An Extension of Loop Deactivation in the Behavioural Method

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Abstract

The behavioural method is an important technique for identifying the dominant feedback loops for a variable of interest. The core mechanism of this approach is that deactivating different loops influences the behaviour of the selected variable to various degrees. Through assessing the variance of the behaviour between the reference model and the modified model for all feedback loops, we are able to identify the loops which exert the most significant influence on the variable, i.e., the dominant loops. An important step in the behavioural method is to deactivate a loop by fixing its control variable or a unique edge. However, a drawback is where neither the control variable nor the unique edge is identified. This paper presents another loop deactivation method which is applicable when such circumstance happens. The new method deactivates a loop by modifying its unique consecutive two edges which are able to distinguish this loop from other loops. The long wave model is used to demonstrate the loop deactivation approach and compare the analysis result with other dominant loop identification methods.

1 Introduction

Exploring the feedback structure in order to explain the behaviour of complex dynamical systems lies at the heart of the system dynamics. Formal analysis reveals the underlying feedback mechanism that gives rise to the observed system behaviour.

A dominant loop analysis method which is much in line with the classic methods that rely on hypothesis testing is the behavioural method. The core mechanism of this method is that deactivating different loops affects the behaviour of the variable of interest to a various degree, hence the one considered as a dominant loop should exert most significant influence to the behaviour, i.e., when the dominant loop is deactivated, the behaviour diverts most from its original trajectory.

A crucial step in the behavioural method is to deactivate the candidate loop. Ford proposed to deactivate a feedback loop by fixing the value of its control variable, i.e., the variable uniquely belongs to a loop. A drawback is that it does not guarantee that every loop has a control variable, therefore those loops cannot be deactivated to assess their roles in the behaviour of the variable. Another factor in its application is Ford did not specify how to select the feedback loops with a given model. Recently, Phaff (2008) suggested we adopt the shortest independent loop set (SILS) (Oliva, 2004) as the candidate loop set and deactivate the loop by its unique edge instead of its control variable. A unique edge is a looser constraint than a control variable for a loop to satisfy, since the existence of a control variable indicates two unique edges. When a loop does not have a control variable, it is highly likely to have a unique edge. However, this improvement does not promise every loop in SILS can be deactivated either, as there are still loops that may even have no unique edges.

The main contribution of this paper arises from the circumstance when the feedback loop does not have a *control variable* or a *unique edge* which can be deactivated. We propose a loop deactivation method by modifying its unique consecutive two edges which are able to distinguish this loop from other loops. We organize the paper as follows: first related research work is introduced in Section 2. After which we clarify the circumstance when the current loop deactivation fails, (a loop does not have a unique edge) and put forward a three-step methodology with illustration. With the proposed method, we then demonstrate it on the Long Wave model (Sterman, 1985) and compare the analysis result with other dominant loop identification methods, i.e., eigenvalue elasticity analysis and the original version of the behavioural method. Finally we end with conclusions and recommendations for further research.

2 Literature review

We can classify the existing dominant loop identification approaches into three categories: the behavioural method (Ford, 1999), pathway participation method (PPM) (Mojtahedzadeh, 1997; Mojtahedzadeh et al., 2004) and eigenvalue elasticity analysis (EEA) (Forrest, 1983; Saleh and Davidsen, 2000; Güneralp, 2005). The first method is the focus of this paper. It deactivates each feedback loop and assess its role by comparing the behaviour of the variable of interest in the reference model with in the modified model. The second one, PPM, starts from the variable of interest and traces the pathways which contribute most to the selected variable until they forms a loop. This method is implemented by a software package (Mojtahedzadeh, 2001): DIGEST. The last one, EEA, establishes the relationship between the system behaviour and the structure elements (e.g., links or feedback loops) by the eigenvalues. Then it uses the eigenvalue response to the perturbation of those elements to assess their influence on the system behaviour. EEA is considered as a highly mathematical approach and it applies only to the linear model or the linearized model while the other two methods are not constrained if the model is nonlinear. Furthermore, the behavioural method and PPM are variable-based while EEA is variable-independent.

One important contribution in the development of the behavioural method made by Ford is formulating the behaviour patterns. He identified three atomic behaviour pattern (ABP) based on the absolute net rates of the variable of interest (see Eq. (1)), i.e., convergent/logarithmic behaviour results in an ABP less than zero, divergent/exponential behaviour results in an ABP greater than zero, linear behaviour results in an ABP equals to zero. Based on the ABP, Ford can divide the behaviour of the selected variable into intervals for individual analysis. Moreover, ABP is a qualitative indicator used to assess the role of a loop. Decision on whether it is a dominant loop is made by checking if the ABP of the variable of interest in the reference model and the modified model differs.

$$ABP = sign\left(\partial\left(\left|\frac{\partial x}{\partial t}\right|\right)/\partial t\right) \tag{1}$$

Recently, extended version of the behavioural approach was introduced (Phaff, 2008). It is referred to as the Generalized Loop Deactivation Method (GLDM). It improves the method from the following two aspects:

- Make use of the shortest independent loop set developed by Oliva (2004) and let it be the candidate loop set to be analyzed. As in the original version of the behavioural method, Ford did not provide a systematic method to select the candidate feedback loops. Kampmann (1996) addressed that "the number of loops $S_{n,p}$ in a maximally connected system with n state variables and p auxiliary variables grows as $2^{np}(n-1)!$". Therefore, it is necessary to limit the number of feedback loops. SILS controls the number of loops at a reasonable size and more importantly, Oliva and Mojtahedzadeh (2004) found SILS captures the core dynamics, i.e., contains dominant feedback loops.

- Deactivate a feedback loop by fixing its unique edge instead of a control variable. A control variable is a variable which uniquely belongs to a feedback loop and used in the original behavioural method. Similarly, a unique edge is an edge that belongs to only one feedback loop. It is easy to verify that a control variable suggests two unique edges, hence identifying a unique edge is a less strict constraint for a feedback loop. This is an improvement which makes the behavioural method applicable to a much wider range of models. In conjunction with the regulation of how to set up the candidate loop set, GLDM offers us an opportunity to automate the behavioural method.

