SENSITIVITY ANALYSIS OF OSCILLATORY SYSTEM DYNAMICS MODELS

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Abstract

Sensitivity analysis of oscillatory models is very difficult with standard statistical methods, such as correlation-based screening. On the other hand, behavior pattern sensitivity analysis, which focuses on the sensitivity of pattern characteristics such as equilibrium level or oscillation amplitude, is appropriate for oscillatory models. This approach also provides insights for controlling the pattern characteristics of oscillations. In this article an analysis procedure is suggested for pattern sensitivity of system dynamics models and this procedure is applied to the inventory workforce model described by Sterman(2000), using regression method.

Key Words: *Behavior pattern sensitivity, sensitivity analysis, oscillatory models, regression*

1 Introduction

In simulation studies, various types of model information cannot be estimated precisely because of impossibilities or insufficient resources. Therefore, modelers make assumptions that are subject to uncertainty (Sterman, 2000). Effects of changes in assumed information on simulation models can be explored by sensitivity analysis which is necessary for reliability of results.

Input sensitivity of simulation results can be defined as the effect of perturbation in model input on simulation output, other things being equal (Saltelli et al., 2000). This definition of sensitivity is mostly appropriate for *uni-variate* sensitivity analysis applications in which input values are changed *one-at-a time* . On the other hand, if simultaneous changes of parameters are important, input sensitivity should be defined as variability in output that can "be apportioned to the model inputs" (Saltelli et al., 2000). In other words, model parameters, which are the most capable of explaining output variability, are the highest sensitivity parameters.

Different types of sensitivities can be defined according to the purpose of the simulation model. Namely, *numerical sensitivity, behavior mode sensitivity* and *policy sensitivity* are different sensitivity types for simulation models. Particularly, *numerical sensitivity* is relevant for models that deal with physical phenomena and working with great numerical precision (Sterman, 2000). On the other hand, *behavior mode sensitivity* focuses on behavior mode changes, while *policy sensitivity* analyzes variations in optimal policy when some of the model assumptions are changed. Furthermore, one more sensitivity type, called *behavior pattern sensitivity* (Hekimoglu, Barlas, 2010), should be added to this list to cover the sensitivity of specific behavior patterns to model inputs. Especially for system dynamics models, the researcher should analyze the effect of parameter uncertainty on behavior *pattern* measures, such as equilibrium level or oscillation period and amplitude, in order to make comprehensive sensitivity analysis.

In system dynamics literature, several researchers consider the sensitivity analysis of simulation models in their studies. Powell et al. (2005) provide a generic seven-step sensitivity analysis procedure and present the analysis of an infectious disease model. Ford and Flynn (2005) suggest Pearson correlation coefficients for quick sensitivity analysis of system dynamics models, called *screening* . They calculate correlation coefficients between parameter and variable values at different time points. Nevertheless, the authors admit that *screening* method is useless for oscillatory models because of difficulty of correlation analysis for oscillatory patterns. Therefore, special characteristics of oscillations should be analyzed rather than the variable values at time points. Moizer et al. (2001) and Ozbas et al. (2008) both consider the *pattern measures* for sensitivity analysis of oscillatory models. Moizer et al. (2001) suggest a *uni-variate* sensitivity analysis procedure while Ozbas et al. (2008) present the sensitivity analysis of a real estate price model using behavior pattern measures.

In this study, sensitivity analysis procedure of Powell et al. (2005) is modified for behavior pattern sensitivity of oscillatory models. Moreover, behavior measures and their estimation procedures are discussed and *multi-variate* sensitivity analysis of an oscillatory system dynamics model is conducted. Behavior pattern sensitivity procedure is discussed in section 2 and the application of the procedure to inventory workforce model is given in section 3.

2 Pattern Sensitivity of Oscillatory Models

Analysis of oscillatory system dynamics models requires special approaches because of nonlinear cyclic behavior patterns. Particularly, sensitivity analysis using values of variables at time points is impossible for oscillations (Ford and Flynn, 2005). So, analysts should consider pattern sensitivity analysis for which we modify the sensitivity analysis procedure of Powell et al.(2005). This modified procedure is given in Figure 1.

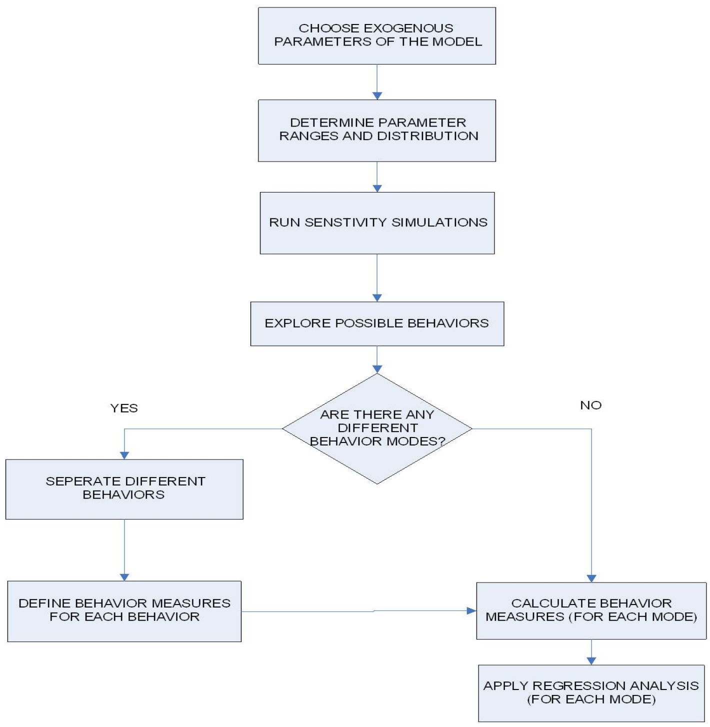

Figure 1: Sensitivity Analysis Algorithm for Behavior Pattern Sensitivity

Pattern sensitivity analysis procedure starts with the selection of model parameters that are subject to uncertainty. In small or medium size models, all parameters may be included into the analysis easily. On the other hand in large models, sensitivity analysis with all parameters may be cumbersome so, analysts may seek to choose a sub set of parameters. After parameter selection, the following step is identification of parameter distribution functions and ranges for generating input sample.

