the end of 2010 in scenario II and 10.4 per cent in scenario III and grows further (in scenario II – until 2013, in scenario III – until 2012). In 2008–2020 the employment ratio is lower on average in the two scenarios (Table 7a, Figure 10, panel 1) than the average CBO's magnitude 0.937 (CBO, January 2010). Scenario I (opportunity lost by labour) would be closest to CBO important social targets: employment ratio of 0.95 and average labour compensation share for 1979–2008 (0.7023). The scenario III is ranked second in closeness to these targets, being superior to scenario II.

CBO expects that net output will grow at an annual rate of 2.9 per cent, profit at an annual rate of 3.3, total wages and salaries at an annual rate of 2.9–3.1 per cent (depending on deflator) on average during the 2010–2020 period (CBO January 2010; Table 8). Net output, profit and total labour compensation will grow at an annual rate of 2.3, 2.3 and 2.3 per cent on average in scenario I, of 2.3, 5.4 and 0.2 per cent – in scenario II, of 2.4, 3.4 and 2.0 per cent – in scenario III (Table 8). The gaps between employment ratios in the CBO projection and in scenarios I–III will be narrow to the end of this period, growth rates of net output in the CBO projection and in scenarios I–III will be closer to each other at the end of this period as well (Figure 10, panel 2).

CBO projects the average annual growth rate of potential hours worked to be at 0.7 per cent from 2010 to 2020, scenarios I, II and III offer the same annual growth rate of labour force on average. CBO anticipates the pace of capital accumulation averaging 2.9 per cent annually during the period, whereas the rate of growth of net fixed capital will be annually 2.4 per cent in the inertia scenario I, of 2.3 per cent – in scenario II, of 2.5 per cent – in the profit enhancing scenario III. Potential labour productivity

[11] According to CBO (January 2010, p.23-25), the natural rate of unemployment is an estimate of the rate of unemployment arising from sources other than fluctuations in the business cycle.

expected by CBO to grow at 1.6 per cent a year, whereas scenarios I and III projects growth rate of output per worker at 1.2, and scenario II – 1.3 per cent a year (Table 8).

Table 7a. Summary statistics of main labour variables in three scenarios for 2008–2020

Scenario	Mean			Normalised standard deviation (variation)		
	Employment ratio (v)	Relative labour compensation (u)	Labour compensation (w)	Employment ratio (v)	Relative labour compensation (u)	Labour compensation (w)
I	0.949	0.687	0.060	0.015	0.005	0.050
II	0.890	0.603	0.052	0.035	0.075	0.031
III	0.929	0.633	0.055	0.036	0.042	0.051

Table 7b. Summary statistics of the main capital variables in three scenarios for 2008–2020

Scenario	Mean			Normalised standard deviation		
	Surplus value, $(1 - u)L$	Profit, $(1-u)P$	Profit rate, $(1 - u)/s$	Surplus value, $(1 - u)L$	Profit, $(1 - u)P$	Profit rate, $(1 - u)/s$
I	48321.4	4228.5	0.155	0.028	0.071	0.018
II	57240.5	5030.9	0.190	0.156	0.201	0.114
III	55409.5	4794.9	0.182	0.101	0.133	0.079

Table 8. Economic indicators in three scenarios and in CBO projection (January 2010) for 2010–2020

Scenario	Average growth rate of					
	output per worker (a)	total labour compensation (wL)	profit	net output	labour force	fixed capital
I	0.012	0.023	0.023	0.023	0.007	0.024
II	0.013	0.002	0.054	0.023	0.007	0.023
III	0.012	0.020	0.034	0.024	0.007	0.025
CBO	0.016 (potential labour productivity)	0.031 deflated by PCE price index, 0.032 deflated by GDP price index, 0.029 deflated by CPI	0.033 deflated by GDP price index	0.029 (for GDP)	0.007	0.029 (non-farm business sector)

The CBO targets could be hardly achieved as projected in the recent report (CBO January 2010). Experimental simulations (skipped) demonstrated that achieving employment ratio of 0.949 in 2015 (for $v_c = 0.8$) requires a greater aggressiveness of stabilisation policy ($c_2 = 1.6$ instead of 0.883, $n_3 = 0.568$ instead of 0.28) and deeper cut in relative labour compensation (of about 15 percentage points in 2009–2015) than assumed in that report – it takes for granted that the share of wages and salaries in GDP after decline by 0.8 percentage points in 2011 compared with 2009 will be higher by 0.2 percentage point in 2015 than in 2009. This internal inconsistency reflects a superfluous treatment of the structural crisis of capitalism by the economic mainstream that bears blinders in front of socio-economic contradictions.