3 Proposed loop deactivating approach

3.1 Problem statement

In our previous discussion over the behavioural method, we adopt the SILS as its candidate loop set. However, there is another issue when attempting to deactivate a loop, i.e., the independent loop set may contain loops that have no unique edges. The definition of ILS from Kampmann (1996) states: "An independent loop set of a digraph is a maximal set of loops whose incidence vectors are linearly independent, i.e. every other loop is linearly dependent upon the loops in it". From the procedure of constructing SILS (Oliva, 2004), we know that every newly added loop has the least number of new edges which have not occur in the loop set. Therefore, this process does not prevent the new loop diminishing the "existing" unique edges for the loops which are already in the set.

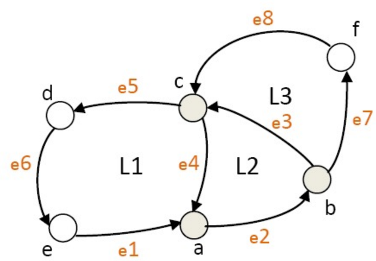


Figure 1: A simple model

To clarify under what circumstance this problem occurs, we use an example which is shown in

Figure 1. In a strongly connected graph [1], the relationship between the total number of independent loops u, nodes n and edges e are:

$$u = e - n + 1$$

Hence, the example shown in Figure 1 has 3 candidate loops: $8 - 6 + 1 = 3$. Table 1 lists the loop matrix describing the loops and their edges. We observe that the incidence vectors of each loop (in rows) are linearly independent whereas $L2$ does not have a unique edge. This is a simple scenario which renders inefficient to both Ford's behavioural method and the GLDM analysis. We can imagine a complex system will cause more loops have no unique edge. Therefore, it is necessary to find a way to deactivate the loop under such circumstances. In the following section, we introduce a method to overcome this problem and use the shown example to demonstrate it step by step.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
L1	1	1	1	0	1	1	0	0
L2	0	1	1	1	0	0	0	0
L3	0	1	0	1	0	0	1	1

Table 1: Loop matrix

3.2 Loop deactivation approach with illustration

The idea of deactivating a loop with no unique edge is derived from GLDM (Phaff, 2008). We consider using the **unique consecutive two edges** to deactivate the candidate loop. The unique consecutive two edges refer to two consecutive edges which belong to a loop but not to any other loops in SILS. We present the procedure of how to deactivate a loop with no unique edge in the behavioural method as follows. Three steps of deactivating a loop with no unique edge:

1. Identify all the loops using SILS algorithm, and let these be our candidate loops.

2. Select the loops that do not have a unique edge and identify the unique consecutive two edges which do not lie in any other loops in each selected loops. In order to deactivate these loops (no unique edge), we follow the steps below to modify the model:

   (a) Create two nodes which serve as the representatives of the two variables of the first edge in pair. Link these two new nodes with the same order as their counterparts in the reference model.

   (b) For the second edge in the unique consecutive edges, switch its tail, i.e., the middle variable in that edge pair, to its newly created representative while keep the head of the edge unchanged.

   (c) Copy all the edges who end with the middle variable in the unique consecutive edges (except the edge which is one of the unique two edges) to the variable of its new representative.

[1] A strongly connected digraph G is one in which, for any pair of nodes x, y ∈ G, there is both a directed path from x to y and a directed path from y to x.

3. Set the new variable who is the representative of the origin of the unique consecutive edges to be a fixed value. The choice of this fixed value is consistent with the Ford's behavioural method, i.e., the value at its deactivation.

To clarify the methodology stated above, we use the model in Figure 1 to demonstrate the approach.

1. The loops in SILS are depicted in Table 1.
2. Table 1 shows that $L2$ shares all its constituent edges with the other two loops and does not have a unique edge. Besides, only the *combination* of $e3$ and $e4$ can determine this loop. Other combinations, e.g., $(e2, e3) \in \{L1, L2\}$, $(e4, e2) \in \{L2, L3\}$, do not uniquely belong to one loop. Therefore, the unique consecutive two edges are identified as e_3, e_4 or denoted by nodes in sequence: b, c, a.
   (a) Create two nodes b', c' as the representatives of b, c. We link these two new nodes $b' \rightarrow c'$ in the same direction in the original model $b \rightarrow c$. This is illustrated in Figure 2.

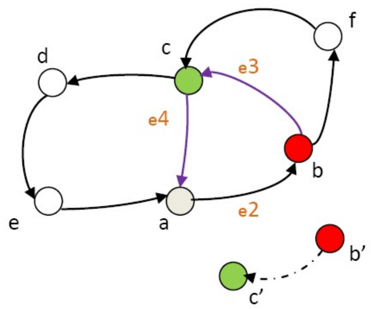


Figure 2: Loop deactivating process (a)

   (b) Switch the tail of the second edge in the unique edges, i.e., c in $e4$, to its representative, c'. Figure 3 shows that $e4$ is removed and replaced by a dash line $e_{c'a}$.
   (c) Copy all the edges where the variable c is a tail to c' except e_{bc}. There is only one edge satisfying this condition: $f \rightarrow c$. In this case, the variable c' is initialized. It is constructed in the same way as its counterpart c, except for one edge, $e3$. The final modified model for deactivating $L2$ is depicted in Figure 4.
3. Set node b' to be a constant at the beginning of each phase. Then, the behavioural method can be performed to analyze the influence of this particular loop to the behaviour of the variable of interest.

Compare the model after deactivating (Figure 4) with the reference model (Figure 1), we can see the removal of $e4$ impacts $L2$ and $L3$ simultaneously. What we do to recover $L3$ and maintain $L2$ deactivated is as follows:

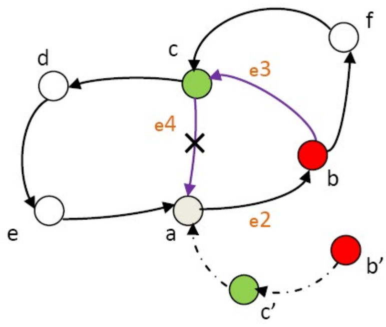


Figure 3: Loop deactivating process (b)

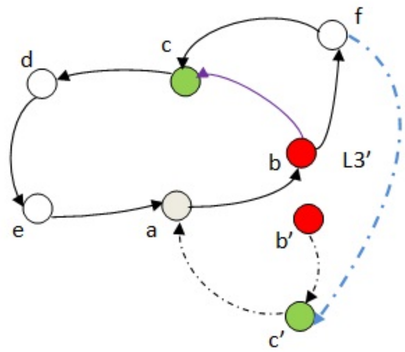


Figure 4: Loop deactivating process (c)

1. We identify that c is a joint point of $L2$ and $L3$. It is a function of two "inputs", say, $c = G(b, f)$, nodes b and f lie in $L2$ and $L3$ respectively. In order not to impact $L3$, we retain all the inputs to c. However, we have to deactivate $L2$, so a node b' is created to be a constant value of b. Another variable c' which is a representative of c is created for the purpose of propagating the information flow passed through the constant b' when deactivating $L2$.

