Feedback analysis of speculation in a foreign currency market[*]

Nadja Dwenger
Deutsches Institut für Wirtschaftsforschung Berlin
Department Public Economics
Berlin, Germany
Email: ndwenger@diw.de

Oleg V. Pavlov
Worcester Polytechnic Institute
Worcester, MA
USA
Email: opavlov@wpi.edu

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This paper investigates the impact of speculative trading on foreign currency markets. A review of economic literature reveals that there is still no agreement to whether speculators amplify or tame fluctuations of exchange rates. Relevant system dynamics literature suggests that trading by speculators contributes to the formation of price bubbles. However, very few system dynamics papers exist that analyze financial markets at the micro-level of traders. Hence, we turn to the field of computational economics and adapt a well-known heterogeneous agent model. Our new system dynamics model is used to analyze the role of speculation in foreign currency markets.

Introduction

We rely on feedback analysis to understand the impact of speculative trading on the dynamics of a foreign currency market.

Because system dynamics literature that takes into account strategies of traders is rather sparse, we adapt and extend the heterogeneous agent model of De Grauwe et al (2005). The De Grauwe et al. model is a popular financial model within the field of computational economics (Hommes 2006). The model includes fundamentalists and trend traders. In the design of their model, DeGrauwe et al. adhere to the framework by Brock and Hommes (1998), which was originally developed for stock markets.

Even though De Grauwe et al. and the follow up studies do not typically talk about feedback in their models, we show that the original heterogeneous agent model

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incorporates a great deal of feedback structure, which makes it amenable to the system dynamics adaptation and analysis.

Preliminary experiments suggest that stability properties of our heterogeneous agent system dynamics model are enhanced by the speculative behavior of some traders.

Understanding why the model converges after a certain time will lead to a better understanding of the underlying processes in currency markets.

The view of speculation and feedback in financial economics

Several authors have emphasized the importance of feedback in asset markets. A typical point of analysis is the role of trend traders, also known as chartists, speculators, or feedback traders, on the stability of a financial system.

Arguing in the 1950s for flexible exchange rates, Friedman (Friedman 1953: 175) thought that speculation in foreign currencies would likely have a stabilizing effect on exchange rates. Before Friedman, Kaldor (1939: 34) thought that speculation destabilizes prices within a certain range of price-oscillations but then has a stabilizing effect outside of the range. He thought that each market has a specific range. De Long et al. (1990) suggest that feedback trading destabilizes asset prices. Cutler et al. (1990) support De Long's argument.

Feedback is important in the model of Hirshleifer et al. (2006). Hirshleifer et al. present a model in which irrational investors may increase trading activity thereby positively affecting cash inflows, which can lead to abnormal returns for irrational investors. This forms a feedback loop between prices and investments. They show that irrational investors benefit from the feedback effect even though the investors themselves are ignorant of the feedback.

Heemeijer et al. (2007) echo Merton’s (1948) view of the mechanism responsible for self-fulfilling prophecies: “in social systems individual expectations of beliefs can affect the aggregate outcome.” Hence, they argue, “a market, like other social environments, may be viewed as an expectations feedback system: past market behaviour determines individual expectations which, in turn, determine current market behaviour and so on” (2007: 2). Heemeijer et al. (2007) distinguish positive and negative feedback between price and price expectation. Speculation on part of buyers plays out through the positive feedback: the expectation of high price by speculators causes a shift in demand, which produces a hike in price, thus confirming the speculators’ expectations. The negative feedback, they suggest, would be due to the suppliers in the market: if suppliers expect a rise in prices, they crowd the market, thus shifting the supply curve to the left and causing a drop in actual prices. Heemeijer et al. (2007) design an experiment to test the effects of positive and negative feedbacks on the market outcomes. They find that in the case of the negative feedback, participants slowly coordinate their predictive strategies but the market price quickly converges to the equilibrium value. In the positive feedback case, market prices showed more oscillations and slow convergence to the steady state; the participants, however, coordinated their forecasting strategies very fast.