Sampling from parameter ranges can be performed by using various strategies, such as random sampling, stratified sampling, latin hypercube sampling (LHS) or Taguchi methods. Among different sampling strategies, LHS is the most appropriate one for the sensitivity analysis of simulation models (McKay et al., 1976). Furthermore, Clemson et al. (1995) conclude that LHS is efficient for the cases in which Taguchi method is "impractical". In this study, LHS is utilized for generating input sample for sensitivity simulations.

Once sensitivity simulation data is obtained, different types of oscillation patterns should be separated from each other. In literature, there are four oscillation types, named stable (damping), unstable (growing), limit-cycle and chaotic (Sterman, 2000). Behavior pattern measures of each oscillation mode are discussed in this article except chaos of which the analysis is beyond the scope. The behavior pattern measures of the damping oscillation, which is the most common oscillatory pattern (Sterman, 2000), can be listed as follows:

1. Period
2. Maximum Amplitude
3. Amplitude Slope

Period can be defined as the distance between two peak points of successive oscillations (Figure 2). The estimation of this pattern measure can be done either autocorrelation or spectral density functions. Both estimation procedures are discussed in great detail in Barlas (1997). Periods of simulation runs are estimated by using autocorrelation function in Behavior Validity Test Software (BTS)(Barlas, 1997) in this study.

Furthermore, maximum amplitude of damping oscillation pattern is another important measure of oscillatory behavior. Once a random shock arrives to the system, a corrective action emerges from the negative feedback loops and system overshoots (undershoots) its equilibrium because of time delays. So, maximum amplitude is the initial response of the stable oscillatory system (Figure 2).

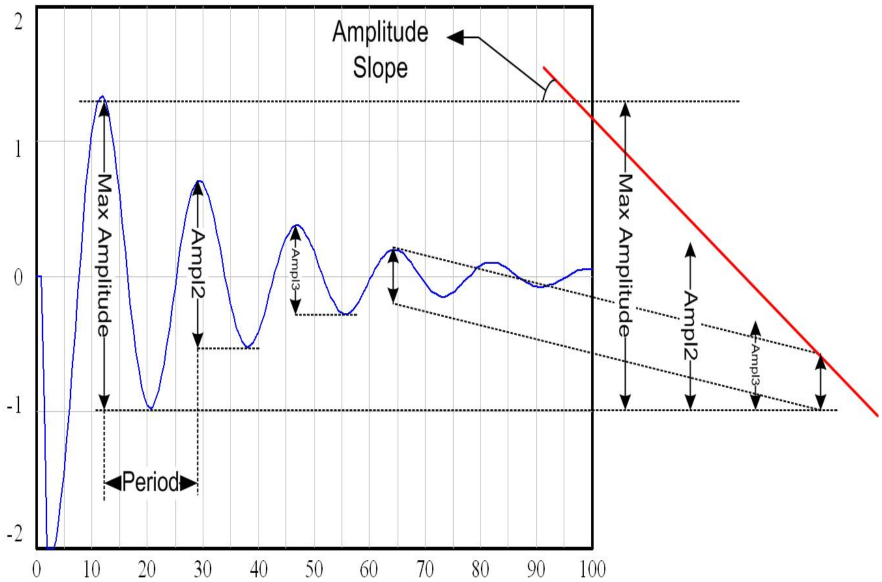

Figure 2: Pattern Measures for Damping Oscillation

Amplitude slope is the last pattern measure that is analyzed in this article. For damping oscillations, the stability character of oscillations can be represented with amplitude slope which is the slope of a straight line fit to successive amplitudes (Figure 2). In particular, high values of amplitude slope indicate more stable systems while low amplitude slopes imply unstable ones which tend to oscillate longer. On the other hand in many applications, amplitudes of stable oscillations do not follow a straight line (See Figure 2). In such cases, the natural logarithm of amplitudes should be taken before the calculation of amplitude slope. Log-amplitude slopes of two different stable oscillations are given in Figure 3.

For growing oscillations, on the other hand, minimum amplitude should be used instead of maximum amplitude. Specifically, maximum amplitude is a function of simulation time for growing oscillations since these behavior patterns tend to depart from equilibrium point as time proceeds (Sterman, 2000). Therefore, minimum amplitude is more appropriate for the analysis of the first response of unstable oscillations. Period and log-amplitude slope measures are useful for the analysis growing oscillations.

Limit cycle is a special type of unstable oscillation pattern which "the system follows a specific trajectory in state space" (Sterman, 2000). In other words, the system tends to oscillate with the same amplitude forever. Thus, the height of an amplitude and period can be used as behavior measure for limit cycles.