2. c' is connected to a while $e4$ is deleted. Therefore $L2$ is deactivated and its modified version $L2'$ is: $b' \to c' \to a$. At the same time, $L3$ is cut by removing $e4$.

3. In order to recover $L3$, we add the edge $f \to c'$ which makes $c' = G(b', f)$. In comparison with the reference model where $c = G(b, f)$, we find c' takes the role of c and flows back to a with $L2$ deactivated but f in $L3$ unchanged. A new loop is formed: $a \to b \to f \to c' \to a$. This is $L3'$ which we consider as a equivalent of $L3$ in the modified model after deactivating $L2$.

In summary, Figure 4 shows the final modified model, compared with the reference model, $L1$ is not changed. Though $L3$ becomes a new loop $L3'$, which is regarded as "not deactivated" by the definition of the original way of deactivating a loop (Ford, 1999), but $L2$ is indeed deactivated as $L2'$. Therefore, we believe deactivating the unique consecutive two edges is a reasonable and reliable approach to deactivate a loop.

4 Application to the Simple Long Wave model

To demonstrate the proposed methodology with a more complex model, we test our approach on the Simple Long Wave model. This is a nonlinear economic model developed by Sterman (1985) to explain the long term economic cycles caused by capital self-ordering in the simplest possible terms. The model has three state variables (capital, supply and backlog), yet it is highly interconnected and contains 16 independent feedback loops. Its stock and flow diagram is shown in Figure 5. Appendix A contains the model equations.

4.1 Why we choose this model?

We choose this model because 1) both Kampmann (1996) and Ford (1999) had studied this model and reached the identical general conclusion with regard to which loops dominate the behaviour of variable *Desired Production*. Therefore we can take their result as a benchmark to test the validity of our loop deactivation approach; 2) though Ford used the behavioural method to analyze it where he deactivated all the feedback loops by fixing the values of their control variables but he did not provide a complete candidate loop set. A drawback is this would affect the validity of the choice of the control variable. In other words, when we deactivate one loop, we may simultaneously deactivate other loops as well. As a consequence, the changed behaviour may be caused by more than the loop as we think of; 3) under the assumption that the SILS is selected as the complete candidate loop set, some loops including the dominant loops (identified by the behavioural method from Ford and EEA from Kampmann) do not have a unique edge, thus we cannot use either Ford's behavioural method or GLDM analysis. Nevertheless, our loop deactivation approach is applicable and it is developed to enable the behavioural method under such a particular circumstance.

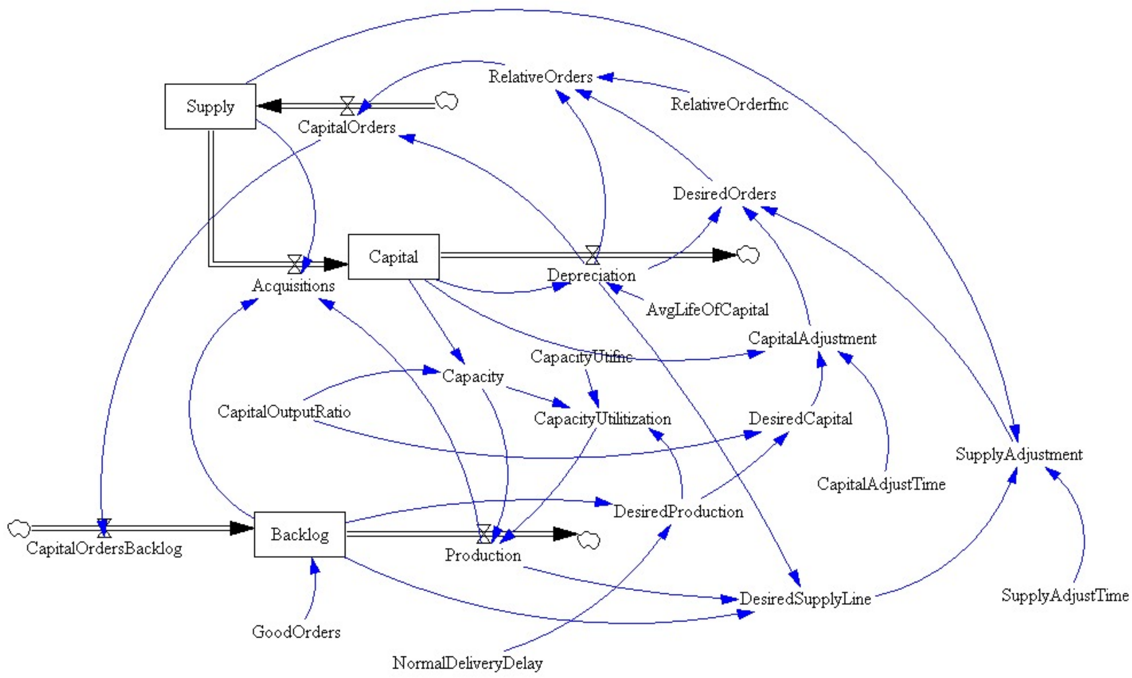


Figure 5: Stock and flow diagram – the Simple Long Wave model

4.2 Modify the model by the new loop deactivation method

Based on the behavioural method, the first step is to select the variable of interest. In order the compare our result with EEA, we will not randomly pick a variable of interest. We know that EEA is variable-independent, i.e., the dominant loops are the same for all the state variables in the model. However, in the behavioural method, different variables of interest give rise to different phase partitions. In terms of the similarities of the behaviour pattern in each phase and the durations of the corresponding phases (indicated by Table 4), we find those in EEA (Kampmann, 1996) coincide more closely with the variable of *Desired Production* than with others. Furthermore, Ford (1999) also chose it as the variable of interest in order to compare the behavioural analysis result with EEA more accurately. Therefore *Desired Production* is chosen here as well.

