Sensitivity Analysis of a Real Estate Price Oscillations Model

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In system dynamics methodology, formal output analysis can create a basis for improving the structure, so it is important to determine the variables to which the model is sensitive, by using a formal experimental design method. Moreover, such a research can allow comparing different methods for parametric sensitivity analysis. The purpose of this study is to understand different reactions of real estate prices to changes in different input factors. The study is carried out on a system dynamics model previously developed by the authors on real estate prices in Istanbul. The sensitivity of period, amplitude and mean of price oscillations to the changes in some selected variables of the model are analyzed. Two experimental sensitivity analysis designs, namely fractional factorial design and Latin hypercube sampling are used to measure the sensitivity of the model. The study shows that the factors that turn out to be most significant in the two designs are not the same. But the factors and interactions found significant by both techniques can be safely assumed to be highly influential.

Keywords: real estate dynamics, sensitivity analysis, output analysis, experimental design, fractional factorial design, Latin hypercube sampling

1. Introduction

This study aims to compare different methods for parameter sensitivity analysis: factorial designs and Latin hypercube sampling. In factorial design orthogonal arrays are used for experimental design. Factorial design is divided into two categories: full factorial design and fractional factorial design. Full factorial design considers all parameter combinations at all levels, so one is confident that the range of possible inputs is fully explored. Also in full factorial design all interaction effects are taken into consideration. However, if we have P parameters each to be tried at l levels, l^P runs would be required, which can be an enormous number for even moderate number of model parameters P, even with just 3 levels (Ford, 2002).

Fractional factorial design uses some pre-specified orthogonal design arrays and by using only a small fraction of all possible runs, obtains relevant information about the experimental problem. This method assumes that "higher order" interactions such as quadruple are negligible. The main disadvantage is that it is not easy for the non-statistician to apply and interpret. Furthermore, for statistical estimation, each experimental condition must have several replications, using different noise seeds.

Latin hypercube sampling (LHS) is a sampling scheme that ensures that all regions of the sampling space are explored without trying all possible combinations of parameters.

Clemson et al. (1995) explains the Latin hypercube sampling process as follows:

- "For each input parameter, divide its range of possible values into *N* strata of equal probability.
- For each input parameter, randomly pick one value from each of the *N* strata.
- From the *N* possibilities for each input parameter, randomly select one. This set of values constitutes the set of parameter values for the first trial.
- Repeat previous step without replacement for *N* iterations. This generates *N* sets of parameter values for *N* trials.

A typical value for *N* is 40, but larger numbers can be chosen as desired. LHS combines many advantages of the full factorial design and requires far fewer trials."

However; it assumes that the input parameters are independent of each other (i.e. no interaction effects exist). It is useful in the sense that the entire region of the experimental space is represented homogeneously if all interactions can be safely ignored. Or else analyst must use some method other than LHS to determine interactions among inputs.

For Factorial Design and LHS, Design-Expert® 7.0.2 (Stat-Ease Inc., 2004) and Stella® 7.0.3 (isee systems, 2002) software are used in this research.

The rest of this paper is organized as follows: section 2 describes the system dynamics model used in this study and explains how it is modified to fit in the sensitivity analysis framework. Section 3 gives the seven model parameters used in analysis, explains the specific design structures used and presents the outputs. Section 4 summarizes the final results and mentions the future research directions.

2. The Model

2.1. Overview of the Model

This study is carried out on a system dynamics model previously developed by the authors (Barlas et al., 2007). The purpose of the model is to build up an understanding of the dynamics of the real estate market in Istanbul, Turkey. There has been a considerable immigration to Istanbul in the last few decades, which has created significant demand for houses. Like it is the case in all other big cities, the house supply has been increased in Istanbul. The real estate market shows booms and busts. The demand increase creates an increase in the prices. With this price increase, the construction companies see the potential profit of constructing new buildings and start new projects. The constructions start to turning into new buildings, which can meet the excess demand. However, due to the delay in constructing buildings, the houses continue to be built when the demand also decreases.