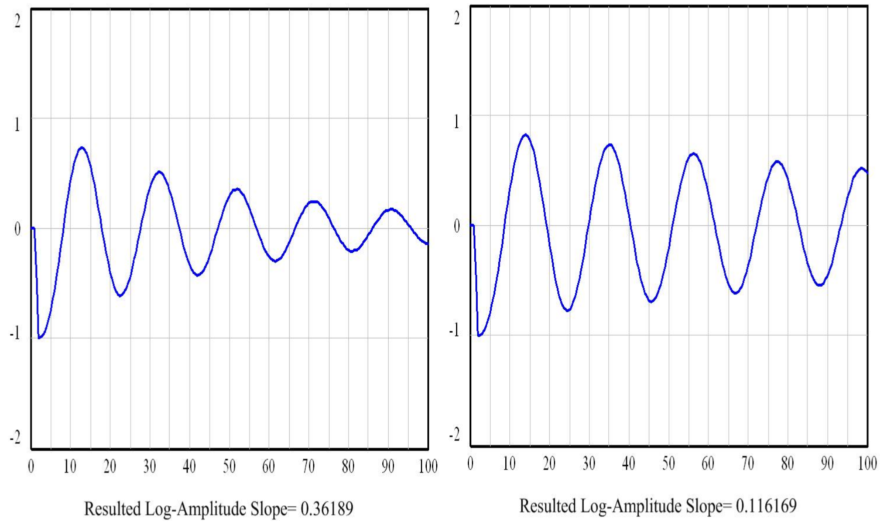

Figure 3: Slopes of Two Different Stable Oscillations

The completion of behavior measure estimation should be followed by statistical analysis such as regression or correlation. Each analysis technique has pros and cons at the same time so, the researcher should evaluate all possible procedures and select the appropriate one for analysis of sensitivity data. Specifically, analysis methods can be classified according to their assumptions on the functional relationship among dependent and independent variables. For instance, Pearson correlation coefficients, given in Equation 1, measure the strength of linear relationship between dependent (y) and independent (x) variable (Saltelli et al., 2000).

$$\hat{\rho}_{x_j y} = \frac{\sum_{i=1}^{N}(x_{ij} - \overline{x}_j)(y_i - \overline{y})}{\left[\sum_{i=1}^{N}(x_{ij} - \overline{x}_j)^2\right]^{1/2}\left[\sum_{i=1}^{N}(y_i - \overline{y})^2\right]^{1/2}} \tag{1}$$

Correlation coefficients do not provide any information about the functional relationship between dependent and independent variables. Particularly, zero correlation implies that there is no linear relationship between variables but does not indicate whether these variables are statistically independent or there is nonlinear relationship. In regression methodology, on the other hand, it is possible to have information about the appropriateness of linear statistical model with different procedures. Regression model is a mathematical approximation to functional relationship between dependent and independent variables. For sensitivity analysis purposes, standardized regression coefficients (Equation 3) point out the most important independent variable (Saltelli et al., 2000). Furthermore, coefficient of determination (Equation 4) of a linear regression model indicates the amount of dependent variation explained by the whole regression model.

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + e_i \tag{2}$$

$$b_i = \frac{\beta_i \hat{\sigma_x}}{\hat{\sigma_y}} \tag{3}$$

$$R^2 = \frac{\sum_i^m (\hat{y}_i - \overline{y})^2}{\sum_i^m (y_i - \overline{y})^2} \tag{4}$$

In these formulations, β_i, b_i and e represent regression coefficient, standardized regression coefficient and residual term respectively. Furthermore, there are three critical assumptions of regression methodology which are zero expectation, constant variance and normality of residual terms (Draper and Smith, 1998). Diagnoses of these assumptions are usually performed with residual plots and histograms. Specifically, the residual terms should be randomly distributed in scatter plot and their histogram should be bell-shaped. If both of these hold, the linear regression model is accepted as an appropriate approximation to the true functional relationship. Otherwise, one should suspect about nonlinear relationship among dependent and independent variables. In this study, the scatter plots of regression analysis are included after each regression model.

If there is no prior knowledge about the relationship between the dependent and independent variables, analysts should start with methods assuming linearity since these methods are simpler and more intuitive. If the violation of linearity assumption is detected, such as in Figure 8, the analysis can be extended with more general methods that cover the nonlinear functional relationship (Kleijnen and Helton, 1999[b]).

Nonlinear relationship between dependent and independent variable can be analyzed in many different ways. Adding interaction terms to the regression model, rank transformation or transformation on regression variables are only some examples of possible approaches. Kleijnen and Helton (1999[a]) provide some statistical analysis methods that can be used for sensitivity analysis of simulation models. In this study, on the other hand, Box-Cox transformation on dependent variable (Draper and Smith, 1998) and rank transformation (Saltelli et al., 2000) are utilized. Rank transformation is concluded to be ineffective for dealing with nonlinear relationship while Box-Cox transformation gives promising results. Only results of Box-Cox method are presented in this article due to the space constraints.

To sum up, the sensitivity analysis procedure by Powell et al. (2005) is modified for behavior pattern sensitivity analysis. Specifically, separation of different modes and estimation of pattern measures are added to sensitivity analysis procedure before the data analysis process. Moreover, the fulfillment of regression assumptions must be checked before the evaluation of analysis results. If any nonlinear relationship is detected, one must extend the statistical analysis using various approaches. In the following section, the application of sensitivity analysis procedure to a medium size oscillatory model, called the inventory-workforce model, is presented.

3 Sensitivity Analysis of Inventory Workforce Model

Manufacturing process is a typical example for supply line structures. *Supply line* is simply a material flow (Yasarcan, 2003), which includes negative feedback loops and time delays. Sterman (2000) models two interacting supply lines of a manufacturing firm, which are production and labor management. Production subsystem gives the required number of employee in order to satisfy incoming demand while amount of workforce determines production rate. Both of these subsystems include supply line structures that consist of negative feedback loops and time delays.