The next step is to set up the candidate loop set and identify the unique edges or the unique consecutive two edges. We use SILS algorithm to generate the candidate loop set (see Table 2 and Table 3). Then the unique edge for each loop has to be identified. Within SILS of the Simple Long Wave model, we find that four loops do not have unique edges. Table 2 and Table 3 list these two loop categories with their unique edges and the unique consecutive two edges respectively.

We participate the time intervals in the third step. The behaviour of *Desired Production* and its atomic behaviour pattern over the entire simulation are presented in Figure 6. We can see it is periodic and the cycle is approximately 45 years. The cycle in the middle is chosen for its clarity and completeness. We divide its behaviour into five phases based on the atomic behaviour pattern. Figure 7 plots the behaviour in that cycle and the value of the ABP according to Eq. 1. Specifically, we list the time intervals in Table 4 compared with the phase information in Kampmann's analysis.

Loop no.	Variable sequence **(Loop name)**	Unique edge
L1	Acquisitions, S **(Supply line-order control)**	Acquisition, S
L2	Depreciation, C **(Capital decay)**	Depreciation, C
L4	RelativeOrders, CapitalOrders, S, Acquisitions, C, Depreciation **(Steady state capital)**	Depreciation, RelativeOrders
L5	DesiredOrders, RelativeOrders, CapitalOrders, S, Acquisitions, C, Depreciation **(Capital replenishment)**	Depreciation, DesiredOrders
L6	RelativeOrders, CapitalOrders, S, SupplyAdjustment, DesiredOrders **(Supply line adjustment)**	S, SupplyAdjustment
L7	RelativeOrders, CapitalOrders, S, Acquisitions, C, CapitalAdjustment, DesiredOrders **(Capital adjustment)**	C, CapitalAdjustment
L8	Acquisitions, Capital, Depreciation, DesiredSupplyLine, SupplyAdjustment, Desiredorders, Relativeorders, CapitalOrders, S **(Steady-state Supply line)**	Depreciation, DesiredSupplyLine
L10	DesiredOrders, RelativeOrders, CapitalOrders, S, Acquisitions, C, Capacity, Production, DesiredSupplyLine, SupplyAdjustment **(Supply line adjustment via Production)**	Production, DesiredSupplyLine
L12	Acquisitions, C, Capacity, CapacityUtilization, Production **(Demand balancing)**	Capacity, CapacityUtilization
L13	DesiredProduction, CapacityUtilization, Production, B **(Production scheduling)**	DesiredProduction, CapacityUtilization
L15	RelativeOrders, CapitalOrders, CapitalOrdersBacklog, B, DesiredSupplyLine, SupplyAdjustment, DesiredOrders **(Hoarding)**	B, DesiredSupplyLine
L16	DesiredProduction, DesiredCapital, CapitalAdjustment, DesiredOrders, RelativeOrders, CapitalOrders, CapitalOrdersBacklog, B **(Capital Self-ordering)**	DesiredProduction, DesiredCapital
	NOTE: B-Backlog, C-Capital, S-Supply	

Table 2: Feedback loops in SILS – the Simple Long Wave model (a)

Loop no.	Variable sequence (Loop name)	Unique consecutive edges
L3	Acquisitions, C, Depreciation, CapitalOrders,S **(Capital expansion)**	Depreciation, CapitalOrders, S
L9	Acquisitions, C, Capacity, Production **(Economic growth)**	Capacity, Production, Acquisitions
L11	Acquisitions, C, Capacity, Production, B **(Order fulfillment)**	Production, B, Acquisitions
L14	Acquisitions, C, Depreciation, CapitalOrders, CapitalOrdersBacklog, B **(Backlog expansion)**	CapitalOrdersBacklog, B, Acquisitions

Table 3: Feedback loops in SILS – the Simple Long Wave model (b)

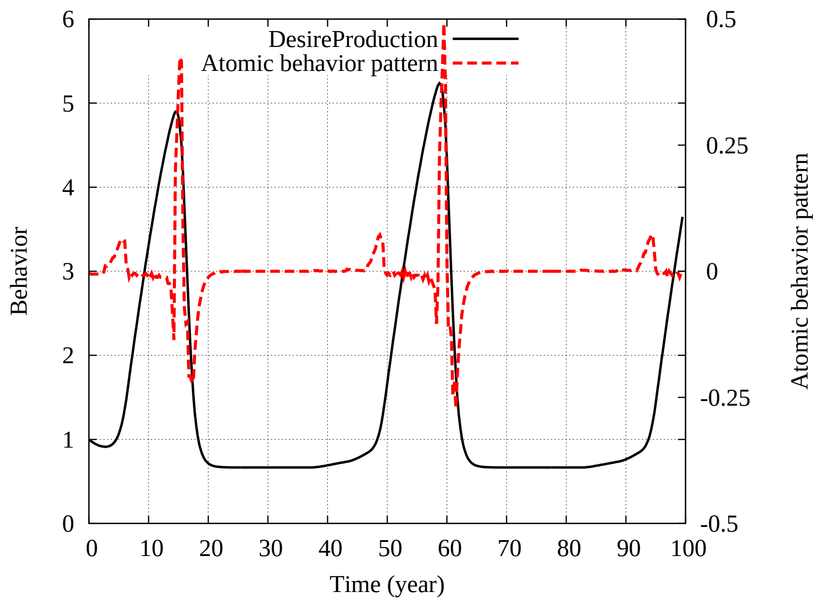


Figure 6: The behaviour of Desired Production

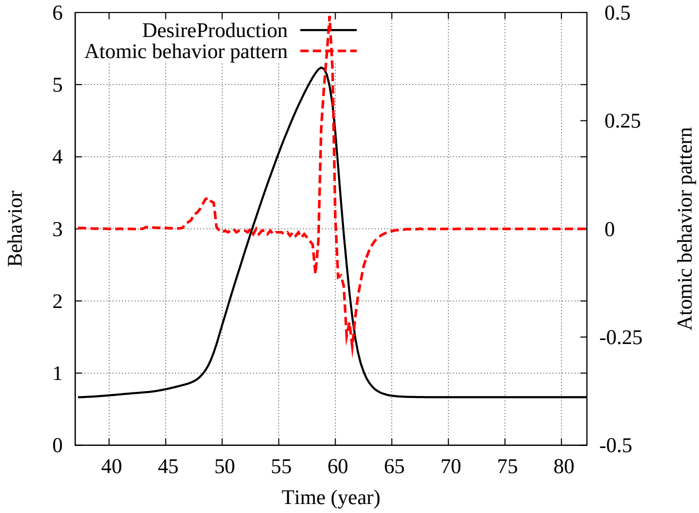


Figure 7: A cycle of Desired Production

Though the timing of shifts in dominance are different [2], the behaviour patterns concur with each other in each phase and the durations of the corresponding phases are close. One discrepancy occurs in the end of the cycle, we identify a fifth linear phase while Kampmann attributed it to be convergent. This can be attributed to their different definitions of the phase.