This pushes the prices down, which decreases the motivation of constructors to build new houses and the construction rate. In this period, the demand for houses is accumulated, and the loop is closed. This loop is very typical, seen in all over the world. So, this system dynamics model has been developed to understand the reasons of oscillations in real estate prices in Istanbul from the perspective of an important construction company (Barlas et al., 2007). The model mainly focuses on the economic balance side of the problem, and pays less attention to the dynamics of demand creation, the effects of costs and interest rates. The time unit of the model is selected to be years since the major time delays and the rates of changes are measured in terms of years. The time horizon was selected as 40 years between 1980 and 2020. The motivation for selecting a range of 40 years was to be able to observe a few real estate cycles. But in sensitivity study time horizon was selected as 120 years between 1980 and 2100 in order to have enough data in analysis and see the effect of interest rates, which are assumed to decrease with mortgage application.

The model includes the following elements of the real estate market:

- The houses for sale and under construction
- The demand for houses
- The current and customer price
- The profit and its effect on the construction start rate of the company and the competitors

The model does not include the dynamics of the demand creation. The population increase rate is taken as an external input to the model and any effect of housing on migration is excluded. Also, the effect of interest rates on customer purchasing power is modeled by a single variable. The cost is taken to be an input that changes with time. Moreover, the effect of national economy on the constructor companies is not considered.

2.2. Dynamic Behavior and Model Structure

The real estate prices show oscillatory behavior. The basic reasons behind it are the balancing loops affecting the price with delays. The demand-price loop balances the demand by the increased price, which is created by the excess demand. That is, when the demand increases, the price swells, which in turn decreases the demand. Another major loop is the supply-price balance loop. When the price increases, the constructions increase, meaning the supply will boost and the prices will eventually fall down. Also, a negative loop between demand and supply exists in the model. When the demand increases, the supply will increase and thus the demand falls down.

Apart from these major loops, there are some minor loops in the model. These are price adjustment loop and sales-demand balance loop. The causal relationships the feedback loops existing in the model are shown in the causal loop diagram in Figure 1 and stock-flow diagram in Figure 2.

