A platform to produce hybrid models in order to manage risks and uncertainties on financial and actuarial organizations[i]

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Abstract

Planning under uncertainty requires reliable tools and methods for dynamic financial analysis. A financial and actuarial problem is how to consider returns of investment forecasts and foresees based on uncertain incomes and liabilities. Lifetime and demographic studies focus on the population dynamics of a financial and actuarial model that has, among others, rates of mortality, withdrawal, disability and death that must be considered in costs assessments and cash flows. Thus , to adress asset and liability management (ALM) problem one must model stochastic liabilities based on stochastic assets to set more reliability to investment policies simulators. This articles analyses a combination of methods and techniques to better model uncertainty. Delphi technique, factor analysis and SD methods were used to identify factors, model causation between variables and to describe the structure and the behaviour of a financial and actuarial system. Stochastic processes and Monte Carlo simulation is used to manage probabilities distributions and to predict what future could be; and, in order to give to financial and actuarial analysts and actuaries a way to perform more realistic analysis of the solvency and liquidity, fuzzy logic and agent based modeling. The conclusion propose a framework where a combination of these techniques are possible and useful to integrate all these approaches and techniques and, on an open system perspective as a way to manage long term investment policy based on current liabilities, both stochastic in nature.

KEY WORDS: System Dynamics, uncertainty methods, fuzzy logic, agent based modeling, Strategic Planning, Delphi technique, factor analysis, Relevant information

Introduction

A dynamic ALM´s model problem is how to manage credit, market, operational and image risks to estimate returns over long-term investments based on uncertain liabilities. Thus, planning under uncertainty requires reliable tools to get better dynamic financial analysis and to manage actuarial assumptions in order to set policies that assure good solvency and liquidity to financial and actuarial funds.

Thus, combining SD methods to agent-based modeling applies simulation to cope some aspects of the social-economic and political environment under the financial and actuarial perspective, using fuzzy logic to model the behavior of socio-economic and political agents.

To place the issues into perspective, this paper has four sections. First, it observes the background to contextualize the theme. Next, discusses a preliminary framework to model financial and actuarial dynamics. Follows considerations about a detailed methodology and agent based modeling. Finally, in the conclusion, propose a framework where a combination of these techniques are possible and useful to integrate all these approaches and techniques and, on an open system perspective as a way to manage long term investment policy based on current liabilities, both stochastic in nature.

1 **Background**

Brazilian legislation states that a pension fund could implement defined benefits or defined contribution plans or a mixture of the two. Figure 1 demonstrate the dynamic of the accumulation of a pension fund that with the last participant must extinguish.

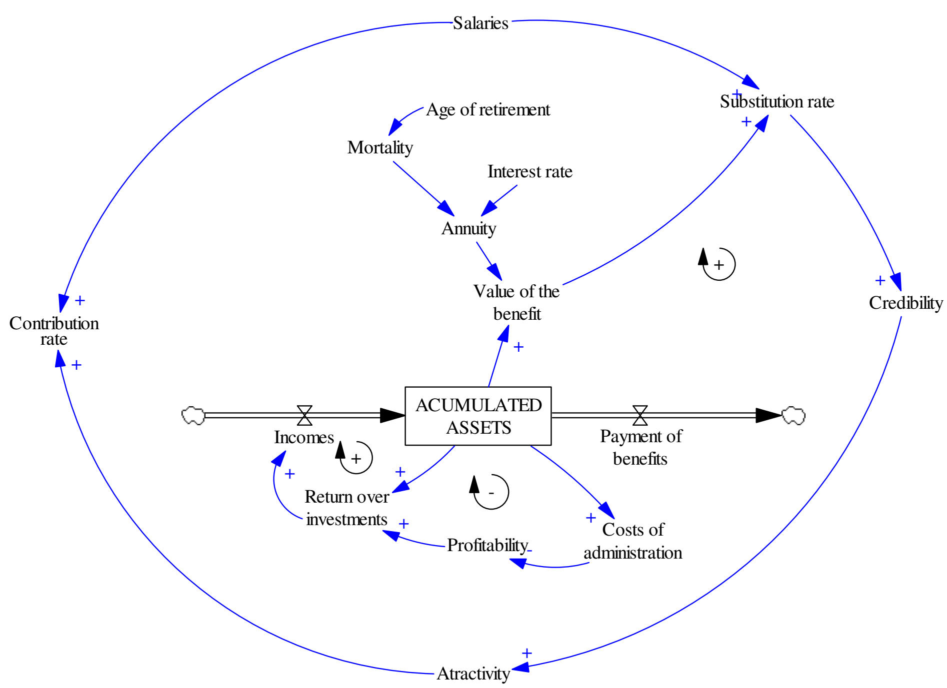


Figure 1: The accumulation dynamics of a generic pension fund.

There are two reinforcing loops because money generate more money and because the strength of the accumulation generates more aggregated value and thus more credibility, particularly if the funds are well administered. The balancing loop is due the costs of the pension system that reduces profitability and thus, subtract money from its stock.

Many actuarial assumptions are used to calculate pension costs and liabilities. Their dynamics includes various rates of decrements applicable to plan participants, future salary estimates in case of plans with benefits came from the salaries, and futures interest returns on plan assets.

The calculation comes from an income that be presumably used to be the reference to estimates of the values to pay and to receive by benefits of disability, retirement, withdrawal or by participant´s dependents. Longevity observed on the plan could generate more payments to a lesser and insufficient accumulation.

2 **A preliminary framework to model Financial and actuarial dynamics**

System Dynamics modellling (appendix 1) gives a way to address financial and actuarial problems, e.g., how to manage asset and liabilities for a financial organization (Chaim, 2006) and to explain phenomena and the structure of the system via stock and flow representation and the causation between factors. Because financial problems are a balance-sheet oriented approach, system dynamics gives the capability to better manage risk factors and policies to prevent agains losses and to maintain plan´s equilibrium.

A balance-sheet oriented methodology can help managers to better know the debt structure, the comprehension of business growth, the results of assets allocations and the wealth of the company. Balance sheet allows them to identify and analyze trends. A stochastic programming model for a financial and actuarial problem is dynamic since the information on the actual value of uncertain parameters is revealed in stages.

The agent-based approach can model the behavior of financial and actuarial participants, e.g, the populational dynamics of a financial and actuarial model and the socio-economical agents of the environment under analysis. The fuzzy logic was chosen to model the behavior of the agents. The information to model the behavior agents considers the integration of two qualitative research methods: (i) content analysis research; and, (ii) in-depth interviews.

They can provide data to model behavior by means of fuzzy logic rules and a systematic collection and interpretation of data produced in textual form as well as knowledge from experts, modeled by triangular distributions. Three rounds of Delphi technique gave conditions to structure the causality between relevant variables by factors obtained by many declarations of actuaries and practices statements from financial managers. The diagram (appendix 2) shows reinforcing loops as good solvency and as balancing loops there are credibility and the good wealth of the plan.

The content analysis is a research method that uses a group of procedures to validate inferences from a text (Weber, 1990). It is used to describe and to interpret the content of an entire class of documents and texts. The central idea of the content analysis is the classification of texts in categories, to reach a better understanding of their semantic contents. The words, sentences, text segments or other analysis units that are classified in the same category have similar or close meanings. The content analysis is used to reveal the attitudes and behaviours of the agents that interact in the context of the system under analysis.

When the content analysis is finished the main elements and assumptions required for the conceptual model design have been mapped. However, the model still needs to be verified in order to revise the assumptions and to fill the informational gaps that can remain. The conceptual model refining is accomplished by in-depth interviews with specialists that act in the system's environment of analysis.

The in-depth interview is a data collection technique that is closer to a conversation than to a formal and structured interview (Marshal and Rossman, 1989). It is a no-structured interview, in which a single respondent is tested by one interviewer in relation to a subject (Malhotra, 1999). It is convenient that an in-depth interview should also be carried out before the content analysis for the initial characterization of the environment to be modeled.

The process of carrying out the actions of the agents is based on fuzzy logic. The agent's beliefs are defined on the antecedent term of the fuzzy rules (IF side), while the term relative to the agent's deliberation is found on the consequent side (THEN side). The main definition step of the model is associated with the selection of the production rules to model the agents' behavior. For instance, the fuzzy rule "IF inflation is high AND inflation variation increases THEN exert moderate pressure for interest rate reduction " indicates, for example, that there is an agent belief that inflation is high and, also, there is a tendency towards increased inflation. Then, the agent deliberation will exert a moderate pressure into another agent for an interest rate reduction. The value resulting from the pressure will depend on the degree of truth of the input variables 'inflation' and 'inflation variation' to the fuzzy sets 'high' and 'increases', respectively.