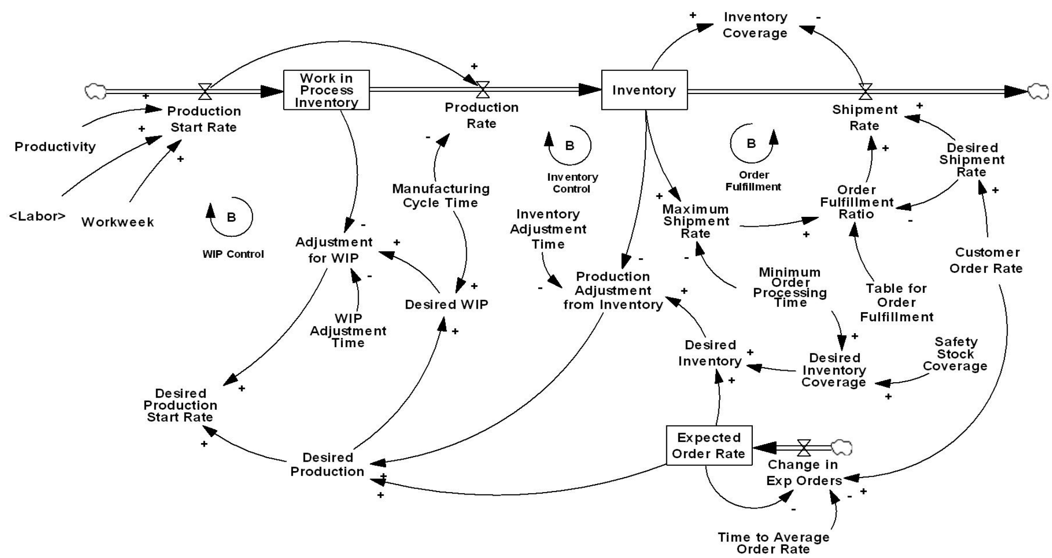

Figure 4: Production Sector of Inventory Workforce Model (Sterman, 2000)

Production sector (Figure 4) of the model consists of *inventory*,*work in process inventory (WIP)*,*expected order rate* stocks. Finished goods are shipped from inventory once demand arrives to the manufacturing firm then, new production order is opened according to the demand expectation which is represented with another stock, called expected order rate, in this model. Furthermore,*desired production rate* affects not only production start rate, but also *desired labor* in labor sector of the model.

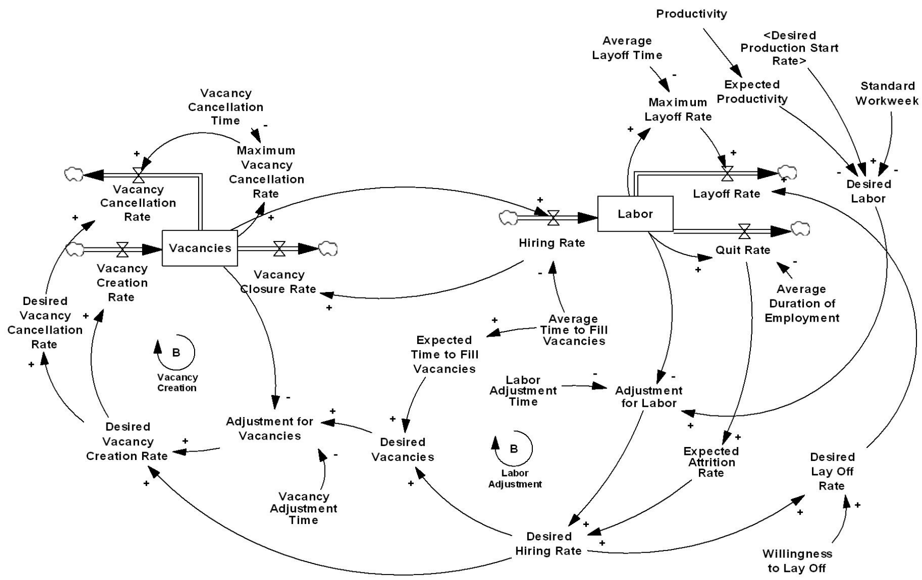

Figure 5: Labor Sector of Inventory Workforce Model (Sterman, 2000)

Labor sector (Figure 5) consists of two stocks, named *labor* and *vacancies* , and feedback loops that aim to keep these stocks at their desired levels. Once new labor requirement emerges, its position, responsibilities and duties are defined. Then a new vacancy is created and announced for this employment. All of these processes cause time lags in hiring process so, they represented with vacancies stock. Further explanation about the structure of this model is given in Sterman (2000). In this article, sensitivity analysis of inventory workforce model will be discussed using behavior pattern measures.

Inventory workforce model includes two interacting supply lines so, model variables are prone to oscillate even with small perturbations. In order to analyze the sensitivity of oscillations in inventory variable, customer order rate (Figure 4), is perturbed artificially with STEP function in Vensim (Figure 6). Sensitivity simulations are run with 14 parameters of which distribution ragnes and functions are given in Table 1.

200 simulation runs, including 193 damping oscillations and 7 limit cycles, are

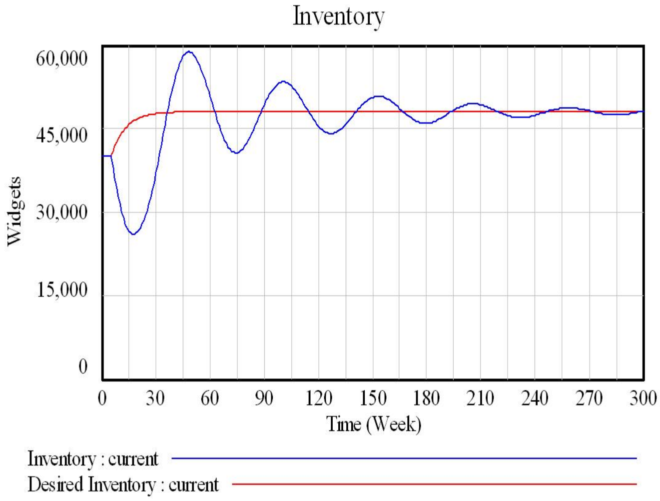

Figure 6: Inventory Stock Behavior After External Perturbation

exported to an Excel spreadsheet. Due to degrees of freedom constraint, analysis of limit cycles are omitted and only stable oscillations are subject to sensitivity analysis using regression method.