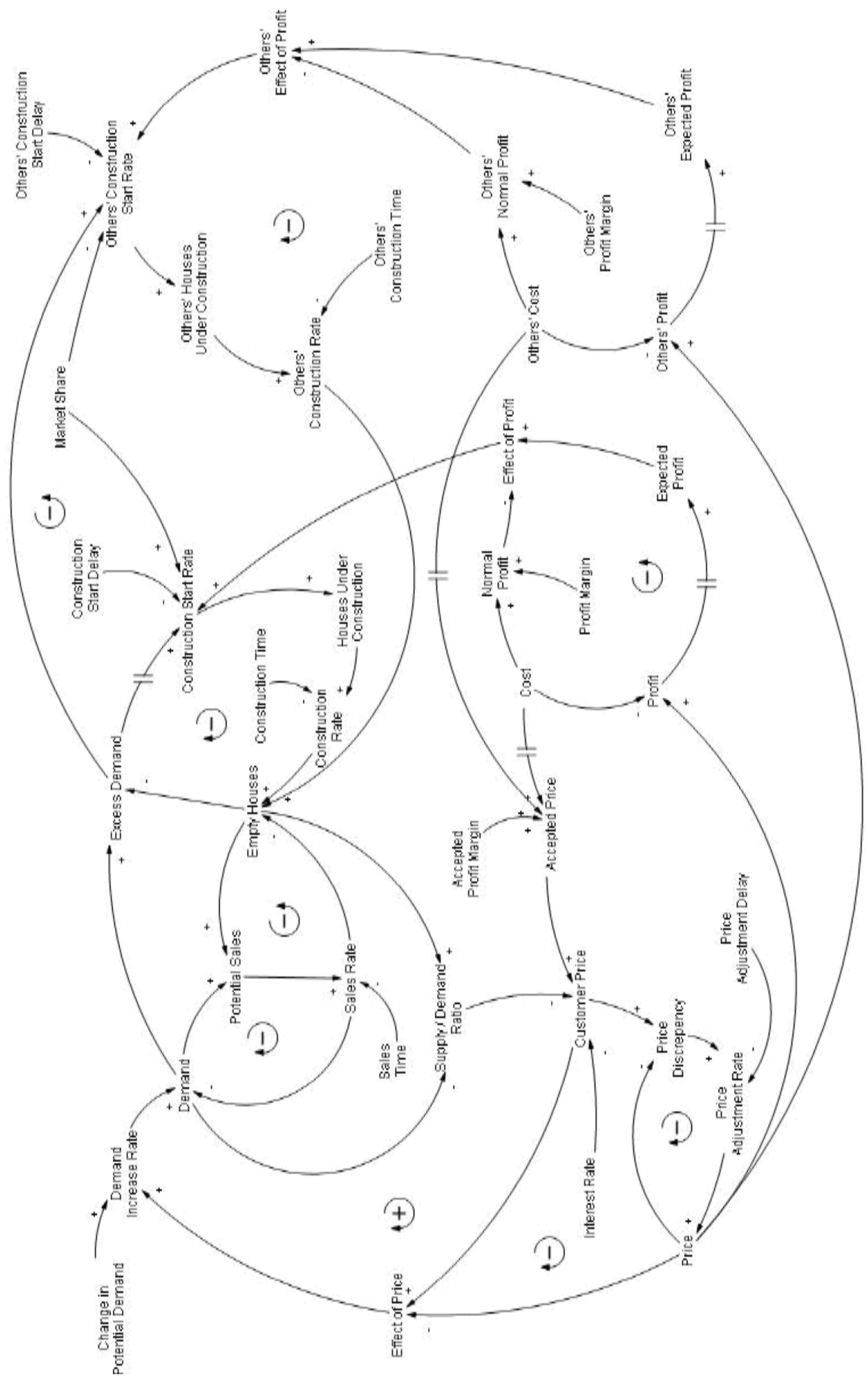

Figure 1- Causal loop diagram of the model

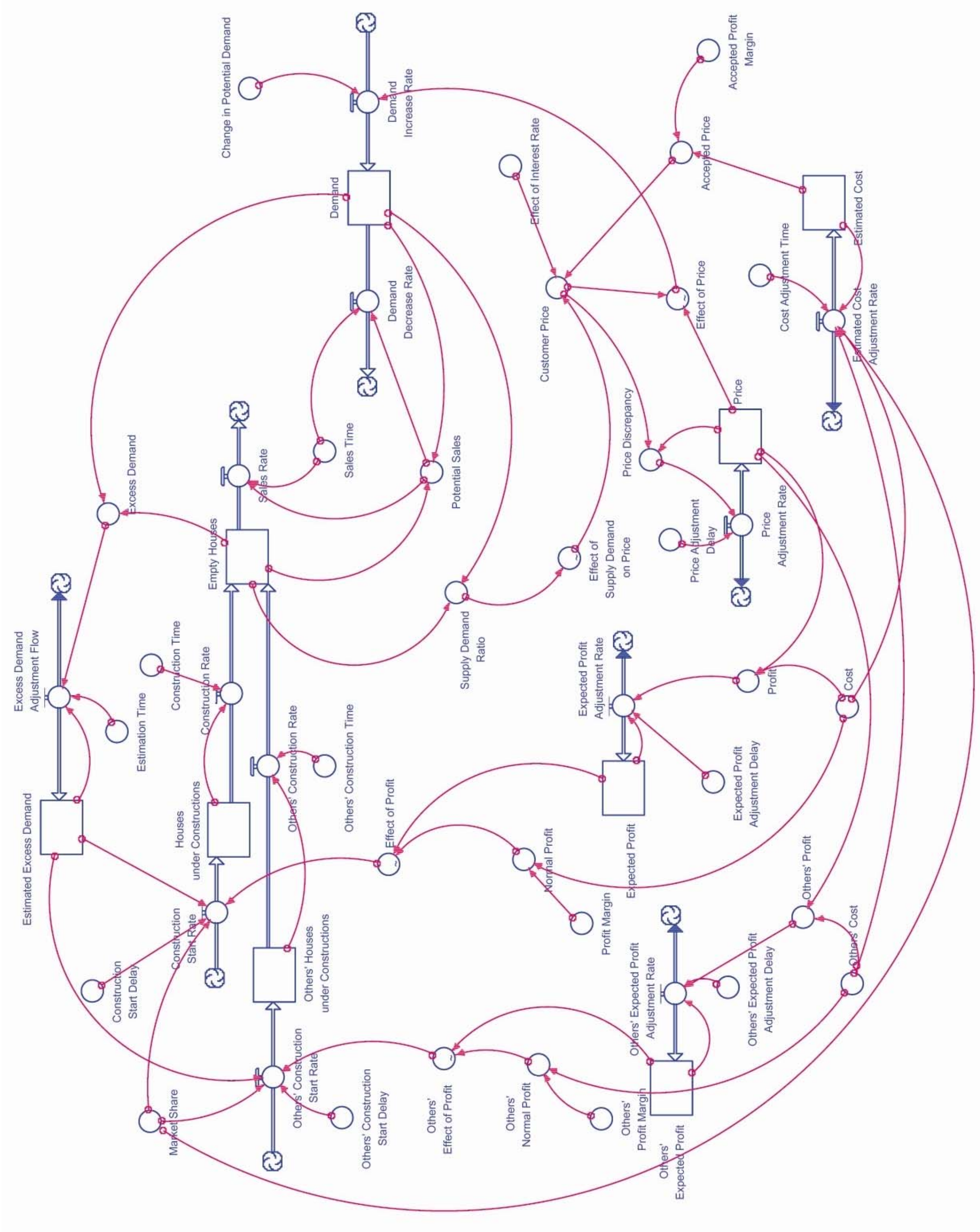

Figure 2 - Stock-Flow diagram of the model

The demand in the model is determined according to the change in potential demand in Istanbul. This change creates an increase in demand together with the effect of price. The stock of awaiting demand is decreased by the house sales. The sales occur when both empty houses and the demand is available. The ratio of empty houses to the demand gives the supply / demand ratio, which indicates the balance between supply and demand. The supply / demand ratio has an effect on the customer price. That is, when there is excess supply, the prices that customers are willing to pay will decrease and vice versa. The price that customers are willing to pay is also affected by the interest rate and the accepted price. Accepted price is the price that customers agree to pay under normal supply / demand and interest rate conditions. It is assumed that, this is 1.1 times the perceived cost of building a house (which is a smoothed version of the average building cost in the market), a 10% profit margin is considered to be normal. Customer price enters an information delay structure and turns into the market price gradually. The difference between the current market price and the customer price creates the effect of price that affects the demand increase rate. This closes the balancing loop between the demand and the price (Figure 1).

The supply side of the model is identical for the hypothetical company under consideration and for all other firms (modeled as a single sector). The market price also has an effect on the supply side of the model. The cost (which is taken to be an external input that changes with economic conditions) and the price determine the profit of the hypothetical firm and the other firms. Since it is not possible to know the exact profit that can be obtained, an information delay is used to represent the estimation of the profit by the firms. In addition