The notion of a fuzzy set was introduced by Zadeh (1965 apud Rizzi et al., 2003, p. 365), in the decade of 60. The objective is to represent mathematically uncertainties and to supply formal tools to deal with the inherent imprecision of many problems. The main idea is the revision of the classical theory of the sets. The traditional way of representing elements u of a set A is through the characteristic function (Kasabov, 1998):

$\mu_A(u) = 1$, if u is an element of set A, and
$\mu_A(u) = 0$, if u is not an element of set A,

that is, an object u either belongs or does not belong to a given set. In fuzzy sets theory an object can belong to a set partially. The degree of membership is defined through a generalized characteristic function called membership function: $\mu_A(u): U \rightarrow [0 \quad 1]$, where U is the universe and A is a subset of U. The values of the membership function are real numbers in the interval $[0 \quad 1]$, where 0 means that the object is not a member of the set and 1 means that it belongs entirely.

The fuzzy logic has been considered useful when the process (system under analysis) is difficult to forecast or model using traditional methods (Mohammadian and Kingham, 2004). This paradigm allows the modeling of complex systems by the use of simple rules that are defined with linguistic variables and terms. The fuzzy logic is versatile because it allows the modeling and manipulation of vague and inexact information mathematically.

A traditional econometric model based on statistical methods is used for the estimates of the model's 'external context parameters'. The main goal is the modeling of the macroeconomic variables and the rules that orchestrate the market dynamic. Therefore, in order to perform long term predictions, an econometric model is employed to estimate the macroeconomic variables of future cycles. The use of an econometric model at the structural level helps to understand the results of the simulations.

The integration of a consolidated econometric model to the SD and multiagent models increases the explanatory power of the model as a whole. The agent-based model is responsible to model the interactions among agents, their behaviors and the populational dynamics (micro level). The SD can aid to model an integrated cash flow based on the populational dynamics (agent model) and the external influences (econometric model), while the econometric model is responsible for the economic estimates at the structural level of the model (macro level). Econometric models have tradition in predicting and simulating economic phenomena.

The choice of the econometric model can be based on two basic criteria: (i) simplicity to implement it computationally; and, (ii) didactically informative to understand its macroeconomic estimates. The main target of the model is the scenarios creation and cause and effect relationships in the economic environment from the macroeconomic variables.

During the agent and SD modeling, variables and messages should be chosen and defined for the integration of the models from different paradigms. Since the behavior of the agents is modeled by fuzzy logic, some of the macroeconomic variables of the econometric model will be defined as fuzzy variables. These variables are used in the production rules that define the action of the agents (IF-THEN rules).

As we can see at figure 3, the three simulation models work together. The econometric model defines the economic environment by means of macroeconomic estimations for each simulation cycle. The multiagent model executes and interacts based on the economic scenario and the SD model condition, e.g., financial and actuarial condition. Also, the SD model executes one simulation cycle based on the environment and behavior of the agents (population dynamic). In this version of the model we are not considering the interaction with the econometric model, e.g., the use of data from the multiagent model and SD model as an input to the econometric model to influence the macroeconomic estimations for the next period.

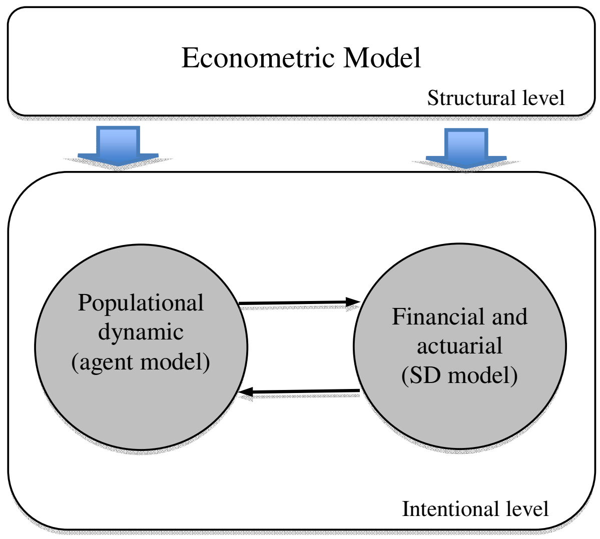


Figure 3: Econometric, agent-based and SD integrated model

Solvency risks is concerned with the long-term ability to pay pensions to participants. It may be simulated by the integration of the three models at figure 3 to programe recursive "reward" alghoritms and thus calculate automatically the utility of each stage.

The system must behave based on objectives and by the optimazation of costs estimates, uncertain assets and liabilities though the join of these two aspects lead to a difficult to implement this king of algorithm. Then, the objective could be a function where the sum of all rewards are maximum and expressed by the following function:

$$R_T = E\left[\sum_{\tau=1}^{T} \gamma^{\tau} r_{t+\tau}\right]$$

Where $r_{t+\tau}$ represents the value of the reward in time, t | τ between the time periods t and $t+T$. The factor γ represents one discount factor that belongs to the interval [0;1] and which value is a parameter of the problem. For instance if the γ factor is low, the reward get lower, giving priority to other interactions on the model.

4 The detailed Methodology

Risk management in a financial and actuarial organization is a decision problem. As an external factor, interest rates can give insights over the decisions and to comprehend the behavior of the system over a fixed value or by a probability distribution that could explain it.

(a) SD Approach

Follows a diagram constructed over a system dynamic approach, after identifying factors with delphi technique perspective .

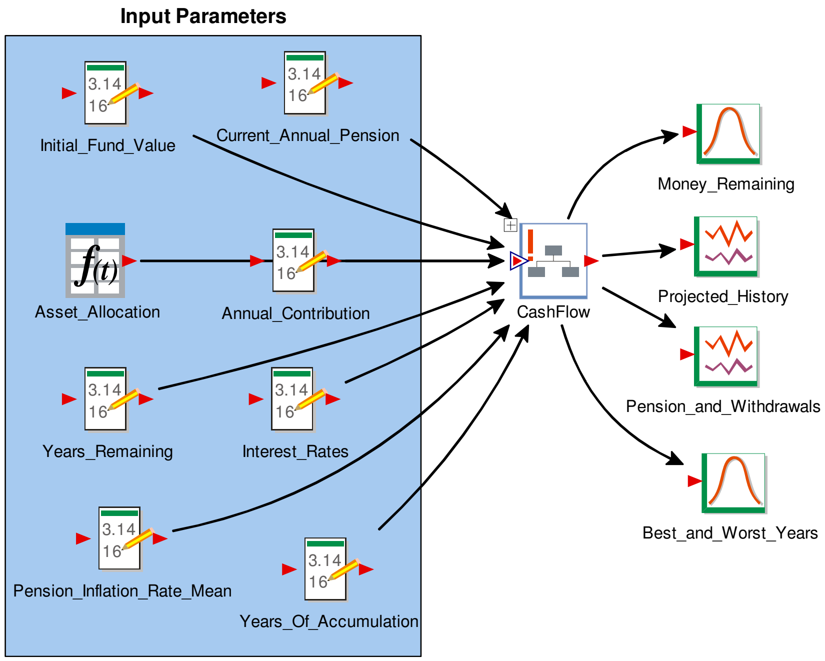


Figure4: Dashboard of a system dynamic model to adress ALM problem

Figure 4 shows a financial and actuarial model that will generate a prospective cash flow and conclusions based on a fixed interest rate.

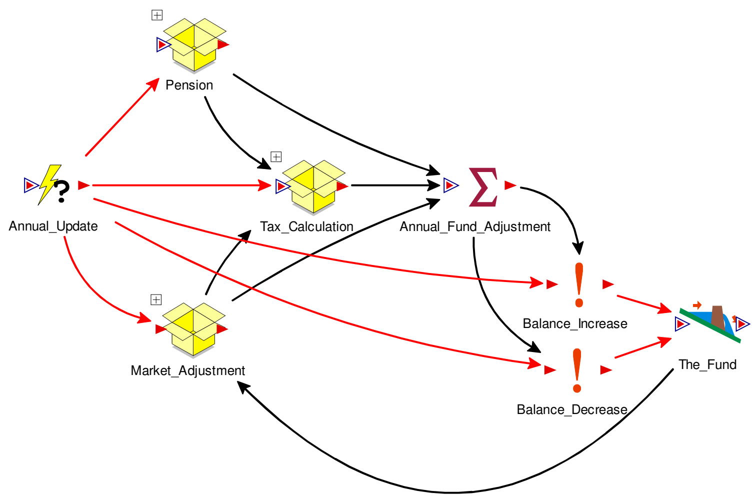


Figure 5: A detailed SD Model to calculate assets and liabilities over time

Triggered by a timed event, changes in fund value are updated annually. Sub-models aid to calculate the effects of fund changes due to market conditions, pensions paid, and many taxes.