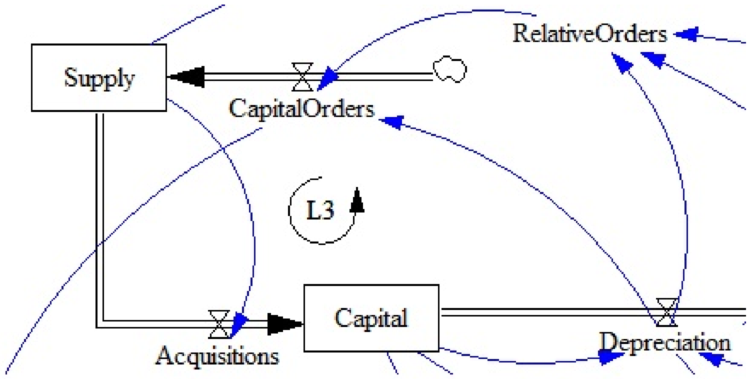
Phase name	I	II	III	IV	V
behaviour pattern	divergent	convergent	divergent	convergent	linear
Time interval *(ours)*	(46.5-49.25)	(49.50-58.5)	(58.75-60)	(60.25-69.0)	(69.25-90)
Time interval *(EEA)*	(50-53)	(54-62)	(63-66)	(67-95)	/

Table 4: Phases and behaviour pattern of Desired Production

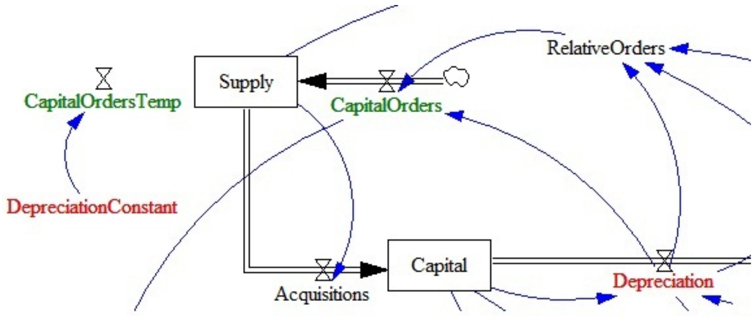
We will highlight the analysis on the loops that have no unique edges as other loops can be analyzed by the current behavioural method. Let us focus on the loops in Table 3 and demonstrate the new deactivation procedure step by step. *L*3 is first selected as the candidate loop. The consecutive unique edges are *Depreciation*→*CapitalOrder* and *CapitalOrder*→*Supply*. Its deactivation is carried out as follows:

1. Create two nodes which serve as the representatives of the two variables of the first edge in the unique consecutive edges. Two nodes are named *DepreciationConstant* and *CapitalOrderTemp* and correspond with *Depreciation* and *CapitalOrder* respectively. Link the new nodes with the same direction as their counterparts in the reference model. The screenshot of the added edge is shown by Figure 8(b).

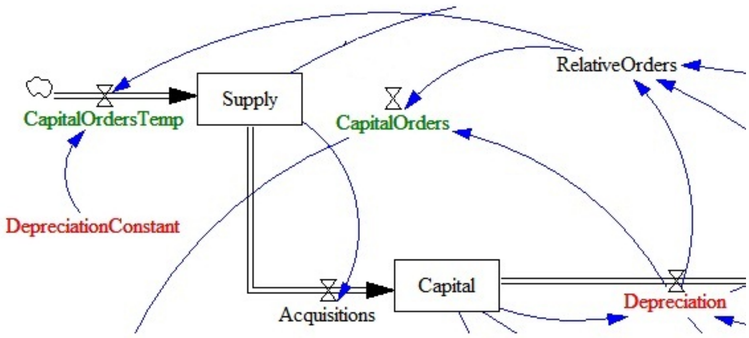
[2]The intervals in our analysis are approximately 4 years earlier than those in EEA, we can adjust our start time of the simulation 4 years behind the current start time to make the intervals match



(a)



(b)



(c)

Figure 8: Loop capital expansion ($L3$) is deactivated

2. Switch the tail of the edge from the middle variable in the unique edge pair to its representative while keeping the head of the edge unchanged. In this model, we delete $CapitalOrders \rightarrow Supply$ and add an edge $CapitalOrdersTemp \rightarrow Supply$ (see Figure 8(c)).

3. Add new edges starting from the variables which have links to variable in the middle of the unique edge pair (except the variable in the unique edge pair), and ending to the new representative of that middle variable. We identify only one edge pointing to *CapitalOrders* except *Depreciation*→*CapitalOrders*, i.e., *RelativeOrders*→*CapitalOrders*. Therefore, we add a new edge *RelativeOrders*→*CapitalOrdersTemp*. Figure 8(c) shows the finally modified model after deactivating $L3$.

The structure of $L3$ has been modified and the feedback from *Capital* to *Depreciation* is cut and represented by a new loop: $DepreciationConstant \rightarrow CapitalOrdersTemp \rightarrow S \rightarrow Acquisitions \rightarrow C$. Then we have to set *DepreciationConstant* which is the representative of the origin in the unique edge pair to be a fixed value and modify equations when simulating the model with $L3$ deactivated:

$$\begin{aligned} CapitalOrdersTemp &= DepreciationConstant * RelativeOrders \\ Supply &= INTEG(CapitalOrdersTemp - Acquisitions, Supply) \end{aligned}$$

where *DepreciationConstant* refers to the value of *Depreciation* at the deactivation time.

Repeated procedures are applied to deactivate $L9$, $L11$ and $L14$. The resulting model is illustrated in a set of graphs (Figure 10, 12, and 14 respectively). Meanwhile, the reference model which highlights the unique consecutive two edges is also displayed for the purpose of clarification. We will outline the deactivation procedure on these loops.