Owhadi (Owhadi 2004) confirms the importance of feedback for stability by showing that an asset market with very simple behavioral rules that do not include price feedback can lead to a market collapse if wealth concentration reaches a certain level.

System dynamics analysis of financial markets

Shimada (Shimada 1978) produced the first, to the best of our knowledge, system dynamics study of the effect that the interaction between fundamental investors and speculators has on stock prices*. In his model, fundamental investors make purchasing and selling decisions based on the dividend prospects and current stock price. Speculators decide whether to buy or sell depending on the deviation of the current price from the lowest price in the previous 5 weeks. The model does not allow agents to change trading strategies (speculation versus fundamental analysis). His model keeps track of the total excess demand in the marketplace as generated by all agents. He explicitly identifies four feedback loops: two speculator loops and two fundamentalist loops. Some of the formulations are questionable; for example, certain loops seem to change polarity during simulations. Shimada concludes that it may be difficult to fit a model to the true stock price time series because real price is influenced by random shocks.

Shimada's model was further analyzed by Weerawat (1993) in a Master's Thesis.

A generic structure of the speculative bubble is given in Sterman (2000: 42); it is reproduced in Figure 1. Increases in price above the fundamental value depress the value-based demand, loop B1. However, the same price movements increase demand by speculators through the reinforcing loop R1.

* Shimada names the agents *speculators* and *investors*. In the financial literature, speculators are also called *chartists, trend traders, trend followers,* and *feedback traders*. Investors are also called *fundamental traders,* or *fundamentalists,* and value investors.

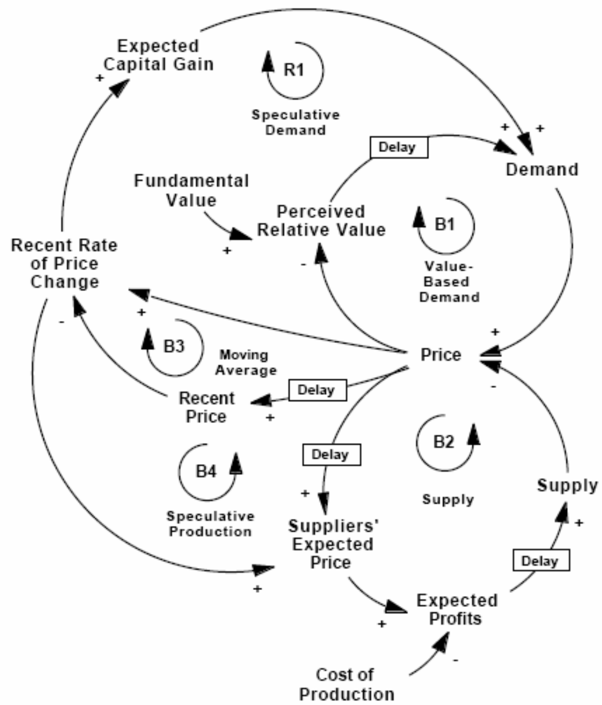

Figure 1: Structure of a speculative bubble. Source: Sterman 2000: 43.

Heterogeneous agent model of exchange rates

In this section, we present the exchange rate model of DeGrauwe et al. (2005). Using a mean-variance utility scheme, the investors set their optimal portfolios. The assumption of rational individuals is moderated as investors do not integrate the whole data set but reduce the complexity of their decisions. The agents can adapt their forecasting formulas if they decide that the other rule is better-performing.