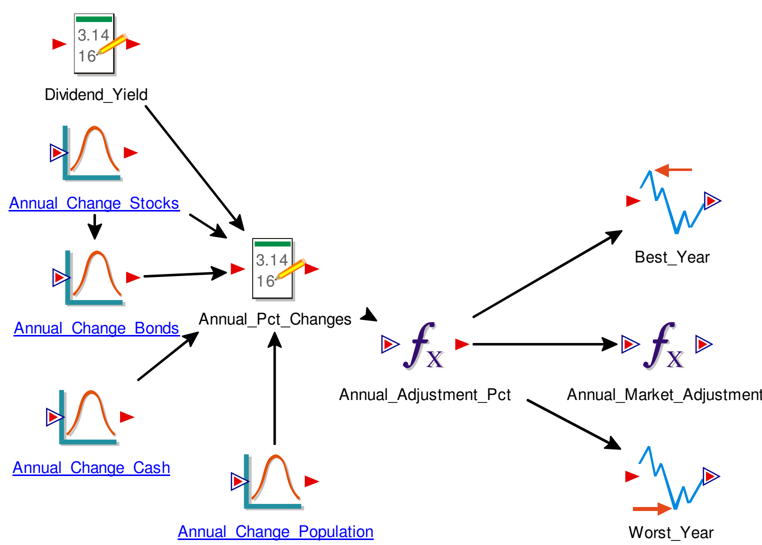


Figure 6: A stochastic approach of ALM model

As may be shown by figure 6, annually there is many adjustments for each asset category and populational parameters due to market factors. The estimates can be enhanced with reliable data on long-term returns for assets and populational data and with many correlations between asset categories.

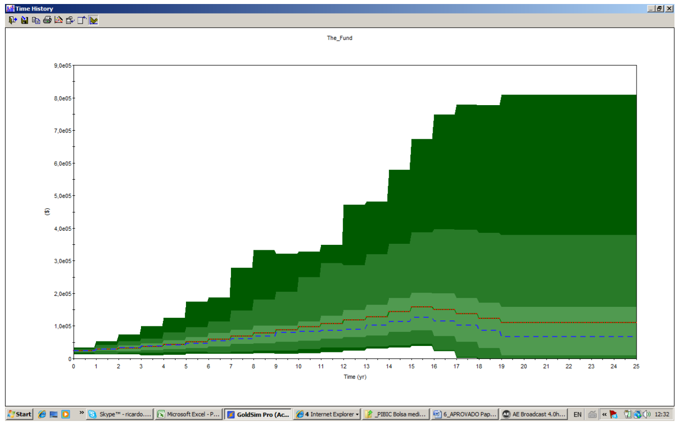


Figure 7: A stochastic result simulated by a SD model

The figure shows the impacts of many factors over the fund. Because the long-term nature of financial institutions, if interest rates are not well estimated, it may cause important distortions over future solvency and liquidity. Another approach must be considered to better calíbrate this variable.

(b) Fuzzy logic and agent based modelling approach combined to Kooiman's econometric model

In order to model interest rates, the research will combine fuzzy logic, agent based modelling and econometric formulations with SD approach to better foresee interest rates impacts over the fund.

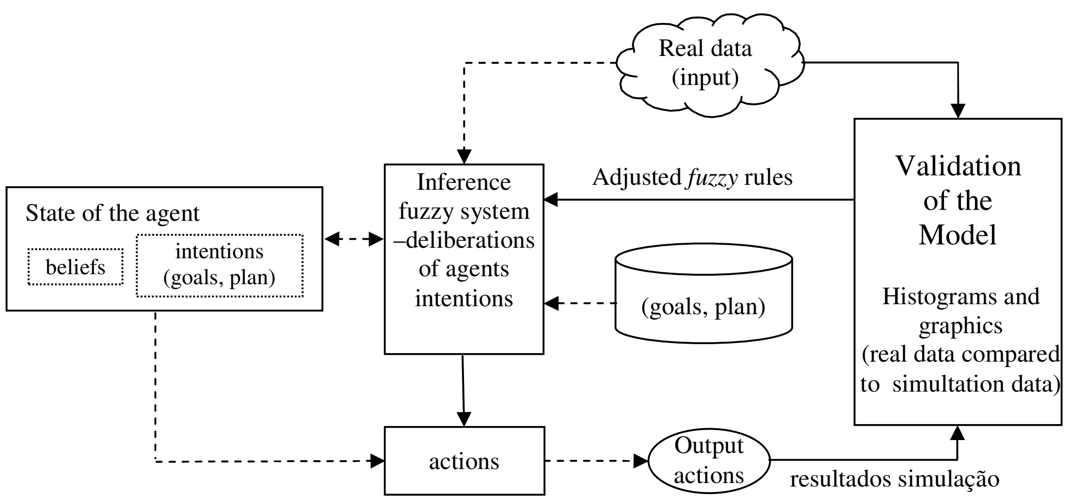


Figure 8: Models adjustment and validation

Figure 8 shows that fuzzy rules is useful to adjust the model in order to better imitate reality. Techniques proposed by Wu et al. (2003) e Ali e Zhang (2001) must be considered to model membership functions. An econometric model must be considered as a way to model agents behaviours. Real data aid to optimize parameters that will lead them to expected behaviour over time.

The identification of fuzzy rules was based on knewspapers publications. Figure 9 shows the results after a simplification analisys process.

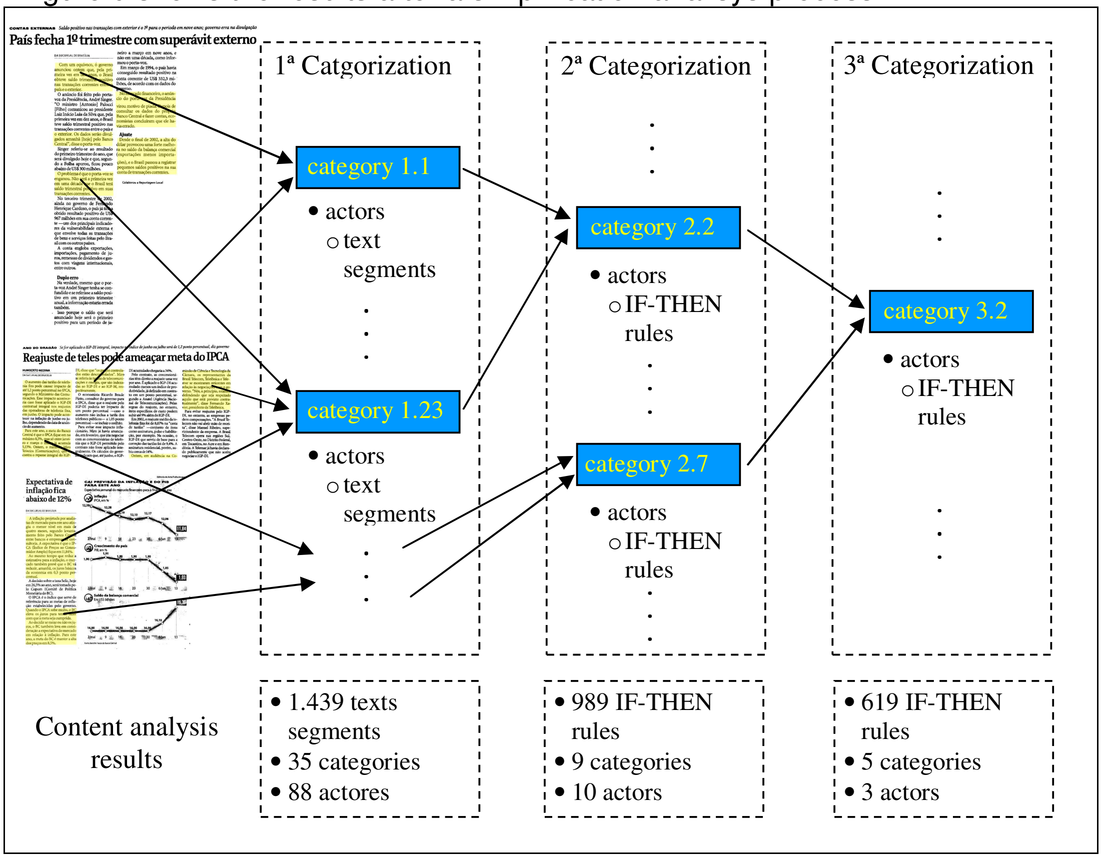


Figure 9: Identifying fuzzy rules from a content analisys approach

After defining fuzzy rules and variables that will influence agents behavior,

It was necessary to decide about an econometric model that could aid to estímate interest rates.