to expected profit, the firms have a normal profit that depends on the cost of the company and the profit margin policy. The difference between the normal profit and the expected profit determines the effect of profit. The firms aim to meet the estimated excess demand for houses by starting new constructions according to their market shares. On doing this, they are affected from the profit. If they have higher profit than the normal profit, their willingness to meet the excess demand increases. The new projects turn into constructions, which become empty houses after a construction delay. Due to the effect of supply / demand ratio on price, a loop is created between the supply and the price.

There is a third loop between the supply and demand. The difference between the empty houses and the demand is the excess demand. This excess demand cannot be known accurately and it is estimated by the construction firms by an information delay structure. As explained above, this estimated excess demand turns into new constructions and the new houses for sale. This decreases the excess demand, which closes the balancing loop (See Barlas et al., 2007).

In the model, there are nine stocks and their related flows. The major stocks of the model are Price, Demand, Empty Houses, Houses under Construction and Others' Houses under Construction. The other four stocks (Estimated Cost, Estimated Excess Demand, Expected Profit, and Others' Expected Profit) are the "OUT" s of the first order information delays of the model (See Figure 2).

The basic stocks of the model show an oscillatory behavior as seen in Figure 3. The main reason behind the oscillation of prices is the delay in starting new houses, which increases the price in that period, and the excess supply when these houses are finished, which decreases the price. The demand increase rate is dependent on the Change in Potential Demand, which is an external function that shows an increasing trend until year 1992 and a decreasing trend thereafter. This trend also affects the other stocks.