As discussed above, linear regression is a useful tool for apportioning the output variability to model parameters. Moreover, linearity assumption of this method can be checked with residual scatter plots easily. If this assumption is not satisfied, Box-Cox transformation is applied on dependent variable. Behavior pattern sensitivity of inventory workforce model is analyzed with this approach in the following sections.

3.1 Period of Damping Oscillations

Oscillatory systems overshoots (undershoots) the equilibrium level repeatedly and the duration between two successive peaks (troughs) is defined as period. Sensitivity of periods to the model parameters indicates potential leverage points for controlling oscillatory behavior. As stated above, sensitivity analysis is conducted with regression of which the summary statistics are given in Table 2.

Regression analysis indicates that first order model manage to explain %90 of the variability in oscillation period (See R^2 at Table 2). In other words, most of output variability is explained by the model parameters. Moreover, the residuals of regression model (Figure 7) do not follow any non-random pattern. Both of these indicators point out that first order model is appropriate for oscillation

Table 1: Parameters of Inventory Workforce Model

	Parameter Name	Actual Value	Min Value	Max Value	Distribution
1	Productivity	0.25	0.2	0.3	Uniform
2	WIP Adjustment Time	6	4.8	7.2	Uniform
3	Manufacturing Cycle Time	8	6.4	9.6	Uniform
4	Inventory Adjustment Time	12	9.6	14.4	Uniform
5	Minimum Order Processing Time	2	1.6	2.4	Uniform
6	Safety Stock Coverage	2	1.6	2.4	Uniform
7	Time to Average Order Rate	8	6.4	9.6	Uniform
8	Vacancy Cancellation Time	2	1.6	2.4	Uniform
9	Average Layoff Time	8	6.4	9.6	Uniform
10	Standard Workweek	40	32	48	Uniform
11	Average Duration of Employment	100	80	120	Uniform
12	Average Time to Fill Vacancies	8	6.4	9.6	Uniform
13	Labor Adjustment Time	13	10.4	15.6	Uniform
14	Vacancy Adjustment Time	4	3.2	4.8	Uniform

Table 2: Summary of Regression Model for Oscillation Period

Model Summary	
R Square	Adjusted R Square
0.901	0.894

period. So, parameter sensitivity of oscillation period can be evaluated using this regression model of which the coefficients are given in Table 3.

Like other regression results in this article, regressors are ordered according to the magnitude of standardized regression coefficients in Table 3. Eight variables are concluded insignificant and they are removed from the regression equation by stepwise regression algorithm of SPSS which use 0.05 and 0.10 as threshold values for regressor entry and removal.

According to the results of regression analysis; *standard workweek*, which represents the work day expectation of production managers, is the most efficient parameter for oscillation period. (Figure 5), Furthermore, negative sign of regression coefficient of this parameter implies that higher values of workweek expectations create shorter period oscillations which probably have more devastating effect on manufacturing firms.

Furthermore, *labor adjustment time* (Figure 5) is the second important parameter for oscillation period. This parameter indicates the responsiveness of labor manager to the changes in required workforce level of the firm. Additionally,*work*

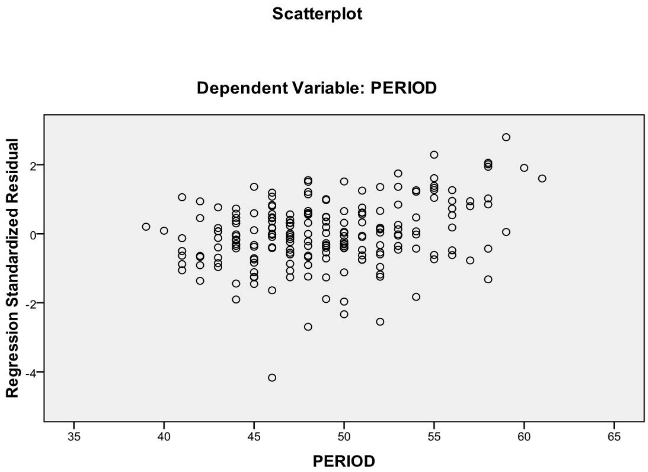

Figure 7: Residual Plot of Regression Model for Oscillation Period

Table 3: Coefficients of Regression Model for Oscillation Period

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
StandardWorkweek	-.855	.038	-.531	-22.576	.000
LaborAdjstTime	1.616	.073	.522	22.192	.000
WIPAdjst	2.877	.158	.429	18.176	.000
SafetyStockCoverage	3.656	.486	.182	7.524	.000
MinimumOrderProcssing	-1.444	.468	-.072	-3.087	.002
VacancyAdjstTime	.588	.238	.058	2.465	.015

in process adjustment time and *safety stock coverage* parameters (Figure 4) are the other important factors for oscillation period.

Oscillation period is the pattern measure that indicates the pace of cyclic behavior. Longer period oscillations overshoot its equilibrium less frequently while shorter period ones rapidly repeat overshooting (undershooting). In fact, for many biological and socio-economic systems longer period oscillations are more preferable since it is easier to tolerate the effects of such kind of behavior (Hekimoglu and Barlas, 2010). The sensitivity analysis of period indicates that standard workweek and labor adjustment time are efficient points for this measure of oscillatory inventory behavior. The second measure of damping oscillation is maximum amplitude which is analyzed in section 3.2.

3.2 Maximum Amplitude of Damping Oscillations

Maximum amplitude is the first response to the incoming shock for stable oscillatory systems. In other words, it is the amount of first overshoot after the arrival of a perturbation. This behavior measure is an important characteristic of real life systems in which "noises never disappear" (Sterman, 2000). The summary statistics and residual plots of first order regression model used for sensitivity analysis of maximum amplitude are given in Table 4 and Figure 8. Although R square statistic is very good, there is an obvious curvature which indicates possible nonlinear relationship between maximum amplitude and model parameters.