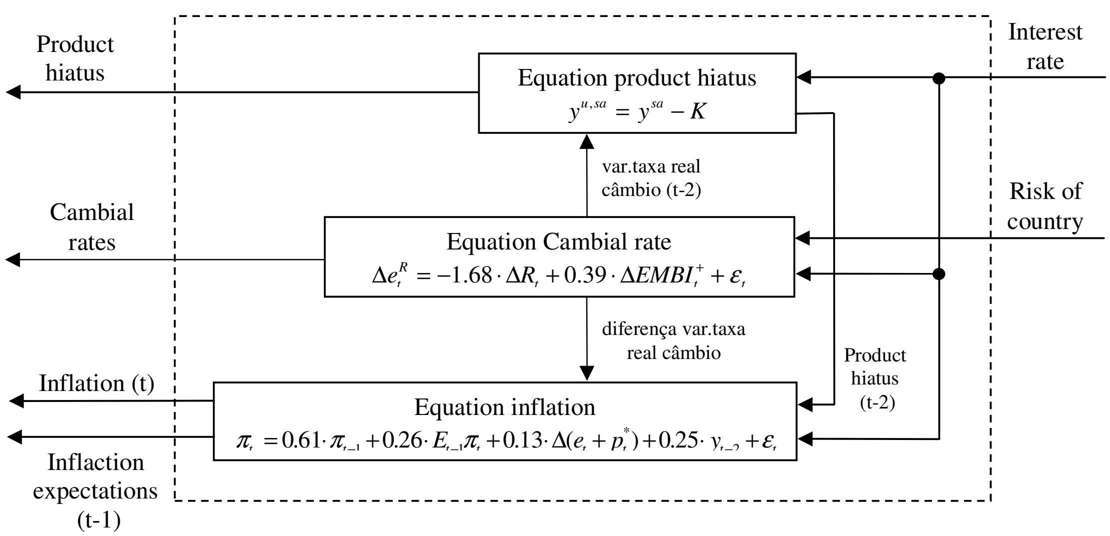


Figure 10: Kooiman's econometric model

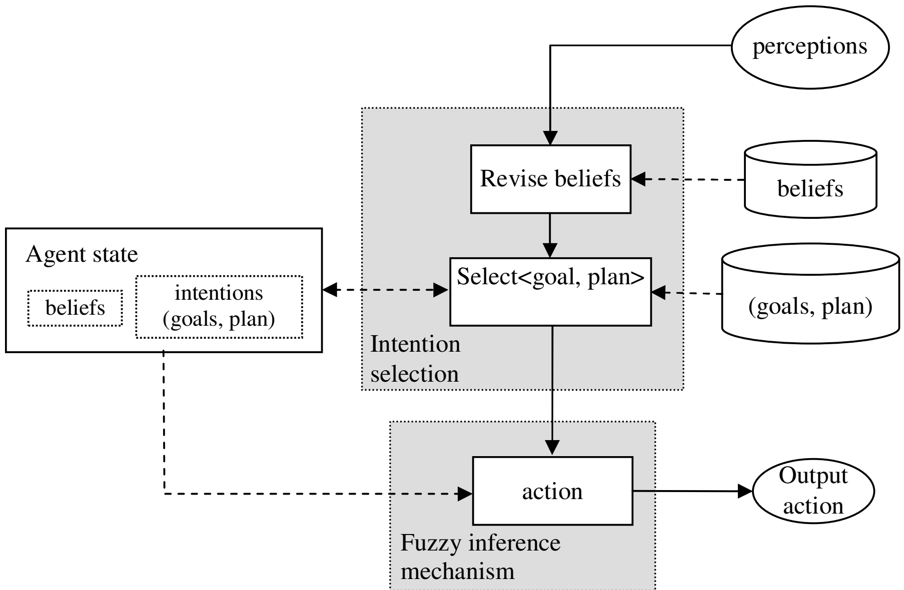


Figure 11: Fuzzy scheme

Figure 11 shows that intentions are based on fuzzy logic. This process is circumscribed on models components and is called fuzzy inference system. In this case, fuzzy rules carry out agents actions over the conception of architecture BDI (beliefs-desires-intentions). Agents beliefs are defined in the antecedent terms of fuzzy rules (IF part), while in the consequent part there is the term related to agent deliberation. Fuzzy rule "IF inflation *high* AND variation of inflation *arise* THEN pressure reduction of interest rates" indicates, for instance, that if there is a belief by the agent that inflation is becoming high and there is a trend of it becoming higher, the deliberation of the agent may be to pressure other agents, particularly "monetary autority" to reduce taxes.

5 Agent Based Modeling

As stated by Chaim and Streit (2008), financial and actuarial schemes on a financial institution context are complex systems so interactions among many components may cause relevant differences in system's performance.

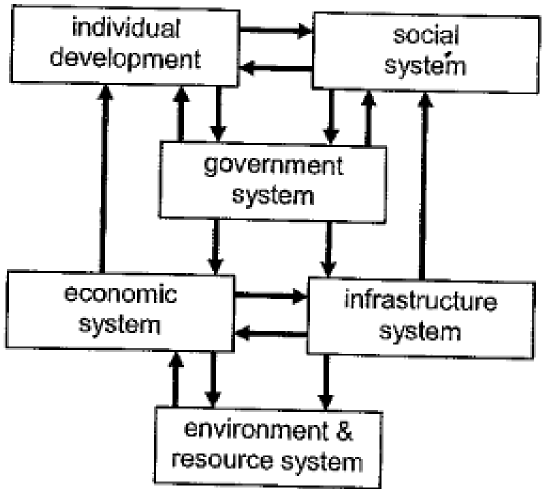


Figure 2 : Six sectors of a societal system and their major relationships
Source: Bossel (2007)

In order to reduce the number of indicators, it seems to Bossel (2007, p. 253) permissible the following aggregation:

Socio-political subsystem: social system+individual development + government system

Support subsystem: infrastructure + economic system

Natural subsystem: resources + environment (BOSSEL, 2007, p. 253)

Particularly to socio-political subsystems, Edmonds (2003) stated that when a study domain is quite complex, approaches based on equations or on other analytical techniques are impracticable or even impossible to be applied. Therefore, Chaim and Streit (2008) proposed the use of agent-based models to represent the behavior of the financial and actuarial participants and the social-economic and political environment to provide deeper insights by simulation experiments.

In order to cope with the peculiarities of pension funds, a type of financial institution, the authors considered the use of an agent-based model to represent the behavior of pension fund participants and the social-economic and political environment to provide deeper insights by simulation experiments. The agent-based models can help to clarify agents' interactions and behaviors (micro level), e.g., the non-linear behaviors of the system that are difficult to be captured with mathematical formalisms.

In their study, Chaim and Streit (2008) combined an agent-based model with a SD model to study pension funds. The agent models are distinguished for relating the heterogeneous behavior of the agents (different information, different decision rules, and different situations) with the macro behavior of the system (Lempert, 2002). The agents have several interaction rules and, by simulation, it is possible to explore the emergent behavior along the time and the space. This modeling technique does not assume a unique component that takes decisions for the system as a whole. Agents are independent entities that establish their own goals and have rules for the decision making process and for the interactions with other agents. The agents' rules can be sufficiently simple, but the behavior of the system can become extremely complex (Gilbert, 1995).

Figure 1 presents a conceptual model to study financial and actuarial governance. Streit (2006) developed this model for regulatory governance analysis of sectors under regulation. The conceptual model is generic and, consequently, it is useful to structure different scenarios.