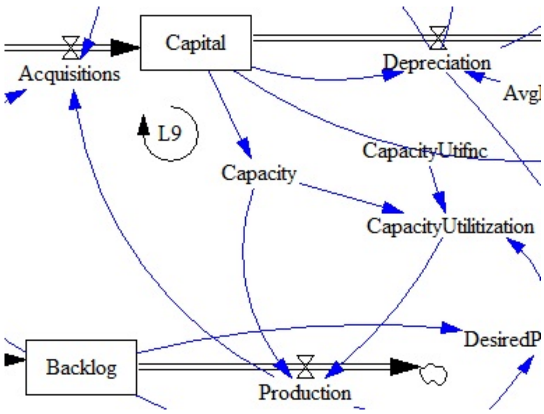
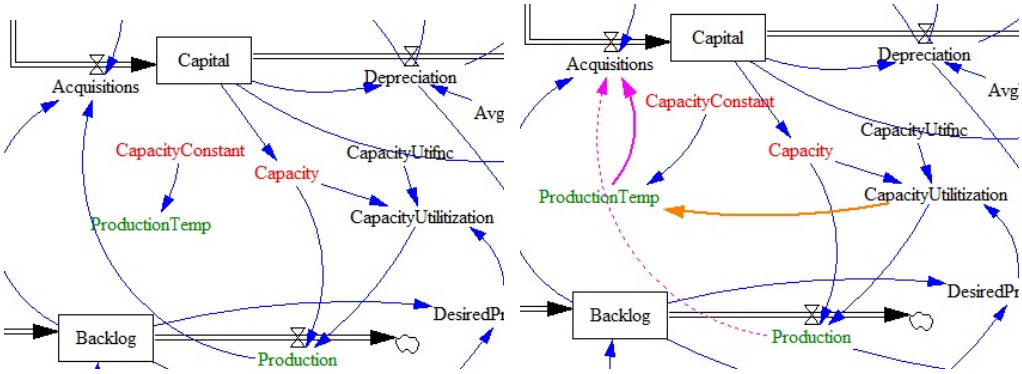


Figure 9: Highlight of loop economic growth ($L9$) in the reference model

The unique consecutive two edges in the economic growth loop $L9$ are *Capacity*→*Production* and *Production*→*Acquisitions*. Create two nodes (*CapacityConstant* and *ProductionTemp*) and link them in the same direction as their counterparts in the reference model (Figure 10(a)). We then remove the edge *Production*→*Acquisitions* (represented by the dash line) and replace it by adding the edge *ProductionTemp*→*Acquisitions* for deactivating $L9$ (Figure 10(b)). Finally, all the variables which have links to *Production* in the reference model are relinked to *ProductionTemp* except for the variable *Capacity* to recover other loops. Hence, one edge from *CapacityUtilitization* to *ProductionTemp* is added.



(a) (b)

Figure 10: Loop economic growth ($L9$) is deactivated

The changed equations in deactivating $L9$ are outlined below:

$$\begin{aligned} ProductionTemp &= CapacityConstant * CapacityUtilization \\ Acquisitions &= Supply * ProductionTemp/Backlog \end{aligned}$$

where *CapacityConstant* is the value of *Capacity* at the shifting time of each phase.

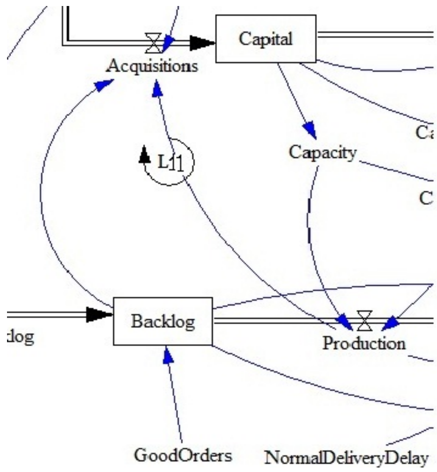
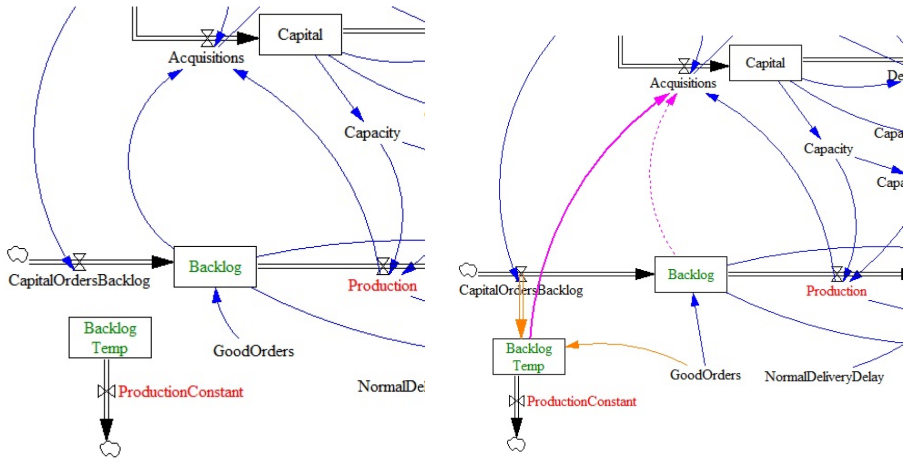


Figure 11: Highlight of loop order fulfillment ($L11$) in the reference model

The third loop order fulfillment $L11$ is selected to be deactivated. *Production*→*Backlog* and *Backlog*→*Acquisitions* are the unique consecutive edges which distinguish $L11$ from other loops in SILS. To avoid the deactivation of edge *Production*→*Backlog* affecting other loops, we maintain the original edge. However, in order to deactivate $L11$, we create a new edge *ProductionConstant*→*BacklogTemp* (Figure 12(a)) to work as a copy of *Production*→*Backlog*. Then, we have



(a) (b)

Figure 12: Loop order fulfillment ($L11$) is deactivated

to add "inputs" (i.e., the terms on the right hand side of equation *BacklogTemp*) to the new node *BacklogTemp*. On account of the fact that *BacklogTemp*'s counterpart is *Backlog*, we add the same "inputs" to *BacklogTemp* (except *Production* as *ProductionConstant* serves as an "input"). Finally, we link *BacklogTemp* to *Acquisitions* and delete the edge *Backlog*→*Acquisitions*.