The optimal portfolio

The model of DeGrauwe et al. is micro-orientated: it is based on heterogeneous agents i in the market, who all have different ways of forming expectations about the future exchange rate. The common underlying calculus of all these investors is the maximization of their utility U. The utility function they optimize can be formally written as follows:

$$U\left(W_{t+1}^{i}\right)=E_{t}^{i}\left[W_{t+1}^{i}\right]-\frac{1}{2}\mu V_{t}^{i}\left[W_{t+1}^{i}\right] \tag{1}$$

Utility depends on the expectations about the wealth in the following period $E_t^i\left[W_{t+1}^i\right]$ and is negatively influenced by the variance of their wealth portfolio $V_t^i\left[W_{t+1}^i\right]$. This variance term is weighted by μ, a coefficient for the risk aversion of the individuals: the less agents are willing to take risks the more negative they perceive uncertainty. The wealth of the individuals at time t+1 can be specified as:

$$W_{t+1}^i = \left(1+r^*\right)s_{t+1}d_t^i + \left(1+r\right)\left(W_t^i - s_t d_t^i\right) \tag{2}$$

Generally, the assets are divided into a foreign and into a domestic portfolio. Variable d_t^i represents the holdings of the foreign assets at time t. Multiplied with the (certain) foreign interest rate r^* and the exchange rate s_{t+1}, we get the value of the foreign holdings at time t+1 expressed in terms of the domestic currency. The second part of the right-hand side of the equation signifies the domestic assets. It is formed by the wealth W_t^i less the amount of money the agent invested in foreign assets before. The interest earned in the home country is defined by the (certain) domestic interest rate r. Substituting (2) into (1) yields:

$$\begin{aligned} U\left(W_{t+1}^i\right) = {} & E_t^i\left[\left(1+r^*\right)s_{t+1}d_t^i + \left(1+r\right)\left(W_t^i - s_t d_t^i\right)\right] \\ & -\frac{1}{2}\mu V_t^i\left[\left(1+r^*\right)s_{t+1}d_t^i + \left(1+r\right)\left(W_t^i - s_t d_t^i\right)\right] \end{aligned} \tag{3}$$

Using the standard rules for the transformation of expectation values and variances we can shorten this equation. After maximizing the utility with respect to d_t^i the optimal holding of foreign assets for the investors can be derived as:

$$d_t^i = \frac{\left(1+r^*\right)E_t^i\left[s_{t+1}\right] - \left(1+r\right)s_t}{\mu\left(1+r^*\right)^2 V_t^i\left[s_{t+1}\right]} \ , \tag{4}$$

with $\mu > 0$.

The optimal amount of foreign currency depends on its expected excess return. As the investors are assumed to be risk averse, this term must be corrected for the risks involved. All individuals together establish the market demand D_t for foreign assets, where n_t^i is the number of investors of type i:

$$n_t^c d_t^c + n_t^f d_t^f = D_t \tag{5}$$

The market supply Z_t is assumed to be exogenous in this model. In the market equilibrium, demand equals supply. Hence, the market clearing condition is:

$$Z_t = D_t \tag{6}$$

We can employ equation (6) and (4) in order to rewrite the market equilibrium in dependence on s_t. Isolating the market exchange rate leads to

$$s_t = \left(\frac{1+r^*}{1+r}\right)\frac{1}{\frac{\omega_t^c}{\sigma_{c,t}^2}+\frac{\omega_t^f}{\sigma_{f,t}^2}}\left[\omega_t^c\,\frac{E_t^c\left(s_{t+1}\right)}{\sigma_{c,t}^2}+\omega_t^i\,\frac{E_t^f\left(s_{t+1}\right)}{\sigma_{f,t}^2}-\Omega_t Z_t\right] \tag{7}$$

with $\omega_t^i = \frac{n_t^i}{\sum_{i=1}^{N} n_t^i}$ as the portion of each type of agent and $\Omega_t = \frac{\mu}{\left(1+r\right)\sum_{i=1}^{N} n_t^i}$. This equation is determined by the fraction of domestic and foreign interest rate as well as by the expectations of the market participants. Owing to the fact that the investors are risk averse, the forecasts are weighted by their respective variance: if an agent expects the exchange rate to rise but has been mistaken several times in the past, he or she will be more careful in making investment choices in the future. Therefore, the influence of these independent predictions on the market exchange rate is lower than if the forecasting revealed to be correct.