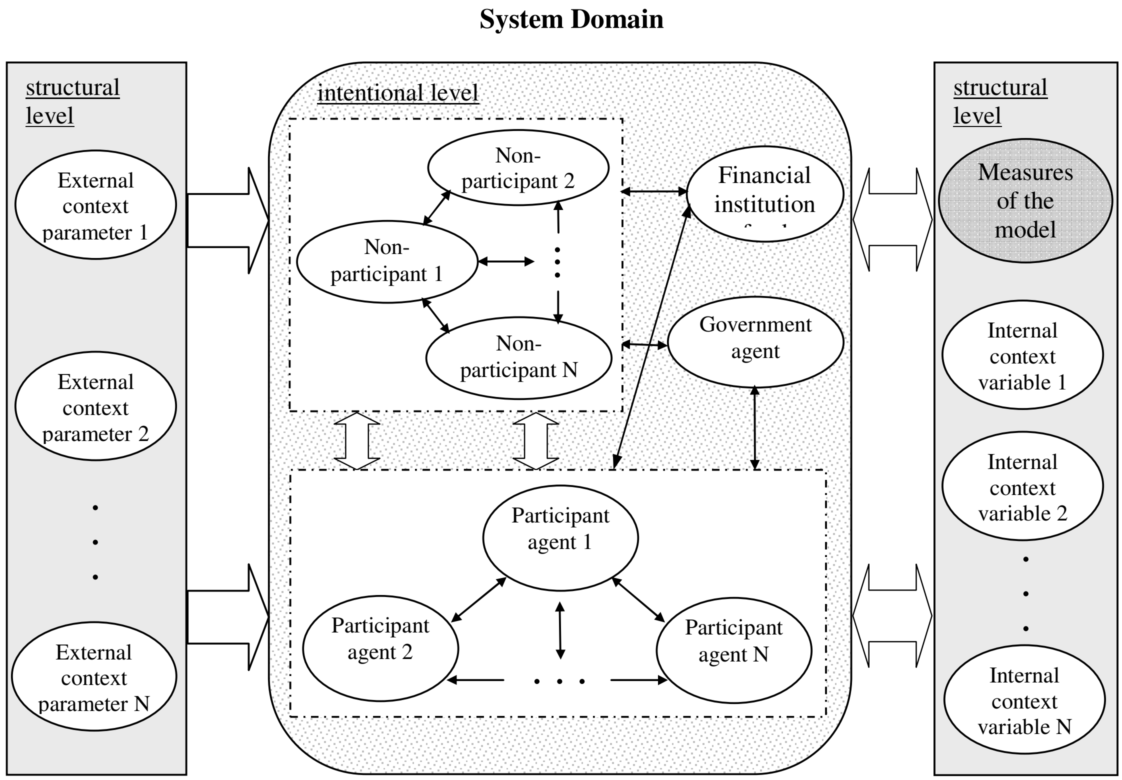


Figure 1 – Generic conceptual model to study financial and actuarial governance.
Source: Chaim & Streit (2008)

Simulation along the time is the strategy to analyze the emergent phenomenon of the model. The intentional level (action level), where the interactions among the agents occur, is differentiated from the structural or contextual level. The structural level indicates the contexts where the interactions happen, e.g., the circumstances that limit, amplify and determine the interactions among the agents and with the environment. Moreover, structural level is the level where the emergent phenomenon takes place. It is a higher level comparing to the intentional level where the agents interact. The basic principle that guides the model is that all interactions have an intention or a set of intentions. For a better understanding, follows the main components of this generic conceptual model:

- Measures of the model: they are the results of the model that make possible the study of the phenomenon for which the model is developed;

- External context parameters: indicate external aspects that may contextualize the model. They are not influenced from the behavior of the model (unidirectional arrow from the structural level to the individual level), but they can influence the interactions among the agents who act in the intentional level of the model (example: international indicators). The external context parameters define the external environment of the system under analysis;
- Internal context variables: represent important external aspects. These variables influence the interactions among the agents and are influenced by them. Thus, during the simulations, the values of these variables are modified depending on the interactions at the intentional level of the model. They express, in its totality, relative external situations of the system under analysis in the environment;
- Government agent: it represents the government at the individual level of the model. The government agent defines the regulation policies of the financial institution;
- Participant agents: they represent the agents who participate in a financial institution. The participant agents are directly influenced by the regulations and situations that impact the Economy as a whole. The amount of “participant agents” in the model will depend on the type of analysis and abstraction desired;
- Non-participant agents: they interact with other agents at the individual level, but they do not participate in a financial institution. The non-participants agents are indirectly influenced by the regulations of the sector and they can indirectly influence the agents who regulate the sector.

The “internal context variables” and the “external context parameters” belong to the structural level of the model (macro level).

The main stage of the agent-based model is the definition of the rules to model agents’ behavior. The criteria that can be used to the rules delimitation is based on the variables used in the dynamic model and the agent-based model. Figure 2 presents the main components of the model and discuss some simulation techniques.

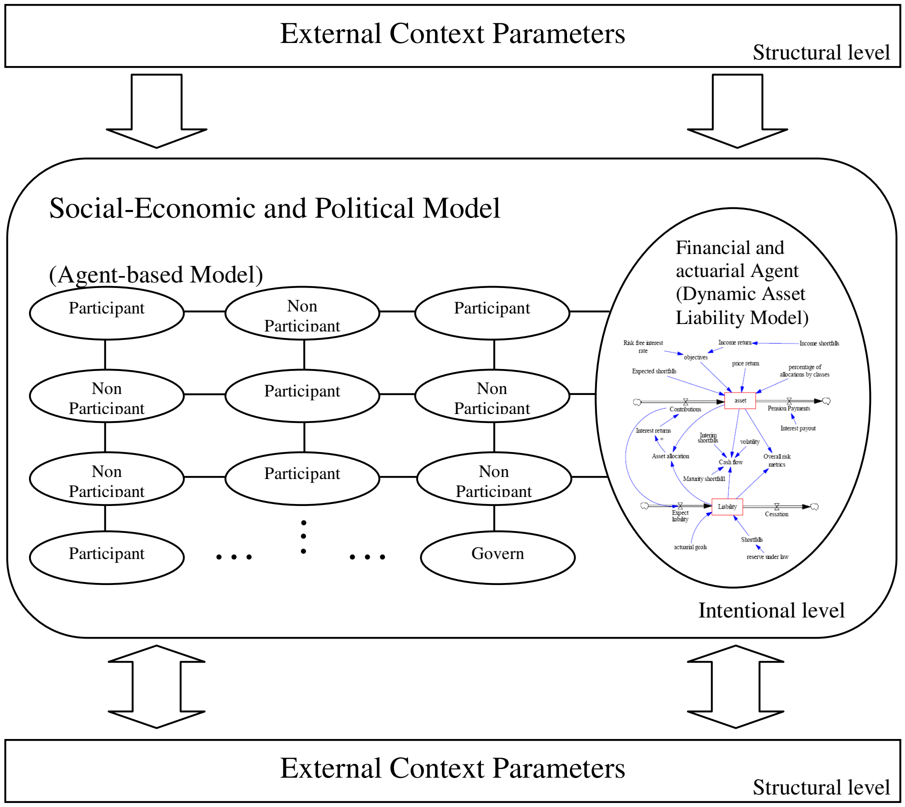


Figure 2: Conceptual and integrated model for financial institutions
Source: Chaim and Streit, 2008

To obtain probability distributions and other information by the interactios of agents cope with the complexity of financial and actuarial systems, one can use many simulation approaches integrated like system dynamics, agent-based and discrete event simulation - econometric model (appendix 3).

4 Conclusions

The multi-paradigm approach is suitable to model sociotechnical factors involved in an ALM problem. Under SD techniques recommendations it was possible to identify the complexity and to characterize many aspects over the problem being modeled, also to model subjective factors and to simulate the complexity of financial and actuarial systems considering their risks and uncertainties and to demonstrate theoretical constructs. The research is multidisciplinary and interdisciplinary by nature and this article presents part of the literature review and methodological strategy to develop the research.

The modeling process of an agent-based model defines its individual components, as a bottom-up approach. The definition of the agents' behaviors is extremely important for a good representation of a pension scheme. Besides, there must be a very good equivalence between the system under analysis and the conceptual model to guarantee great consistency to the agent-based model and reliability from the simulation results.

Lifetime and demographic studies focuses on the population dynamics of a financial and actuarial that has, among others, rates of mortality, withdrawal, disability and retirement that must be considered in assessing costs and to consider credibility in structuring a prospective cash flow. This way, the research is being conducted by the authors and it combines methods and techniques to study financial and actuarial populational models and the influence of subjective factors over it. It is projected to combine structural model and internal model to better imitate the real system.