Table 4: Summary of Regression Model for Maximum Amplitude

Model Summary	
R Square	Adjusted R Square
0.939	0.934

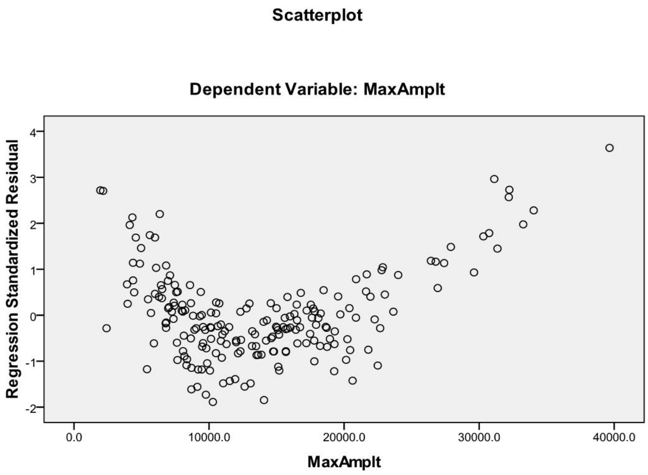

Figure 8: Problematic Residual Plot of Regression Model for Maximum Amplitude

Also, linear regression model, which is obtained by stepwise algorithm in SPSS, is given in Table 5. Stepwise algorithm removes five variables since they are insignificant in regression equation. Like oscillation period,*standard workweek* and *inventory adjustment time* parameters are important for maximum amplitude measure. Nevertheless, one should treat the results of this model with caution because of nonlinearity between parameters and maximum amplitude. In order to deal with this problem, Box-Cox transformation on dependent variable is utilized using software R.

Table 5: Coefficients of First Order Regression Model for Maximum Amplitude

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
(Constant)	69911.450	4078.200		17.143	.000
StandardWorkweek	-1615.606	45.318	-.658	-35.650	.000
InvntAdjstTime	-2227.668	93.923	-.436	-23.718	.000
MnfctrngCycleTime	2302.679	144.653	.300	15.919	.000
SafetyStockCoverage	8693.608	585.886	.283	14.838	.000
Time2AvgOrderRate	-1306.676	141.937	-.170	-9.206	.000
LaborAdjstTime	633.702	88.921	.134	7.127	.000
VacancyAdjstTime	1436.954	287.291	.094	5.002	.000
Time2FillVacancy	526.501	140.268	.069	3.754	.000
AvgDuratnEmploy	-35.012	11.238	-.057	-3.116	.002

Box Cox method is a transformation algorithm that utilize maximum likelihood principle in order to calculate a power value once it is assumed that the power transformation family is appropriate for the data set at hand. The transformed dependent variable (W) can be formulated as follows:

$$W = \begin{cases} (y^{\lambda} - 1)/\lambda & \text{for } \lambda \neq 0 \\ ln(y) & \text{for } \lambda = 0 \end{cases} \tag{5}$$

The value of transformation exponent (λ) is calculated using maximum likelihood principle under the assumption that the residual terms are normally distributed ($e \sim N(0, \sigma^2)$). This procedure is described in greater detail in Draper and Smith (1998). In this study, the Box Cox transformation algorithm in statistical package R is utilized. The search algorithm of this software gives the λ value that maximizes log-likelihood function of regression model. Sensitivity of maximum amplitude is analyzed with the regression model including dependent variable transformed with this λ value.

Table 6: Summary of Regression Model for Transformed (Box-Cox) Maximum Amplitude

Model Summary	
R Square	Adjusted R Square
0.975	0.973

The transformation exponent, which box-cox search algorithm concludes, is 0.3838 for maximum amplitude. Summary of regression analysis summary with transformed dependent variable is given in Table 6. The R square value of this model indicates a better fit to data set at hand. Moreover, there is no obvious pattern in the residual plot of regression given in Figure 9. In other words, this regression model is more appropriate for sensitivity of maximum amplitude variable to the model parameters.

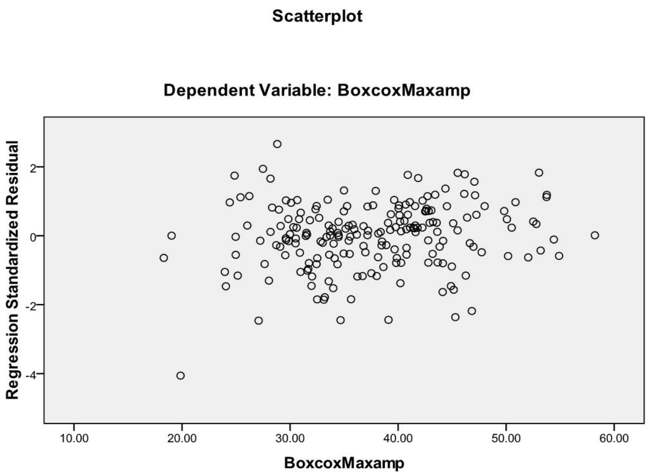

Figure 9: Residual Plot of Regression Model for Transformed (Box-Cox) Maximum Amplitude

Coefficients of regression model for transformed dependent variable are given in Table 7. Firstly, I should note that stepwise regression algorithm is not used for this data set since the transformation exponent is computed for 14 parameter-regression model. In this regression model, standard workweek and inventory adjustment time are the most important parameters. Inventory adjustment time directly represents the response of production manager to the incoming demand changes while standard workweek is the work hour expectation of production planners. Manufacturing cycle time and safety stock coverage are also other important parameters affecting the maximum amplitude of damping oscillation.