Two types of investors: fundamentalists and chartists

There are two types of investors: fundamentalists and chartists. They use different methods in order to form expectations about the future exchange rate. It is important to underline that neither fundamentalists nor chartists take into account the whole amount of information available. They know about the intricacy of price building processes and about their incapability to deal with all details. Thus, they reduce their decision problem to two simplified rules. Fundamentalists rely on the value of the fundamental exchange rate s_t^*. This fundamental exchange rate is assumed to be exogenous and to follow a random walk without drift. It can be described as follows:

$$s_t^* = s_{t-1}^* + \varepsilon_t \tag{8}$$

where ε_t are normally distributed shocks. The fundamentalists compare the exchange rate on the market to the fundamental exchange rate. In the model at hand, they only consider the previous period when they are forming their expectations. The deviation of the market exchange rate from the fundamental is weighted by the factor ψ, where $0 < \psi < 1$ in order to prevent an explosive process. Then, fundamentalists establish their expectations by subtracting the weighted variation from the previous exchange rate:

$$E_t^f\left[s_{t+1}\right]=s_{t-1}-\psi\left(s_{t-1}-s_{t-1}^*\right) \tag{9}$$

This means that fundamentalists generally forecast a return of the future exchange rate to its fundamental value. The speed of this reversion is assumed to be determined by the velocity of adjustments on the goods market. Fundamentalists are supposed to know about these adaptation processes and about the value of fundamental variables. By comparing them with the market exchange rate, they take into account information directly. This process corresponds to a *negative feedback rule*. As we will see in the following, the mean reverting tendency sets a counterpart to the driving up behavior of the chartists.

Note, that the assumption of not fully informed agents has implications on the forecasting rule: the investors do not know the structure of the market. Therefore, they cannot keep track of the current market exchange rate. They only have data of time $t-1$ at disposition. As a result, there is a time gap between the instant the exchange rate is formed on the market and the moment in which it is integrated into the decision rules of the market participants.

Chartists, by contrast, base their forecasting on the past market exchange rate. In considering past movements of s, they extrapolate past trends and transfer them into the future. Hence, they use information indirectly, i.e. through the exchange rate itself, and establish a *positive feedback rule*. Since they entirely neglect information concerning the fundamental variables, they can be regarded as pure noise traders. Their reliance on "market sentiments", i.e. general trends on the market, gives rise to *herding behavior*.

The extent to which noise traders extrapolate past patterns into the future depends on the coefficient β, which measures the agents' inclination of using past changes. DeGrauwe et al. define β to be $0<\beta<1$, i.e. the future exchange rate will never be completely determined by past data. This assumption avoids an explosive process. It must be further pointed out that in the original model chartists integrate five lags into their forecasting rules (H = former exchange rates, here H = 5), whereas fundamentalists do not consider more than one period of the past. We follow this assumption. These elapsed movements of the exchange rate are multiplied by ρ_h, which are geometrically declining weights:

$$\rho_h=\frac{(1-\rho)\rho^{h-1}}{1-\rho^H}$$

where $1-\rho^H=\sum_{h=1}^{H}(1-\rho)\rho^{h-1}$. Thus, the expectations computed by the chartists can be written as:

$$E_t^c\left(s_{t+1}\right)=s_{t-1}+\beta\sum_{h=1}^{H}\rho_h\Delta s_{t-h} \tag{10}$$

Note, that H = 1 and $\rho_h = 1$ are special cases concerning the way of forming expectations: They correspond to investors with static expectations. As presented before, chartists also suffer from a time lag of information of one period. Thus, at time t, they only possess data of the market exchange rate at time $t-1$.