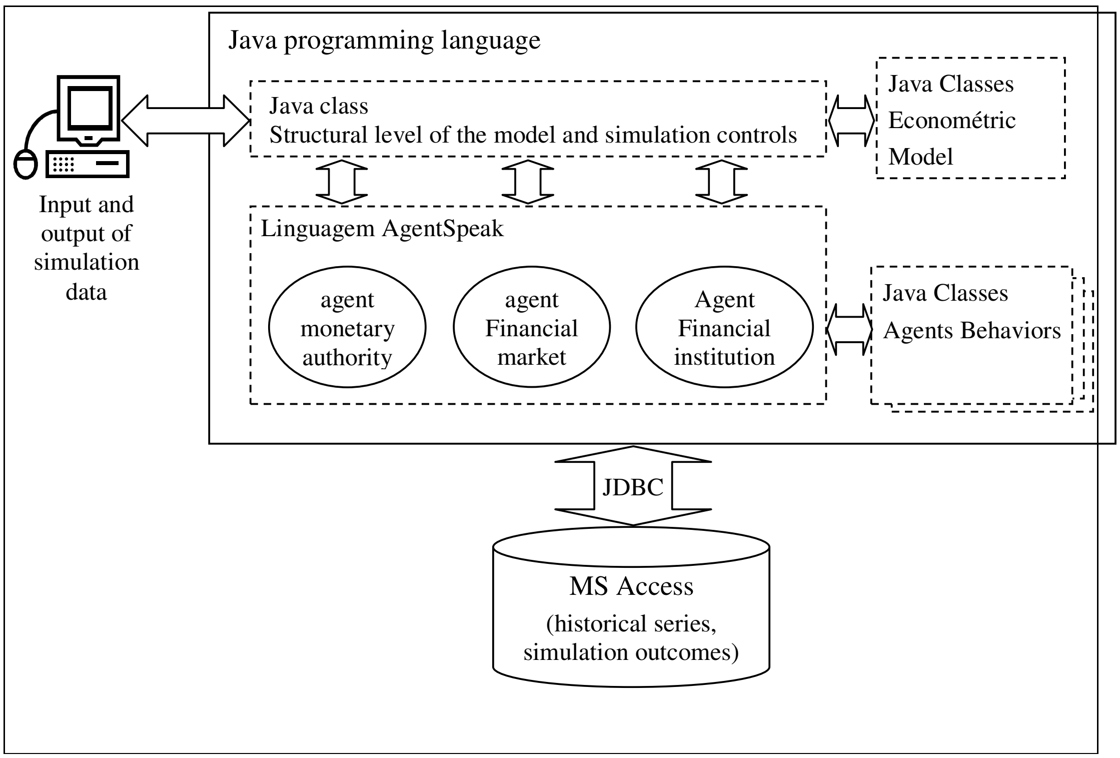


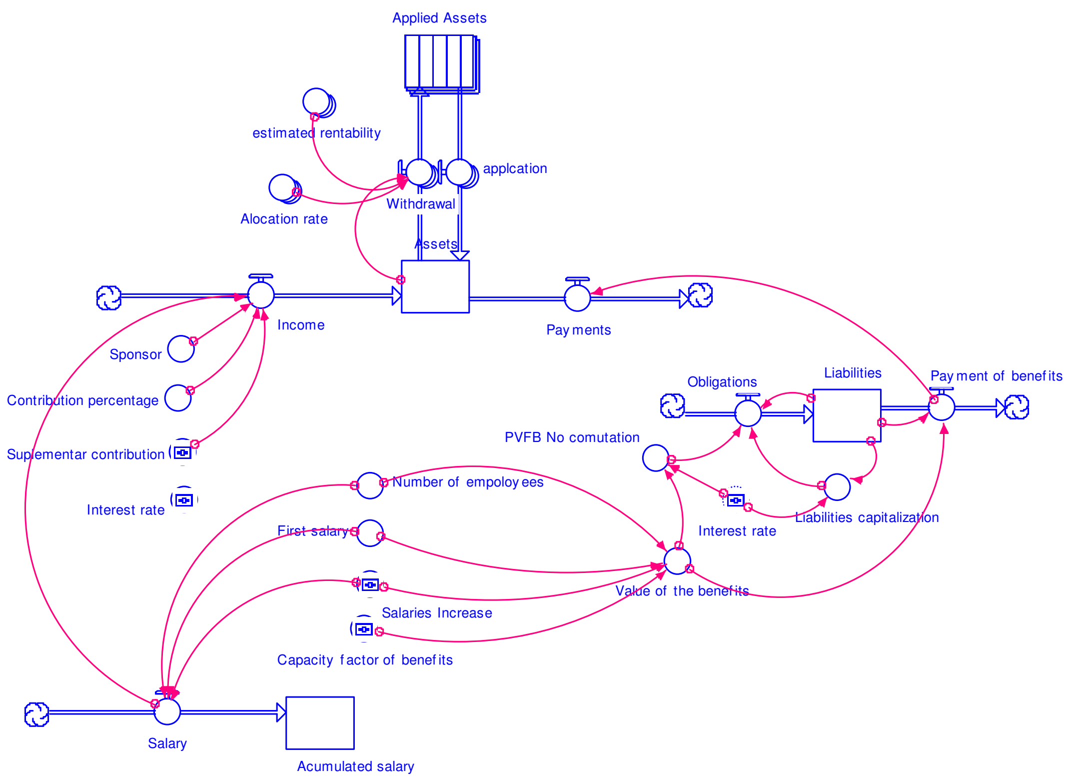
Figure 11: The combination of many technologies and approaches

The research is in progress and figure 11 shows the planned approaches combination to get a multi-paradigm approach on modeling interest rates indexes and their impact over a financial institution. The authors identified the main actors and the methodology to proceed the modeling recommendations identified on the literature review. The software to be produced will consider ages, mortality, withdrawal and mortality rates, assets, liabilities, investments and many other factors from the database of an important Brazilian financial and actuarial company.

Finally, as mentioned by Lee & Padmanabhan (2004), there are evidences that irregular fluctuations on estimative could be derived not only by stochastic fluctuations, but could be related to the own deterministic managing of these subjective factors primarily related to how financial and actuarial organizations manage their market logistics, their rules, policies and capacities. This way, the research must include Bayes Statistics, markov chains and chaotic analysis to better comprehend the probabilistic nature of financial and actuarial decisions and to produce simulators that could imitate more precisely decisions made by a financial and actuarial organization.

APPENDIX I

This model shows an invividual perspective of a particular member and the evolution of the assets and actuarial liabilities over time. Follows formulas and other relations, particularly a comutation table used to better estimate mortality, withdrawals, disability and other probabilities relevant to a financial and actuarial organization asset and liability management problem.



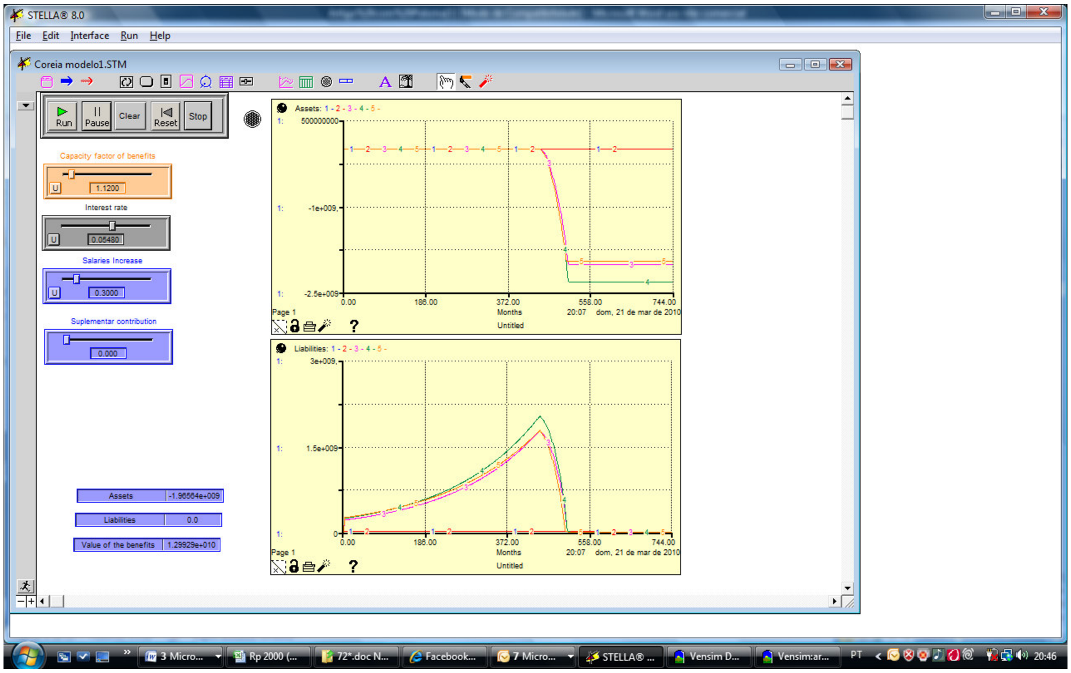
Acumulated_salary(t) = Acumulated_salary(t - dt) + (Salary) * dt
INIT Acumulated_salary = 0
INFLOWS:
Salary = if time > 444 then 0 else
if time = 0 then Number_of_empoloyees*First_salary else
if mod (time,12) = 11 and time = 11 then
Number_of_empoloyees*(2*((First_salary*(1+Salaries_Increase)^int(time/12)))) else
if mod (time,12) = 11 and time > 12 then Number_of_empoloyees*(2*((First_salary*(1+Salaries_Increase)^int(time/12)))) else Number_of_empoloyees*(First_salary*(1+Salaries_Increase)^int(time/12))
Applied_Assets[Alocations](t) = Applied_Assets[Alocations](t - dt) + (Withdrawal[Alocations] - applcation[Alocations]) * dt
INIT Applied_Assets[Alocations] = 12
INFLOWS:
Withdrawal[Disp_Cash] = Alocation_rate[Disp_Cash]*Assets*(1+estimated_profitability[Disp_Cash])
Withdrawal[Stocks] = Assets*Alocation_rate[Stocks]*(1+estimated_profitability[Stocks])
Withdrawal[Real] = Alocation_rate[Real]*Assets*(1+estimated_profitability[Real])
Withdrawal[Real_state] = Alocation_rate[Real_state]*Assets*(1+estimated_profitability[Real_state])
Withdrawal[FI_RF] = Alocation_rate[FI_RF]*Assets*(1+estimated_profitability[FI_RF])
Withdrawal[FI_RV] = Alocation_rate[FI_RV]*Assets*(1+estimated_profitability[FI_RV])