Box-cox transformation solves the nonlinearity problem and does not create any alteration on the importance rank of model parameters. Parameter orders from two regression analyses are given in Table 8. Obviously, box-cox transformation does not create any alteration in the parameter ranking for maximum amplitude. Therefore, it can be concluded that linear regression may be used for sensitivity analysis even if the regression assumptions are not fulfilled. The last pattern measure of damping oscillation is log-amplitude slope of which sensitivity analysis is

Table 7: Coefficients of Regression Model for Transformed (Box-Cox) Maximum Amplitude

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
(Constant)	98.507	3.538		27.846	.000
StandardWorkweek	-1.805	.032	-.687	-57.223	.000
InvntAdjstTime	-2.430	.066	-.444	-36.946	.000
MnfctrngCycleTime	2.615	.101	.319	25.878	.000
SafetyStockCoverage	8.938	.412	.272	21.706	.000
Time2AvgOrderRate	-1.403	.099	-.171	-14.220	.000
LaborAdjstTime	.569	.063	.113	9.008	.000
VacancyAdjstTime	1.785	.202	.109	8.855	.000
Time2FillVacancy	.541	.101	.066	5.349	.000
AvgDuratnEmploy	-.025	.008	-.038	-3.131	.002
AvgLayoffTime	.131	.101	.016	1.294	.197
WIPAdjst	-.125	.132	-.011	-.948	.345
Productivity	2.179	3.208	.008	.679	.498
VacancyCancelTime	.257	.397	.008	.648	.518
MinimumOrderProcssing	.255	.390	.008	.654	.514

presented in the next section.

3.3 Log-Amplitude Slope of Damping Oscillations

Amplitude slope is the third behavior pattern measure that is analyzed in this study. Slopes of successive amplitudes indicate the stability character of the oscillations. However, except rare cases, amplitudes of an oscillation follow a nonlinear curve (Figure 2). Therefore, taking the natural logarithm of amplitudes before fitting a linear line gives more reliable amplitude slopes. The amplitudes and log-amplitudes of oscillations obtained from sensitivity runs of inventory workforce model are plotted in Figure 10. Obviously, taking natural logarithm of amplitudes (right hand side in Figure 10) linearize the pattern of successive amplitudes of a stable oscillation.

Estimated log-amplitude slopes are subject to regression analysis of which the summary statistics are given in Table 9. This regression model is obtained by using stepwise algorithm. R square statistic indicates that %96 of variability in log amplitude slopes is explained by the regression model. In addition to this high coefficient of determination, there is no obvious pattern in residual plots (Figure 11). Therefore, this regression model can be accepted as an sensitivity analysis

Table 8: Comparison of Different Methods For Maximum Amplitude

RANK	STEPWISE REGRESS	BOXCOX TRANSFORM
1	StandardWorkweek	StandardWorkweek
2	InvntAdjstTime	InvntAdjstTime
3	MnfctrngCycleTime	MnfctrngCycleTime
4	SafetyStockCoverage	SafetyStockCoverage
5	Time2AvgOrderRate	Time2AvgOrderRate
6	LaborAdjstTime	LaborAdjstTime
7	VacancyAdjstTime	VacancyAdjstTime
8	Time2FillVacancy	Time2FillVacancy
9	AvgDuratnEmploy	AvgDuratnEmploy
10		AvgLayoffTime
11		WIPAdjst
12		Productivity
13		VacancyCancelTime
14		MinimumOrderProcssing

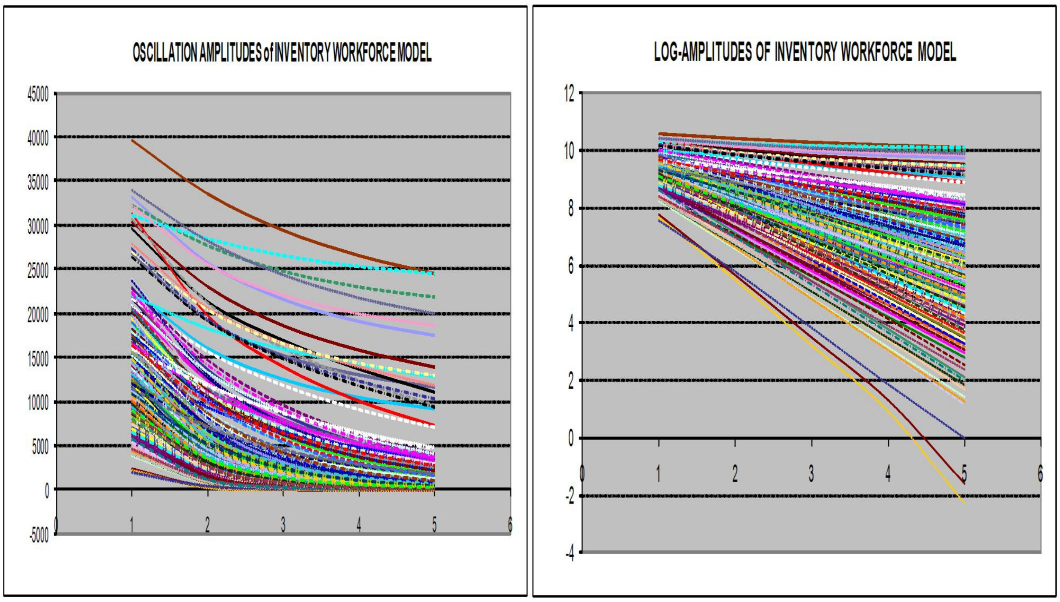

Figure 10: Amplitude Diagrams of Inventory Workforce Model

tool for this behavior measure. The coefficients of regression equation are given in Table 10.