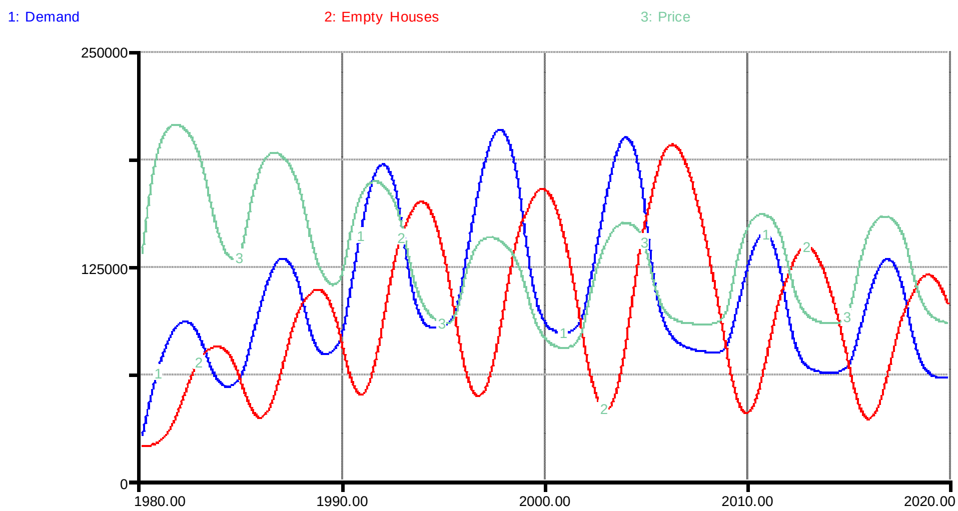

Figure 3 - Dynamics of the system

2.3. Randomness

In order to understand the dynamics generated by the feedback structure, noise is omitted in the initial model. After the dynamics of the model is understood, some important random variations at key points are selected and added to the model. Then their effects are tested.

All noise sources in real life have inertia and attenuate high frequencies. Hence white noise shouldn't be used in models. In this study pink noise structure is used to model randomness. Our best judgment about distribution, standard deviation and correlation time is used as there is no available numerical data (Sterman, 2000). The pink noises which are added to three key places in the model and their details are as follows;

- *Price_Adjustment_Rate = (Price_Discrepancy/Price_Adjustment_Delay) + NOISE_2*
  NOISE_1_ = NORMAL (0, 200000, 1024)
  NOISE_2 = SMTH1 (NOISE_1_, Corr_Time1)
  Corr_Time1_ = 0.45

- *Construction Rate = (Houses_under_Constructions/Construction_Time) + NOISE_6*
  NOISE_5 = NORMAL (0, 100000, 6666)
  NOISE_6 = SMTH1 (NOISE_5, Corr_Time_3)
  Corr_Time_3 = 1

- *Others'_Construction_Rate =*
  (Others'_Houses_under_Constructions/Others'_Construction_Time) + NOISE_4
  NOISE_4 = SMTH1 (NOISE_3, Corr_Time2)
  NOISE_3 = NORMAL (0, 100000, 1058)
  Corr_Time_2 = 1

For statistical estimation, each experimental condition must have several replications, using different noise seeds. In this study, experiments are also performed with three replications and each replication is obtained by changing the seed values of the noise formulations. Three seed sets are used: seed1 = 1024, 8900, 2500 – seed 2 = 1058, 7474, 5454 and seed 3 = 6666, 2222, 1111.

3. Sensitivity Analysis

For experimental designs, selection of appropriate model outputs for observation and response measures is imperative. In this study "Price" stock is selected for observations. Amplitude, period and mean of the price oscillations are selected as the response variables. In sensitivity study, time horizon was selected as 120 years between 1980 and 2100 in order to have enough data in analysis and see the effect of interest rate, which are assumed to decrease with mortgage application.

The model includes several parameters that can be experimented with. Since number of parameters is very large, seven of them are selected; Market Share, Sales Time, Profit Margin, Other's Profit Margin, Construction Time, Other's Construction Time and Change in Potential Demand. The Change in Potential Demand (CPD) is a graph function. Since in the analysis the study mainly focuses on parameter value sensitivity, only the output values of these graph function are changed by multiplying the output by a constant. This constant is called "coefficient of CPD". These seven factors are selected because they seem to have effect on price oscillations in initial experimentations.

In order to see the variations in three responses due to changes in input factors, experiments should be carried out under different levels of these factors. For the high and low levels of factors be meaningful, there should be a sufficient difference between them and the "zero level" of factors. Because of this, some zero levels in the model are updated. That is, the zero levels of some factors used in sensitivity analysis are different from the values used in the previously explained model. To determine a reasonable range for all the parameters, some experiments are done by changing the parameter values simultaneously. A reasonable range has to be wide enough permitting the model to show some numerical sensitivity in response variables, but it also should not cause pattern sensitivity. Then the high and low levels of factors are set as ±50% of the zero levels which seems to be appropriate and enough for both numeric sensitivity and reasonableness. The selected factors and their levels are given in Table 1.