Withdrawal[Borrow_part] = Alocation_rate[Borrow_part]*Assets*(1+estimated_profitability[Borrow_part])
Withdrawal[Fin_Realstate] = Alocation_rate[Fin_Realstate]*Assets*(1+estimated_profitability[Fin_Realstate])
Withdrawal[Debentures] = Alocation_rate[Debentures]*Assets*(1+estimated_profitability[Debentures])
Withdrawal[Tit_Public] = Alocation_rate[Tit_Public]*Assets*(1+estimated_profitability[Tit_Public])
Withdrawal[Others] = Alocation_rate[Others]*Assets*(1+estimated_profitability[Others])
Withdrawal[OpSponsor] = Alocation_rate[OpSponsor]*Assets *(1+estimated_profitability[OpSponsor])
Withdrawal[13] = { Place right hand side of equation here... }
OUTFLOWS:
applcation[Alocations] = CONVEYOR OUTFLOW
Assets(t) = Assets(t - dt) + (Income + applcation[Alocations] + applcation[Disp_Cash] + applcation[Stocks] + applcation[Real] + applcation[Real_state] + applcation[FI_RF] + applcation[FI_RV] + applcation[Borrow_part] + applcation[Fin_Realstate] + applcation[Debentures] + applcation[Tit_Public] + applcation[Others] + applcation[OpSponsor] + applcation[13] - Payments - Withdrawal[Alocations] - Withdrawal[Disp_Cash] - Withdrawal[Stocks] - Withdrawal[Real] - Withdrawal[Real_state] - Withdrawal[FI_RF] - Withdrawal[FI_RV] - Withdrawal[Borrow_part] - Withdrawal[Fin_Realstate] - Withdrawal[Debentures] - Withdrawal[Tit_Public] - Withdrawal[Others] - Withdrawal[OpSponsor] - Withdrawal[13]) * dt
INIT Assets = 0

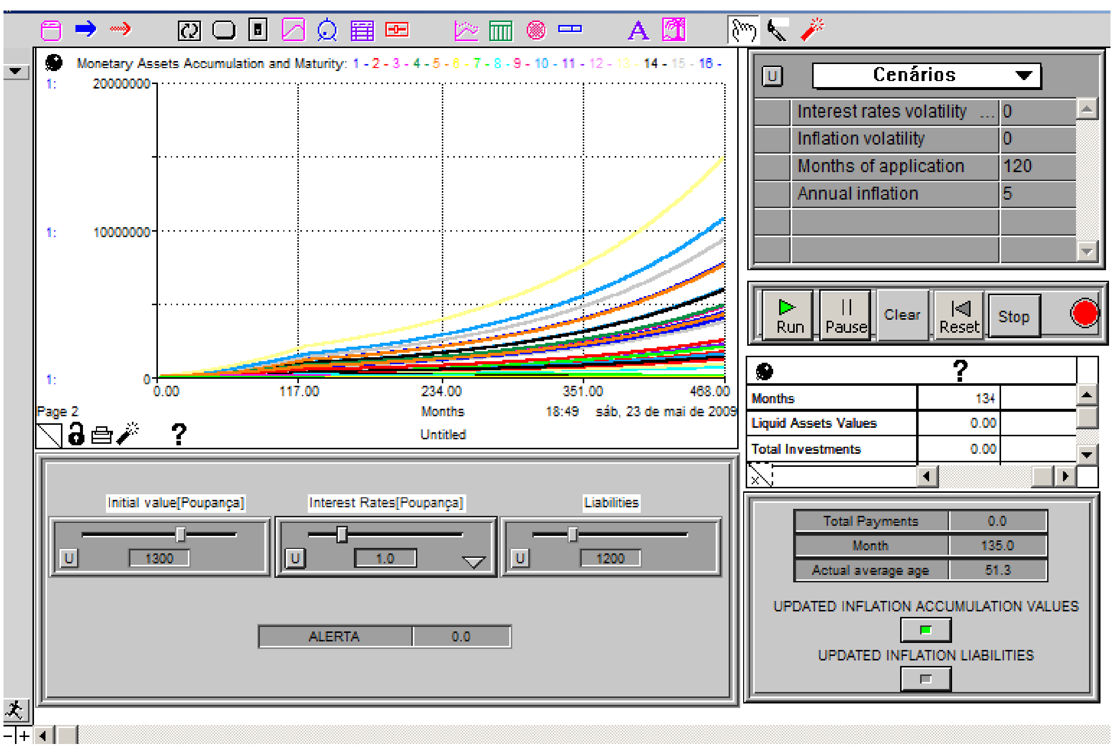
INFLOWS:
Income = if time > 444 then Suplementar_contribution else
((Salary*Contribution_percentage) + (Sponsor*(Salary*Contribution_percentage)) + Suplementar_contribution)
applcation[Alocations] = CONVEYOR OUTFLOW
OUTFLOWS:
Payments = Payment_of_benefits
Withdrawal[Disp_Cash] = Alocation_rate[Disp_Cash]*Assets*(1+estimated_profitability[Disp_Cash])
Withdrawal[Stocks] = Assets*Alocation_rate[Stocks]*(1+estimated_profitability[Stocks])
Withdrawal[Real] = Alocation_rate[Real]*Assets*(1+estimated_profitability[Real])
Withdrawal[Real_state] = Alocation_rate[Real_state]*Assets*(1+estimated_profitability[Real_state])
Withdrawal[FI_RF] = Alocation_rate[FI_RF]*Assets*(1+estimated_profitability[FI_RF])
Withdrawal[FI_RV] = Alocation_rate[FI_RV]*Assets*(1+estimated_profitability[FI_RV])
Withdrawal[Borrow_part] = Alocation_rate[Borrow_part]*Assets*(1+estimated_profitability[Borrow_part])
Withdrawal[Fin_Realstate] = Alocation_rate[Fin_Realstate]*Assets*(1+estimated_profitability[Fin_Realstate])
Withdrawal[Debentures] = Alocation_rate[Debentures]*Assets*(1+estimated_profitability[Debentures])
Withdrawal[Tit_Public] = Alocation_rate[Tit_Public]*Assets*(1+estimated_profitability[Tit_Public])
Withdrawal[Others] = Alocation_rate[Others]*Assets*(1+estimated_profitability[Others])
Withdrawal[OpSponsor] = Alocation_rate[OpSponsor]*Assets *(1+estimated_profitability[OpSponsor])
Withdrawal[13] = { Place right hand side of equation here... }
Liabilities(t) = Liabilities(t - dt) + (Obligations - Payment_of_benefits) * dt
INIT Liabilities = 0

INFLOWS:
Obligations = if time = 0 then PVFB_No_comutation else
if Liabilities <= 0.00 then 0 else Liabilities_capitalization
OUTFLOWS:
Payment_of_benefits = if time <= 444 then 0 else

```
if  Liabilities <= 0 then 0 else
if Liabilities < Value_of_the_benefits then Liabilities else Value_of_the_benefits
Alocation_rate[Disp_Cash] = 0.165405
Alocation_rate[Stocks] = 0.156014987565378
Alocation_rate[Real] = 0.030789385457671
Alocation_rate[Real_state] = 0.00868845449717989
Alocation_rate[FI_RF] = 0.356637645106099
Alocation_rate[FI_RV] = 0.0800482400560538
Alocation_rate[Borrow_part] = 0.0146975353021157
Alocation_rate[Fin_Realstate] = 0.00645911153188553
Alocation_rate[Debentures] = 0.00891737186117745
Alocation_rate[Tit_Public] = 0.0931745698143681
Alocation_rate[Others] = 0.0124265709979127
Alocation_rate[OpSponsor] = 0.000187296025088908
Alocation_rate[13] =  { Place right hand side of equation here... }
Capacity_factor_of_benefits = 0.98
Contribution_percentage = 0.08
estimated_profitability[Disp_Cash] = 0
estimated_profitability[Stocks] = 0.1189
estimated_profitability[Real] = 0.1147
estimated_profitability[Real_state] = 0.068
estimated_profitability[FI_RF] = 0.0680
estimated_profitability[FI_RV] = 0.1189
estimated_profitability[Borrow_part] = 0.062
estimated_profitability[Fin_Realstate] = 0.062
estimated_profitability[Debentures] = 0.1189
estimated_profitability[Tit_Public] = 0.068
estimated_profitability[Others] = 0.05
estimated_profitability[OpSponsor] = 0.05
estimated_profitability[13] =  { Place right hand side of equation here... }
First_salary = 1000
Interest_rate = 0.06
Liabilities_capitalization = if Liabilities <= 0.00 then 0 else Liabilities*((1+Interest_rate)^(1/12)-1)
Number_of_empoloyees = 1
PVFB_No_comutation = if time = 0 then Value_of_the_benefits*((1-((1+Interest_rate)^-25))/Interest_rate) else 0
Salaries_Increase = 0.02
Sponsor = 1.0
Suplementar_contribution = 0
Value_of_the_benefits = if time <= 444 then Number_of_empoloyees*First_salary*((1+Salaries_Increase)^int(444/12))*Capacity_factor_of_benefits else if mod (time,12) = 11 then Number_of_empoloyees*(2*((First_salary*(1+Salaries_Increase)^int(time/12))))*Capacity_factor_of_benefits else
Number_of_empoloyees*First_salary*((1+Salaries_Increase)^int(time/12))*Capacity_factor_of_benefits
```