Inventory adjustment time and standard workweek are the most effective parameters for log-amplitude slope. Then, manufacturing cycle time and vacancy adjustment time parameters take third and fourth places in ranking. Further-

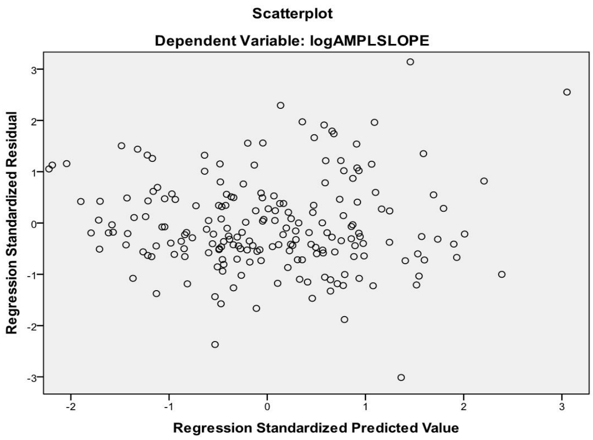

Figure 11: Residual Plot of Regression Model for Log-Amplitude Slope

Table 9: Summary of Regression Model for Log-Amplitude Slope

Model Summary	
R Square	Adjusted R Square
.967	.936

Table 10: Coefficients of Regression Model for Log-Amplitude Slope

Coefficients					
	Regression Coefficients			t value	Sig.
Model	beta	Std. Error	Std. Coef (b)		
(Constant)	-2.172	.216		-10.068	.000
InvntAdjstTime	.205	.006	.664	35.291	.000
StandardWorkweek	.076	.003	.510	26.742	.000
MnfctrngCycleTime	-.216	.009	-.466	-24.221	.000
VacancyAdjstTime	-.166	.018	-.179	-9.347	.000
WIPAdjst	.084	.012	.136	7.104	.000
SafetyStockCoverage	-.194	.037	-.105	-5.284	.000
Time2FillVacancy	-.046	.009	-.099	-5.230	.000

Table 11: Summary Table for Results of Sensitivity Analysis of Inventory Workforce Model

RANK	PERIOD	MAXIMUM AMPLITUDE	LOG-AMPLITUDE SLOPE
1	StandardWorkweek (−)	StandardWorkweek (−)	InvntAdjstTime (+)
2	LaborAdjstTime (+)	InvntAdjstTime (−)	StandardWorkweek (+)
3	WIPAdjst (+)	MnfctrngCycleTime (+)	MnfctrngCycleTime (−)
4	SafetyStockCoverage (+)	SafetyStockCoverage (+)	VacancyAdjstTime (−)
5	MinimumOrderProcssing (−)	Time2AvgOrderRate (−)	WIPAdjst (+)
6	VacancyAdjstTime (+)	LaborAdjstTime (+)	SafetyStockCoverage (−)
7		VacancyAdjstTime (+)	Time2FillVacancy (−)
8		Time2FillVacancy (+)	
9		AvgDuratnEmploy (−)	

more, stepwise algorithm removes seven regressors since they are insignificant in individual t-tests.

To sum up, standard workweek and inventory adjustment time parameters are the two possible leverage points for many pattern measures of oscillatory behaviors. In fact, the importance of standard workweek parameter in these simulation runs is very counter-intuitive and possibly indicates the necessity of deeper analysis on model structure. Moreover, manufacturing cycle time is appeared as another important parameter especially for the stability character of the oscillations. On the other hand, labor *productivity* of a manufacturing firm is concluded as ineffective parameter for different oscillation pattern measures. This parameter is always appeared as insignificant in regression equations. The summary of sensitivity analysis results of inventory workforce model is given in Table 11. In this table, the positive and negative sings near the parameter names indicate the direction of correlation between the pattern measure and model parameter. Namely, increasing inventory adjustment time creates smaller maximum amplitude and greater amplitude slope which points out more stable oscillation pattern.

4 Conclusions

Parameters and functions of system dynamics models involve uncertainty so, analysts must make assumptions. In order to analyze the effects of these assumptions on the model, sensitivity analysis should be conducted after the model building phase. But standard statistical sensitivity tools, such as correlation coefficients, are not applicable to oscillatory models.

Behavior *pattern* measures are used in the problem articulation, dynamic hypothesis and policy analysis phases of system dynamics studies. Moreover, behavior *pattern* sensitivity is a very suitable approach to statistical sensitivity analysis of oscillatory models. In this article, an analysis procedure is proposed for *pattern* sensitivity of oscillatory models to the parameters. In the application of this procedure, regression method is suggested as a formal analysis method with its convenience and rich tools indicating model fit to data. Specifically, if additive functional relationship is not appropriate for sensitivity simulations, several approaches can be used for nonlinear relationships. In this study, Box-Cox transformations and rank transformation procedures are applied two pattern measures of inventory workforce model. In these analyses, rank transformation is found to be ineffective in dealing with nonlinearity while Box-Cox transformation provides promising results.

Pattern sensitivity of inventory workforce model is analyzed using three *pattern* measures of damping oscillations. Analysis results indicate that the workday expectation of production manager (standard workweek) and his/her responsiveness to the immediate changes in demand (inventory adjustment time) are the most important factors for oscillatory patterns. Furthermore in two sensitivity experiments, Box-Cox transformation is applied to the dependent variable. In these experiments, Box-Cox transformation does not create major changes on the importance ranking of model parameters while dealing with nonlinearity.

In short, behavior pattern measures are useful and appropriate tools for formal sensitivity analysis of system dynamics models. They provide information on potential leverage points of the system and guide the additional modeling effort for further analysis. Moreover, regression analysis is found to be an efficient sensitivity tool for system dynamics models. This method is not only a convenient analysis tool, but also it includes efficient capabilities that deal with lack of fit situations. Parameter sensitivity is one step for comprehensive analysis of a system dynamics models. Other sensitivity types described in the literature should be analyzed in order to fully understand the relations between the model structure and its dynamics.

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