One may think that investors who neglect the whole data set about fundamental values cannot persist on the market. At this point, it is important to underline that there have been various studies on the use and the profitability of technical analysis, which have sustained the hypothesis of its widespread popularity in practice. Taylor and Allen (1992) were one of the first who documented the use of technical extrapolation methods among traders on the foreign exchange market. In their survey, approximately 90% of currency traders questioned in London answered that technical trading rules were an important component of their short-investment strategies. 60% of the interviewees judged charts to be at least as important as fundamentals. This result was corroborated by the inquiry conducted by Cheung et al. Using questionnaires, they systematically analyzed the North American, British and Asian exchange market. To the question "select the single most important factor that determines exchange rate movements" the currency traders responded that intraday and in the medium-run the exchange rate is mainly determined by non-fundamental factors. Obviously, most of the dealers think that irrational factors play a key role in the determination of exchange rates in the short and medium-term. In the long-run they indicate a larger impact of fundamental variables, but even then some irrational influences are suggested.

In the following, we will examine if chartists in our model also shape pricing processes on the foreign currency market. It will be seen that these technical traders indeed perform very well. Moreover, we will observe that the chartists' forecasting rule favors herding behavior and the development of bubbles.

Evaluation of the risk involved in the forecasting rules

As individuals are risk averse, they have a stake in dependable expectations about the future. Consequently, the agents have to evaluate the risk involved in their forecasting rules. For the sake of simplicity, all of them measure the risk of their portfolio identically. In the model, every individual draws on the adjusted variance $\sigma_{i,t}^2$:

$$\sigma_{i,t}^2 = \left(1+r^*\right)^2 V_t^i\left(s_{t+1}\right) \tag{11}$$

for $i = c, f$, where

$$V_t^i\left(s_{t+1}\right) = \sum_{k=1}^{K} \theta_k \left[E_{t-k-1}^i\left(s_{t-k}\right) - s_{t-k}\right]^2 \tag{12}$$

The adjusted variance of the agents is mainly established by the weighted average of their squared forecasting errors in the past. In doing so, the variances of the previous periods lose the more influence the more distant they are. These geometrically declining weights are calculated as follows:

$$\theta_k = \frac{\theta(1-\theta)^{k-1}}{\sum_{k=1}^{K} \theta(1-\theta)^{k-1}}$$

We assume $K = 1$.

Exchanges between the two groups of traders

In the model elaborated by DeGrauwe et al. the market participants select one of the two forecasting rules in order to streamline their decision process. At the end of each trading period, chartists and fundamentalists compare ex post the risk adjusted profitability of their rule to the one of the other group. Afterwards, they decide to keep their method or to skip to the more profitable one.

This is close to the idea of evolutionary dynamics. Originally, Brock and Hommes applied the approach by making the weights (i.e. the quantitative proportions of the groups) dependent on their relative profitability. Its general mechanism is in tradition of a *discrete choice framework*, which involves three characteristics of the choice set. First, the alternatives must be mutually exclusive from the decision maker's perspective. Choosing one alternative necessarily excludes any of the other ones. In our model, this is guaranteed by the fact that an agent is either fundamentalist or chartist. Second, the choice set must be exhaustive, meaning all possible alternatives are included: besides chartists and fundamentalists there are no other types of investors on our foreign currency market. Third, the number of alternatives must be finite.

In their model, Brock and Hommes use past realized net profits π_t^i as the publicly available performance measure. These profits are defined as the entire earnings on the optimal foreign portfolio. DeGrauwe et al. slightly adapt the method and outline profits as the one-period earnings of investing one monetary unit in the foreign asset:

$$\pi_t^i = \left[s_{t-1}\left(1+r^*\right) - s_{t-2}\left(1+r\right) \right] \operatorname{sgn}\left[\left(1+r^*\right) E_{t-2}^i\left(s_{t-1}\right) - \left(1+r\right) s_{t-2} \right] \tag{13}$$

with

$$\operatorname{sgn}[x] = \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$$