For saving consistency of CBO projections, it would be necessary to raise the magnitude of the critical parameter v_c, say, to its previous magnitude (0.925). Yet the CBO document does not explain how this parametric alteration could be achieved against opposition of capital.

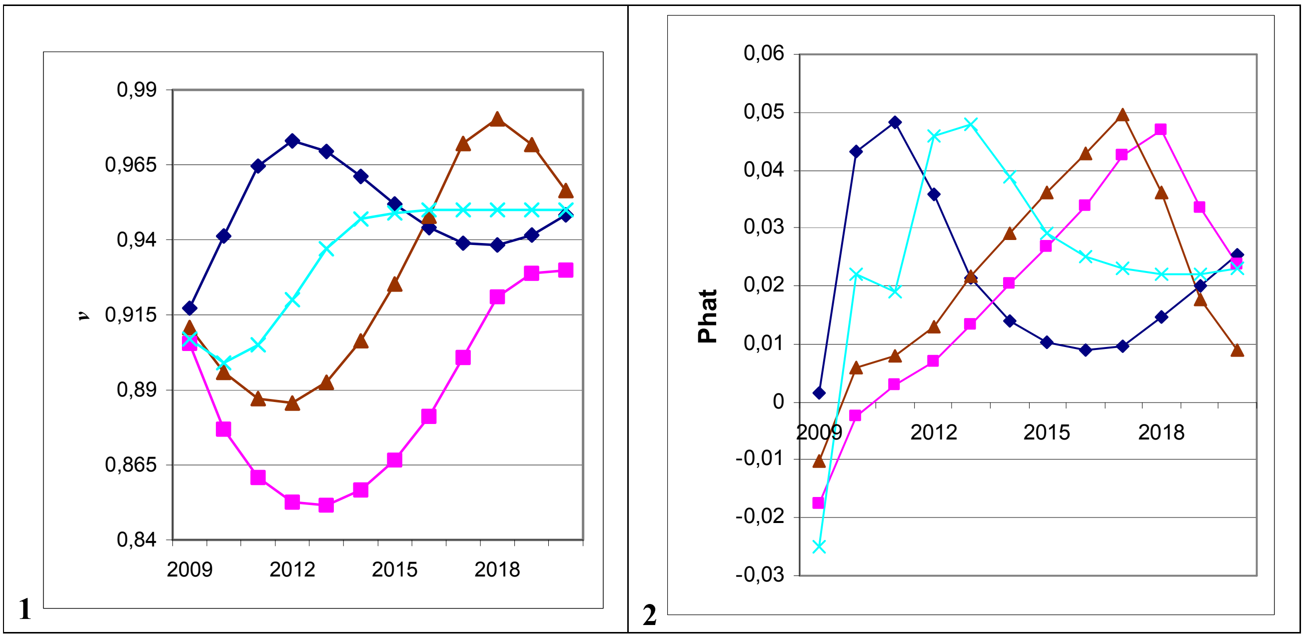

Figure 10. Evolution of the employment ratio (v) on panel 1 and of growth rate of net output (Phat) on panel 2 in three scenarios compared with CBO's projections over 2009–2020 (blue – I, violet – II, brown –III, aqua – CBO January 2010)

The Outlooks Through 2062

A period of perspective fluctuations in scenarios I and II is about 9-10 years that is typical to industrial cycles, in scenario III – 12–13 years. This latter is longer than typical for industrial cycle.

First, compare scenarios from the capital standpoint. The lowest profit share, profitability and capital-output ratio characterise the discarded scenario I. Scenario II has the lowest rate of accumulation, the highest profit share, capital-output ratio and profit rate, on the average. In scenario III, these economic indicators find their intermediate magnitudes between heights of scenario III and bottoms of scenario I. Variation of profitability is maximal in scenario II that provides the greatest gain in relative and absolute wealth for capital; variation of profitability is minimal in scenario I that ranks under other two in capital ability to create and appropriate surplus value.