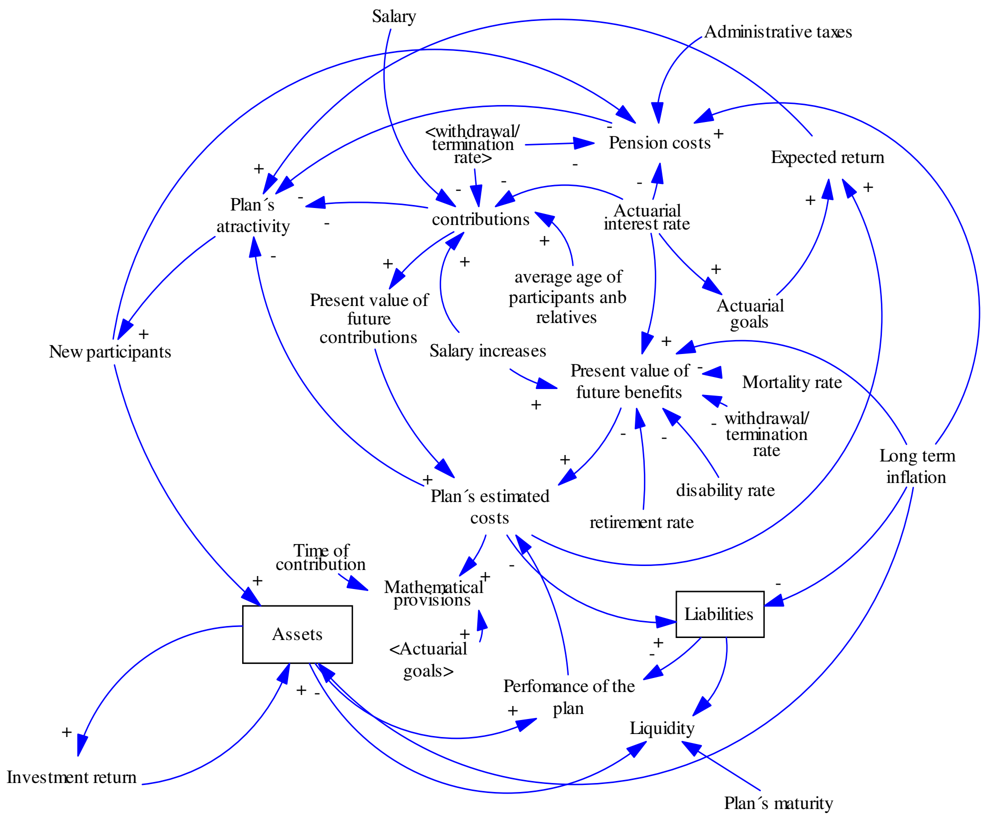
The graph shows assets and liabilities values over time and many factors that could be changed to test different policies and their consequences over the system as a whole.



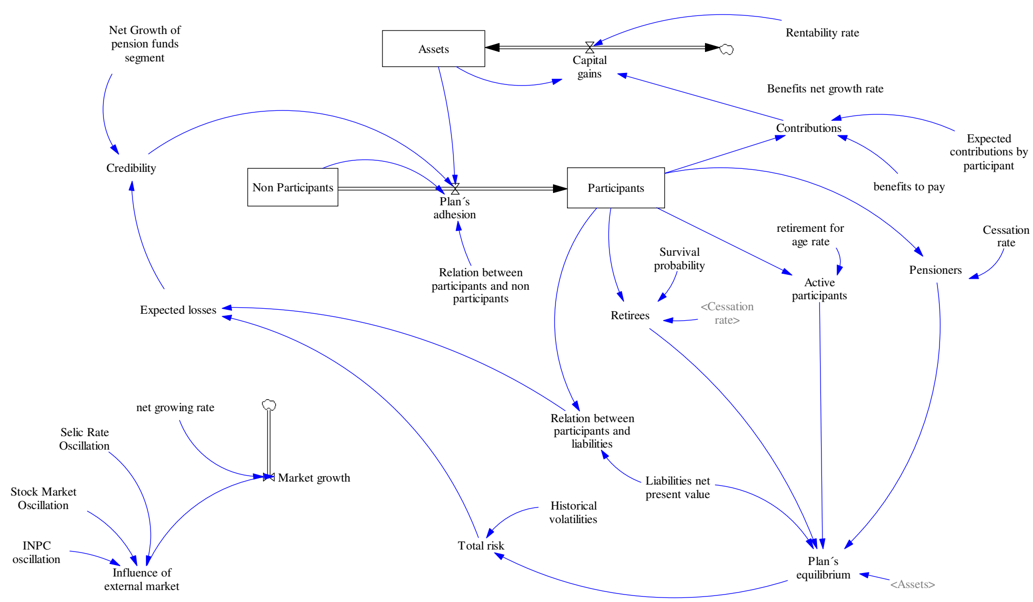
Monte carlo simulation is set by the information of parameters as mean and standard deviation to factors like inflation or interest rates and to analyze their impact over the system as a whole.

APPENDIX 2

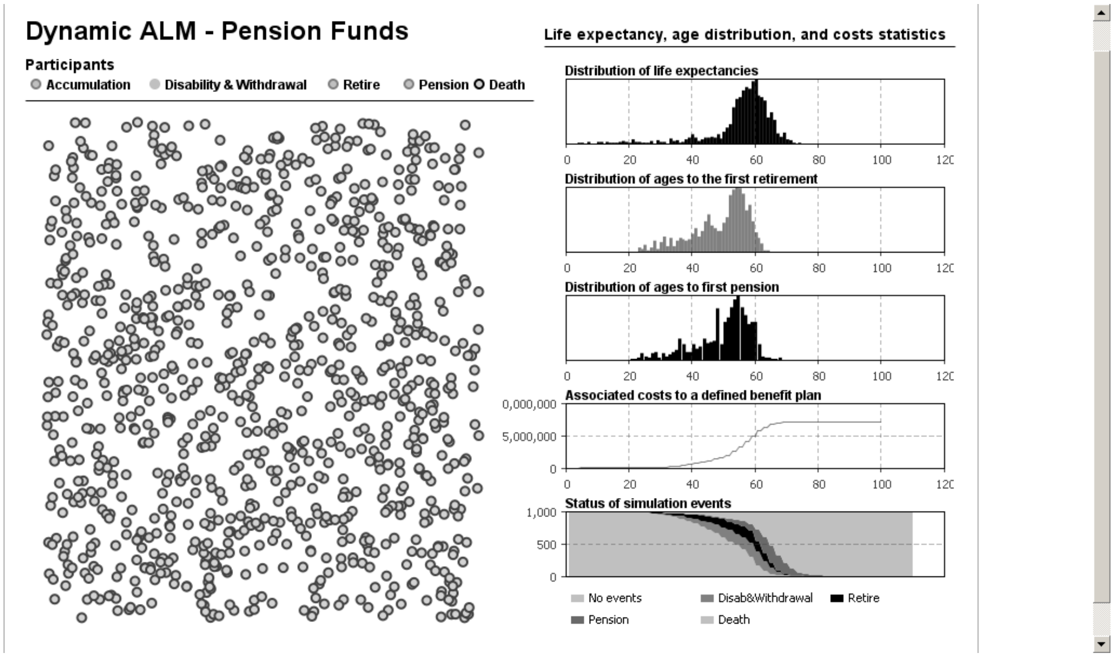
By the application of delphi technique combined to prospective scenarios analysis, these diagram was produced. It shows the complexity of assets and liabilities and the factors that are related to it.



In order to get other indicators the following diagram was useful to simulate the dynamics of credibility over the attractivity of the plan and their consequences to the plan.



APPENDIX 3



Agent based modeling shows the interactions of many agents and their decision rules, giving the distribution of ages and other information useful to come back to system dynamics models. Hybrid models, thus, enable the analyst to combine techniques and approaches to better infere system behavior and to predict what future could be and this way try to cope with uncertainty.

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