The sign function guarantees that the agents only realize a profit if they correctly predict the direction the market exchange rate moves into. For instance, if an investors expects the exchange rate to rise and this forecast is realized indeed, the algebraic sign of the bracket term is positive (sgn[x] = 1). Thus, the profit of this agent equals the observed increase (which must be corrected for the interest differential). If, by contrast, the forecast reveals to be wrong the sign gets negative (sgn[x] = -1) and the investor suffers a loss identical to the change in the exchange rate. Only if the market exchange rate remains at its previous level the agents do not earn or lose money. Then, the relative profitability of the two forecasting rules are defined as a weighted average of past realized profits: $\pi_t^{'c} = \pi_t^c - \mu\sigma_{c,t}^2$ and $\pi_t^{'f} = \pi_t^f - \mu\sigma_{f,t}^2$. These are the net profits corrected for their respective variance. Remember, the agents in the market are unwilling to take risks. That is why the variance is weighted by the risk aversion of the traders. Here, risk aversion is assumed to be a constant factor, which is identical for everybody.

When the number of agents n_t^i in each group tends towards infinity, the fractions of the groups are determined by the Logit discrete choice model probabilities. The Logit concept is derived under the assumption that the unobserved factors (i.e. the fundamental shocks ε_t in our model) are uncorrelated over alternatives, as well as having the same variance for all of them. Using the Logit transformation rules, the fractions of the two groups can then be expressed in the following way:

$$\omega_t^c = \frac{\exp\left[\gamma\pi_t^{'c}\right]}{\exp\left[\gamma\pi_t^{'c}\right] + \exp\left[\gamma\pi_t^{'f}\right]} \tag{14}$$

$$\omega_t^f = \frac{\exp\left[\gamma\pi_t^{'f}\right]}{\exp\left[\gamma\pi_t^{'c}\right] + \exp\left[\gamma\pi_t^{'f}\right]} \tag{15}$$

As in Logit models the fractions ω_t^c and ω_t^f add up to one, the latter can be rewritten as

$$\omega_t^f = 1 - \omega_t^c \tag{16}$$

A crucial feature of the concept by DeGrauwe et al. is that the investors are boundedly rational. This means that on the one hand they do not bear in mind all data available and use simple mental models as decision rules: they are not completely rational. On the other hand they check the fitness of their elementary forecasting rules ex post and switch if advantageous. Hence, the majority of agents choose the predictor which yields most. For instance, if $\pi_t^{'c} > \pi_t^{'f}$ more investors will follow a technical trading rule; thus, ω_t^c increases.

However, profits are not the only factor which determines the weight of each group. In fact, it is common knowledge that habits are important to human beings. In everyday life we constantly rely on psychological patterns that have been formed in the past. Therefore, to a certain extent, the force of habit can entail sluggishness among agents even if the other group of investors earns more money. The coefficient γ specifies this (un)willingness of the individuals to switch their forecasting rule, with $0 < \gamma < \infty$. A low parameter γ reflects certain inertia of the investors concerning the way they form their expectations. If γ takes a value of zero this means that nobody ever changes a pattern of prediction that has been chosen once: the fractions of chartists and fundamentalists remain fixed. If the agents, on the contrary, react strongly to the relative profitability of the forecasting rules, γ is high. The special case of limiting γ to infinity leads to the neoclassical deterministic choice model, where in each period all agents choose the optimal predictor.

To sum up, the model integrates psychological elements of decision making. Cognitive psychologists claim that human beings look for patterns and subsequently generalize them in order to structure the real world. This inductive behavior is represented by chartists as well as fundamentalists. Individuals then apply these general rules to future special cases (deductive proceeding). Ex post, they receive feedback about the accuracy of their expectations. If this response is coherent with their forecasting rule, they keep it; if not, they try to reduce the mental discrepancy by switching their forecasting rule.

Feedback in the exchange rate model

Analysis of the De Grauwe et al. model indicates that it contains a significant number of feedback loops. The causal loop diagram of the exchange rate model is in Figure 2. The Vensim DSS's loop tool counted 22 loops passing through the market-clearing exchange rate. Some of these loops are positive and some are negative (the polarities are not shown in Figure 2 – we will do that in the following drafts of the paper).