Second, compare scenarios from the labour stance. Clearly, scenario I would be best judged by employment ratio and relative labour compensation that are highest among the given opportunities while variation is lowest. Absolute labour compensation would be also the highest in scenario I. Scenario II symbolises heavy toll for workers.

Scenario III has a substantially higher relative and absolute labour compensation than scenario II, yet it ranks third by average employment ratio. Still scenario III is superior to scenario II by employment ratio in the second decade of the 21st century (Table 7a). It may represent a new social compromise.

For 2008 through 2062 the ranking of three scenarios in relation to two CBO important social targets (employment ratio of 0.95 and average labour compensation share for 1979–2008) remains the same as for 2008 through 2020: scenario I would be best, scenario III is second in closeness to these targets, being superior to scenario II again.

The longer projections confirm that the aggressive profit enhancing scenario II is best for capital (Tables 6, 9a and 9b, Figures 6, 9 and 10). Scenario I represents missed opportunity for workers. Scenario II requires a dramatic plunge of labourers' living standard for a protracted period with returning to the level of labour compensation (w) of 2008 only in 2038 (in scenario III – 2018), whereas total labour compensation (wL) will not match the 2008 level until 2026 (in scenario III – 2016), the previous local maximum of the employment ratio of 2007 will be outside reach until 2017 (in scenario III – 2014). Mostly probably scenario II could be not realised on practice (at least without engaging in a grand war – to be avoided at all costs!).

Table 9a. Summary statistics of three main labour variables in three main scenarios for 2008–2062

Scenario	Mean			Normalised standard deviation (variation)		
	Employ-ment ratio (v)	Relative labour compensation (u)	Labour compensa-tion (w)	Employ-ment ratio (v)	Relative labour compensation (u)	Labour compensa-tion (w)
I	0.951	0.685	0.078	0.010	0.004	0.181
II	0.943	0.498	0.056	0.037	0.129	0.110
III	0.939	0.616	0.069	0.025	0.033	0.174

Table 9b. Summary statistics of three main capital variables in three main scenarios for 2008–2062

Scenario	Mean			Normalised standard deviation (variation)		
	Rate of ac-cumulation	Capital-output ratio (s)	Profit rate $((1-u)/s)$	Rate of ac-cumulation	Capital-output ratio (s)	Profit rate $((1-u)/s)$
I	(k) 0.114	2.037	0.155	(k) 0.452	0.008	0.012
II	0.078	2.100	0.239	0.527	0.009	0.125
III	0.090	2.046	0.188	0.573	0.012	0.054

Scenario II is superior for capital than the discarded inertia scenario I and midway scenario III. Workers are destine to lose relatively to capitalists not only in scenario II but to a lesser extent in scenario III as well: even scenario III will extend and exacerbate the secular tendency of relative labour compensation to fall that was a characteristic of the post-war epoch including basal period 1969–2008 (Figures 6 and 9). Still the supposed structural change of the law of capital accumulation in scenario III may help the US economy to overcome the stern crisis at lower social costs than scenario II.

Conclusion

This paper tests the deterministic and probabilistic forms of hypothetical laws of capital accumulation (HL-1 and HL-2) statistically for the basal period of the US economic evolution, 1969–2008. These two laws generate industrial cycles with a period of about 9–10 years; approximations of limit cycles are simulated with a period of fluctuations of about 11 years.

HL-1 reinforces the tendency of profit rate to fall in 1969–1982 due to positive dependencies of growth rate of labour compensation on employment ratio and growth rate of capital intensity. For overcoming this tendency capital transformed HL-1 into HL-2 by subordinating growth of labour compensation to growth of output per worker. The surmised substitution of HL-1 by HL-2, likely, in 1983, drastically improved profitability. As a result the achieved levels of profit rate in 1997–1999, and in 2004 (just before the onset of relative capital over-accumulation) were only slightly lower than the maximal post-war profit rate observed in 1966. Similarly, capital rejected inertia scenario I (from 2008 through 2062 for certainty) based on unaltered HL-2 as a trap as further prospects of capital accumulation would be worse than in the finished industrial cycle.