Based on the above analysis, we need to change equations by:

$$\begin{aligned} BacklogTemp &= INTEG\,(CapitalOrdersBacklog + GoodsOrders - ProductionConstant, BacklogTemp) \\ Acquisitions &= Supply * Production/BacklogTemp \end{aligned}$$

where *DepreciationConstant* and the initial value of *BacklogTemp* refer to the values *Depreciation* and *Backlog* have at the deactivating time.

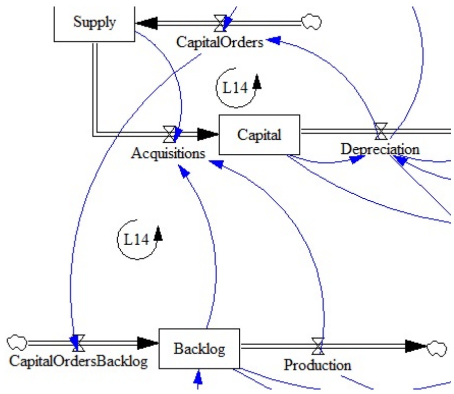
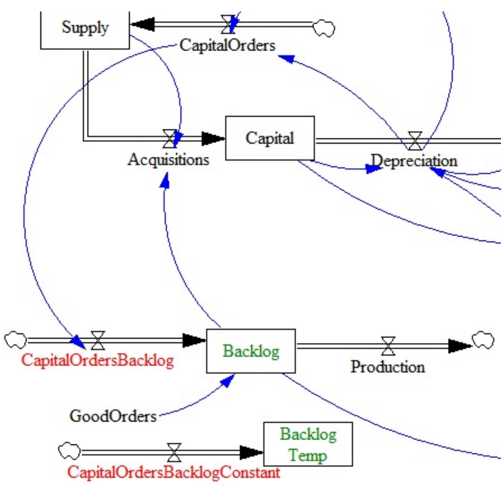
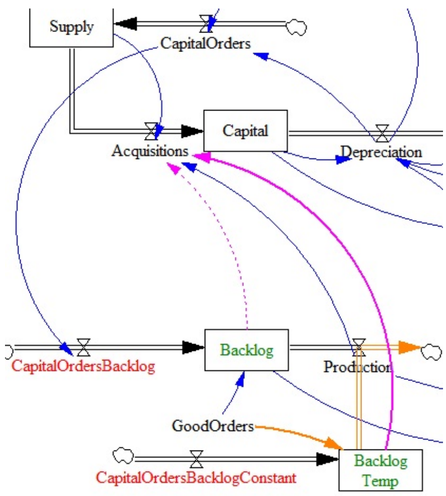


Figure 13: Highlight of loop backlog expansion ($L14$) in the reference model

The final loop is the backlog expansion loop ($L14$). Its unique consecutive two edges are *CapitalOrdersBacklog*→*Backlog* and *Backlog*→*Acquisitions*. A new constant *CapitalOrdersBacklog-Constant* is created to be a fixed inflow of *BacklogTemp* (Figure 14(a)). We add the other "inputs" of *Backlog* to *BacklogTemp*, i.e., *Production* and *GoodOrders*. Finally, this new substructure replaces *Backlog* in the input of *Acquisitions* in Figure 14(b). The deactivation of $L14$ is implemented by passing the constant inflow of *BacklogTemp* to *Acquisitions* without affecting the edge *CapitalOrdersBacklog→Backlog* in the reference model (Figure 13). Modifications are made



(a)



(b)

Figure 14: Loop backlog expansion ($L14$) is deactivated

according to the model in Figure 14(b) by the following equations:

$$\begin{aligned} BacklogTemp &= INTEG\,(CapitalOrdersBacklogConstant + GoodsOrders - Production, BacklogTemp) \\ Acquisitions &= (Supply * Production)/BacklogTemp \end{aligned}$$

where *CapitalOrdersBacklogConstant* refers to the value of *CapitalOrdersBacklog* at its deactivating time and the initial value of *BacklogTemp* is corresponding to *Backlog* at the same time.

4.3 Experiment result

Simulations on the modified model with the above four loops deactivated individually is conducted in phase I. We observe that none of their atomic behaviour patterns changes. This indicates that they are not dominant loops in this phase. Furthermore, deactivating the self-ordering loop ($L16$) gives rise to the change of behaviour pattern from divergent to linear. Therefore, $L16$ is considered to be the dominant loop in phase I. Examine the result from Kampmann's eigenvalue analysis and Ford's behavioural method in the same phase, we find our dominant loop test concurs with the their conclusion, i.e., we all agree that the self-ordering loop $L16$ contributes most to generating the exponential growth of *Desired Production* in phase I.

In phase II, EEA concludes the economic growth loop ($L9$) and capital expansion loop ($L3$) have the most significant influence on *Desired Production* growing increasingly slowly (49.50 - 58.5 year). Thus, we initially test $L9$. The modified model reduces the convergence of *Desired Production*, but still maintains a weak logarithmic growth. Then, the capital expansion loop ($L3$)

is chosen to test. The result is as same as $L9$. The atomic behaviour pattern sustains convergent over phase II, although its value is close to zero. Taking into account the possibility of forming a shadow loop pair, we deactivate $L3$ and $L9$ simultaneously. The behaviour pattern now changes from convergent to roughly linear and we can see the change of the behaviour pattern values in Figure 15. As it is a numerical simulation and the data has precision limitation, it is difficult to observe exactly linear behaviour, so our conclusion is derived based on the comparison with others, it is the most close to zero. Since the dominance test when both loops are inactive is different from when deactivated individually, we conclude that that the capital expansion loop dominates but has a shadow feedback loop (Ford, 1999): the economic growth loop. This result is not identical with Ford's analysis, which considers the economic growth is the only dominant loop. This is attributable to the way of deactivating the loop that differs. Specifically, deactivating $L9$ by its unique consecutive edges is weaker than by a unique edge. However, this is not a contradiction.