Even though De Grauwe et al. do not mention stocks in their paper, their model contains several variables that, in effect, are stocks. Once we identified such variables, it was possible to formulate corresponding rate equations. A preliminary system dynamics model based on the heterogeneous agent model of De Grauwe is shown in Figure 3. One can see from the figure that we denoted expectations, numbers of different types of agents and fundamental exchange rate as stocks. The mathematical formulation of the system dynamics model is closely related to the original formulation in De Grauwe et al.

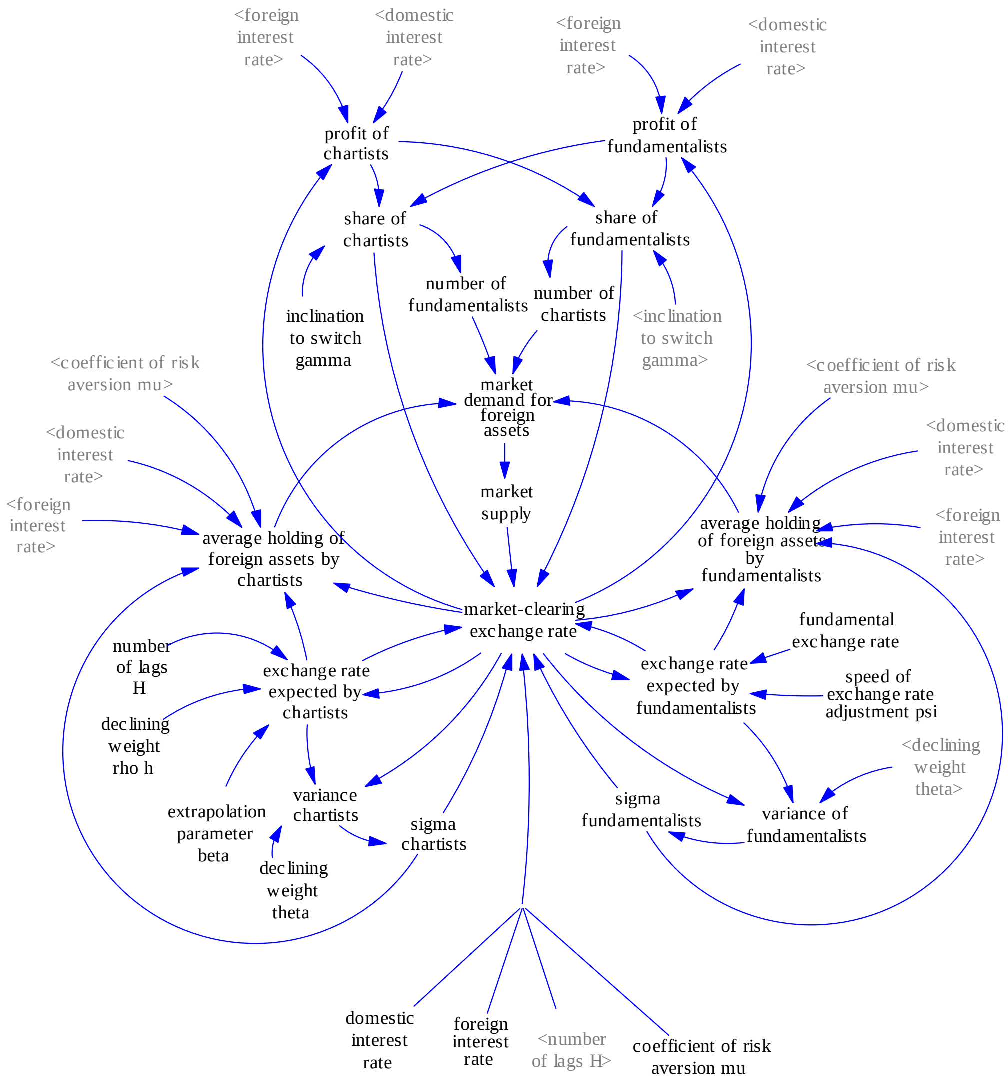

Figure 2: Feedback structure of the exchange rate model

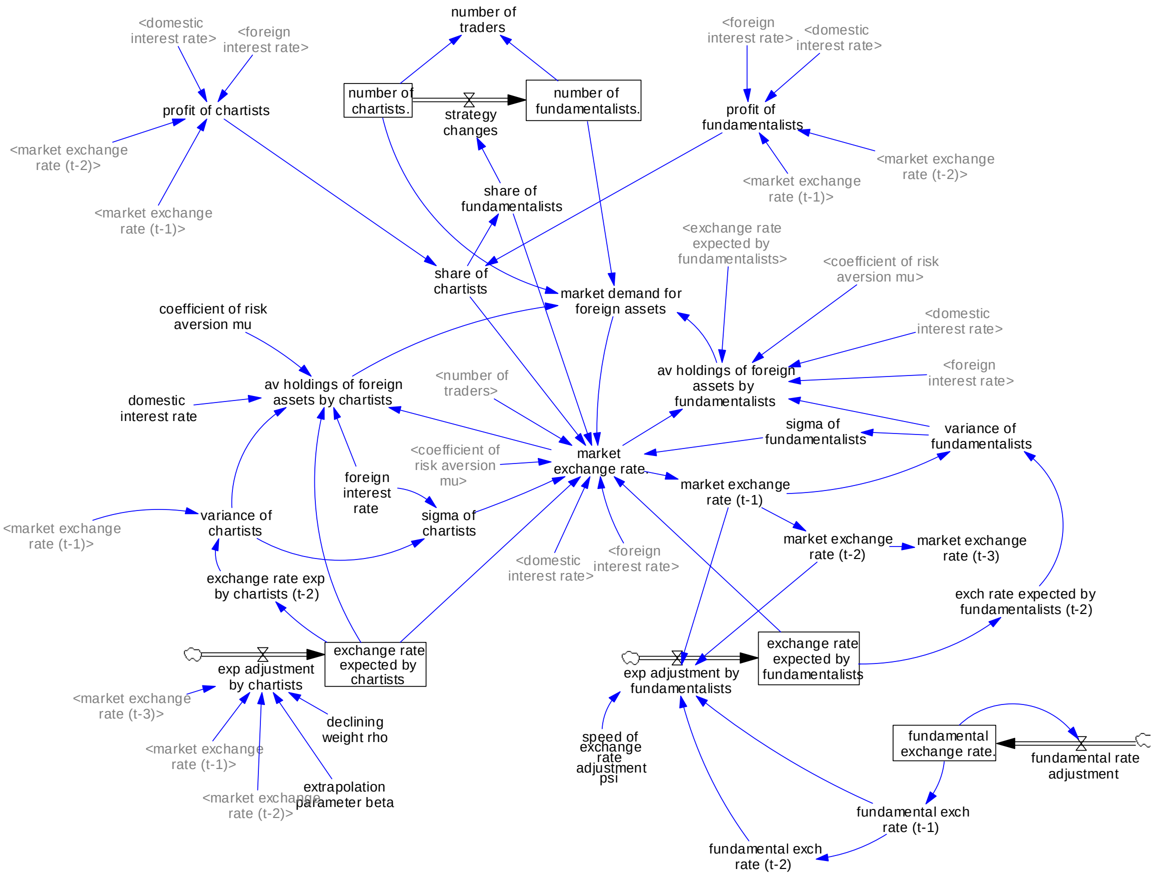

Figure 3: Stock and flow implementation of the exchange rate model

Experiments

Our preliminary runs indicate that chartists contribute to the stability of the system – which augments the popular system dynamics view that speculators create price bubbles. By the time of the conference in Athens, we will perform a critical analysis of this result. We will also do the following: (i) conduct experiments that test the sensitivity of the model to various parameters; (ii) determine whether or not actions by chartists amplify fluctuations of the market exchange rate – specifically, if they create tunnel stability as was suggested by some economists (which may explain our preliminary result); (iii) explore the reaction of the model to sudden changes in the fundamental exchange rate and interest rate. The emerging behavior of the system will be analyzed in terms of feedback loops.

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