The fundamental contradiction between *value* and *use-value* of commodity (especially of *labour power* as commodity) is the most essential. This paper verifies the clue that this *causa prima* is the key to the present structural crisis of capital accumulation. A strive of capital dominated by its relentless financial arm to higher profit and higher profitability hides behind the *explosive and implosive nature* of capitalist reproduction in profit enhancing scenario II based on altered HL-2. The recent sharp macroeconomic decline follows, likely, from this law too. Unlike the conventional wisdom, capital did not choose between a mild recession sooner and a nastier one later but engineered instead the sharp crisis for creating for itself a stable or sound long-term macroeconomic environment.

This paper offers a more socially balanced stabilisation policy aimed at greater equity in income distribution together with higher employment ratio. HL-3 implements closed loop control that supposes a positive linear determination of growth rate of total surplus value by difference between a target and actual employment ratios. Scenario III based on HL-3 is a midway between discarded inertia scenario I and greedy scenario II, it describes the relatively mild socio-economic recovery from the present structural crisis and projects subsequent transient to vicinity of limit cycle with a period of about 12 years.

Apart from scenario I, the recovery from the present structural crisis of the capital accumulation, worst after the World War II, will last in scenarios III and II until 2011–2013 when the pre-crisis maximum of net output is restored and 2014–2017 when the pre-crisis maximum of employment is reached again. These projections differ from the recent official projections prepared by CBO (January 2010). On the other hand, these estimates are different from estimates given in (Ryzhenkov 2009) mainly as belated application of profit enhancing policies is substituted by their earlier onset.

Future research may turn the mentioned exogenous financial flows and stocks into endogenous variables. The HLs boundary could be reasonably extended for including fictitious capital.

References

BEA. 2009. *Concepts and Methods of the U.S. National Income and Product Accounts* / Bureau of Economic Analysis // U.S. Department of Commerce.

BEA. 2010. *A Guide to the U.S. International Transaction Accounts and the U.S. International Position Accounts* / BEA Briefing // Bureau of Economic Analysis.

Boddy R., Crotty J. 1975. Class Conflict and Macro-Policy: The Political Business Cycle / *Review of Radical Political Economics* 7 (1): 1–19.

CBO Memorandum. *An Economic Model for Long-run Budget Simulations*. 1997 (July) / The Congress of the United States // Congressional Budget Office.

CBO. *The Budget and Economic Outlook: Fiscal Years 2010 to 2020 January 2010* / The Congress of the United States // Congressional Budget Office.

Eagly R.V. 1972. A Macro Model of the Endogenous Business Cycle in Marxist Analysis / *The Journal of Political Economy*, 80, No. 3, Part 1: 523–539.

Economic Report of the President 2010. Washington, D.C. GPO.

Galbraith J.K. 2008. *The Predator State.* New York: Free Press.

Kalecki M. 1943. The Political Aspects of Full Employment / *Political Quarterly*, 14 (4): 322–331.

Kotz D.M. 2009. The Financial and Economic Crisis of 2008: A Systemic Crisis of Neoliberal Capitalism / *Review of Radical Political Economics*, 41 (3): 305–317.

Ryzhenkov A.V. 2008. Supposing a Control Law of Capital Accumulation for the Modern Italian Economy // *Proceedings of the 26th International Conference of the System Dynamics Society*. 20–24 July 2008, Athens, Greece / Ed. by B. C. Dangerfield. – Athens: System Dynamics Society, 2008. – P. 99–100. http://www.systemdynamics.org/conferences/2008/proceed/papers/RYZHE475.pdf
http://www.systemdynamics.org/conferences/2008/proceed/supp/475.pdf

Idem. 2009. A System Dynamics Design of the US Economy Exit from the Stern Crisis / *Proceedings of the 27th International System Dynamics Conference*, Albuquerque, New Mexico, USA July 26 – 30, 2009. http://www.systemdynamics.org/conferences/2009/proceed/abstracts/1379.html
http://www.systemdynamics.org/conferences/2009/proceed/papers/P1379.pdf
http://www.systemdynamics.org/conferences/2009/proceed/supp/S1379.pdf

Theil H. 1966. *Applied Economic Forecasting*. Amsterdam: North-Holland Publishing Company.