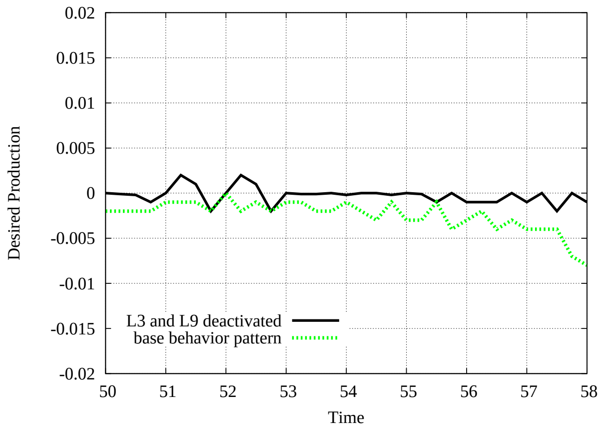


Figure 15: Comparison of atomic behaviour patterns in phase II

The precipitous decrease of *Desired Production* in phase III is 3 years in eigenvalue analysis while we identify it to be much smaller, i.e., 1.25 years. Ford (1999) obtained a similar result as ours where phase III consists of two phases, divergent and convergent. Eigenvalue analysis identifies the self-eroding loop ($L16$) as the dominant loop in the steep decline of *Desired Production*. The behaviour patterns are the same (divergent) when we deactivate the four loops. The dominance test over capital self-ordering loop ($L16$) results in linear and agree with the result with both Kampmann and Ford.

The gradual recovery of *Desired Production* characterizes phase IV. An identical analysis is conducted and the ABP does not indicate any dominant loops identified among these four loops. The behavioural analysis identifies deactivating the capital decay loop ($L2$) generates the convergent behaviour. This result reinforces Ford's and Kampmann's analysis on the initial part of phase IV. Moreover, in the late stage of phase IV, the behaviour pattern implies the demand balance loop ($L12$) gradually become a dominant loop.

Likewise, in the final phase, which is the late period of phase IV (approximately 83-95) in eigenvalue analysis, we repeat the dominance tests. The atomic behaviour pattern suggests the loop capital decay ($L2$) dominates phase V (the atomic behaviour pattern changes from linear to divergent) while the loop demand balancing ($L12$) is also dominant throughout this phase (divergent when loop $L12$ is deactivated). This result does not fully agree with others because the reference behaviour pattern disagrees.

To clearly present the dominant loops identified by different method, we organize them in Table 5. In summary, our method of deactivating a loop with no unique edge by the unique consecutive two edges concurs with the previous analysis using the same model. This helps us to establish confidence in the new loop deactivation approach.

Dominant loops	EEA	*behavioural method* (*Ford*)	*behavioural method* (*new*)
I	$L16$	$L16$	$L16$
II	$L3$, $L9$	$L9$	$L3$, $L9$
III	$L16$	$L2$, 16	$L16$
IV (V)	$L2$	$L2$	$L2$, $L12$

Table 5: Dominant loop analysis by different approaches

5 Conclusions and future work

Based on the work from Ford (1999) and Phaff (2008), we proposed a loop deactivation method by modifying its unique consecutive two edges. The Simple Long Wave model is chosen to demonstrate this method and test its validity. A main contribution of this paper is that we enhance the capability of the behavioural method by allowing it to be applicable when no control variable or no unique edge is identified in a loop (SILS is adopted as the candidate loop set). This improvement makes the behavioural method more robust and helps to automate it, as well as develop a systematic formal analysis software.

There are some limitations in the proposed approach: 1) When more than one pair of the unique consecutive edges are identified, it does not specify if using different pairs to deactivate the same loop would vary the atomic behaviour pattern. 2) It does not guarantee the loop deactivation is universally possible. A failure case is that the candidate loop cannot find even one set of the unique consecutive two edge. Though it is unlikely in most cases, it is a potential problem. 3) The test set is small. Only one example is given to compare the analysis result. It is in need of abundant models and test examples to validate this alternative loop deactivation approach.

Another drawback associated with the behavioural method is using the atomic behaviour pattern to determine whether it is a dominant loop. This works when the behaviour is convergent or divergent, but it becomes vulnerable when the behaviour is linear. It is because the simulation result contains some approximations and varies depend on different simulation tools. Furthermore, we may use different ways to process the data. Therefore, it is unlikely to get a series of continuous zeros for the values of the behaviour pattern. A possible way is to set a range for the linear behaviour pattern according to the application, within which we consider it is linear.

The issues we have stated are ideal avenues for the future research.

6 Acknowledgment

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A Equations for the Simple Long Wave model

timestep dt=0.25 years; start time=0.0; Euler integration method.

Stocks

Backlog = INT(CapitalOrdersBacklog+GoodsOrders-Production, NormalDeliveryDelay)
Supply = INT(CapitalOrders-Acquisitions,(Backlog/Production)*Depreciation)
Capital =INT(Acquisitions-Depreciations,
CapitalOutputRatio*AvgLifeOfCapital/(AvgLifeOfCapital-CapitalOutputRatio))

Flows

CapitalOrders = Depreciation*RelativeOrders
Acquisitions = Supply*Production/Backlog
Depreciation = Capital/AvgLifeOfCapital
CapitalOrdersBacklog = CapitalOrders
Production = Capacity*CapacityUtilitization

Auxiliaries

RelativeOrders = RelativeOrderfnc(DesiredOrders/Depreciation)
DesiredOrders = CapitalAdjustment+Depreciation+SupplyAdjustment
CapitalAdjustment = (DesiredCapital-Capital)/CapitalAdjustTime
Capacity = Capital/CapitalOutputRatio
DesiredCapital = CapitalOutputRatio*DesiredProduction
CapacityUtilitization = CapacityUtifnc(DesiredProduction/Capacity)
SupplyAdjustment = (DesiredSupplyLine-Supply)/SupplyAdjustTime
DesiredProduction = Backlog/NormalDeliveryDelay
DesiredSupplyLine = Depreciation*Backlog/Production
AvgLifeOfCapital = 20
CapitalOutputRatio = 3
CapitalAdjustTime = 1.5
SupplyAdjustTime = 1.5
NormalDeliveryDelay = 1.5
GoodOrders = 1
RelativeOrderfnc = {(-1,0),(-0.5,0),(0,0.2),(0.5,0.5),(1,1),(1.5,1.5),(2,2),(2.5,2.5),
(3,3),(3.5,3.5),(4,4),(4.5,4.4), (5,4.8),(5.5,5.2),(6,5.5),(6.5,5.65),(7,5.7),(7.5,5.75),(8,5.8),(40,6)}
CapacityUtifnc = {(0,0),(0.2,0.3),(0.4,0.6),(0.6,0.8),(0.8,0.9),(1,1),(1.2,1.03),(1.4,1.05),
(1.6,1.07),(1.8,1.09),(2,1.1)}