Table 1- Selected Factors and Their Levels

Factor	Assigned Letter	Unit	Level	
			High	**Low**
Market Share	A	Unitless	0.75	0.25
Sales Time	B	Years	1.5	0.5
Profit Margin	C	Unitless	0.75	0.25
Other's Profit Margin	D	Unitless	0.75	0.25
Construction Time	E	Years	1.8	0.6
Other's Construction Time	F	Years	1.5	0.5
Coefficient Of CPD	G	Houses/year	3	1

3.1. Fractional Factorial Design

Since seven parameters are selected for sensitivity analysis and each factor has two levels, required number of experiments for full factorial design is 128 experiments (with just one replication). would be quite time consuming as there are three response variables (amplitude, period and mean of the price oscillations) to measure, and compare statistically in each run. Three replications for three response variables would require 1152 experiments. Thus, fractional factorial design is preferred instead of full factorial design. A 2^{7-3} resolution-IV fractional factorial design is chosen (main effects are aliased with three way interactions and two way interactions are aliased with two way interactions) which would require 16 experiments for one replication. Since each experiment is performed three times in order to have better estimates of variables, total number of experiments performed for one response is 48. As there are three responses, 144 data sets are analyzed during the fractional factorial design.

The analysis is performed with no response transformation. The design of the experiment that is given according to the 2^{7-3} resolution-IV fractional factorial design by Design-Expert®
and the aliases are given in Table 2 and 3, correspondingly. Aliased effects have a common effect estimate. And the influences of individual aliased terms are indistinguishable. For example, see the first term on Table 3; [A] = A + BCE + BFG + CDG + DEF means that in

a fractional factorial design, we can only observe the combined effect of factor A (market share) and the three-way interactions listed. Since it is less likely to have three-way interactions than the effect of a single factor, we assume that this total effect is the effect of factor A. In this design, two-way interactions are aliased with each other as seen in Table 3. By subjective judgment, we choose the most likely interaction term among the aliases. For example, we think that the interaction between market share (A) and sales time (B) is more likely than the interaction between profit margin (C) and construction time (E) and the interaction between others' construction time (F) and coefficient of "change in potential demand" (G). The outputs (values of price stock) are collected at each quarter by using Stella® for each experiment by setting the responding levels to the parameters. Then the collected outputs are analyzed in BTS II software (Barlas et al., 1997) in order to calculate the amplitude, period and mean of price. At each step, autocorrelation function is used for period calculation. Also, spectral density function is used in order to check calculated period and if multiple periods exist. First moment is used to calculate mean of the price. Then the most appropriate method is selected to calculate amplitude of the oscillations. As mentioned above, for each experiment three replications are performed. In the analysis of outputs, the average values of three replications are used in each experiment and sensitivity analysis of the selected responses to the seven chosen factors are analyzed in Design Expert (Stat-Ease Inc., 2004).

Table 2 - Experiment Design

Exp. No	Market Share	Sales Time	Profit Margin	Other's Prof. Mrg.	Construction Time	Other's Cons. Time	Coeff. of CPD
1	0.25	0.5	0.25	0.25	0.6	0.5	1
2	0.75	0.5	0.25	0.25	1.8	0.5	3
3	0.25	1.5	0.25	0.25	1.8	1.5	1
4	0.75	1.5	0.25	0.25	0.6	1.5	3
5	0.25	0.5	0.75	0.25	1.8	1.5	3
6	0.75	0.5	0.75	0.25	0.6	1.5	1
7	0.25	1.5	0.75	0.25	0.6	0.5	3
8	0.75	1.5	0.75	0.25	1.8	0.5	1
9	0.25	0.5	0.25	0.75	0.6	1.5	3
10	0.75	0.5	0.25	0.75	1.8	1.5	1
11	0.25	1.5	0.25	0.75	1.8	0.5	3
12	0.75	1.5	0.25	0.75	0.6	0.5	1
13	0.25	0.5	0.75	0.75	1.8	0.5	1
14	0.75	0.5	0.75	0.75	0.6	0.5	3
15	0.25	1.5	0.75	0.75	0.6	1.5	1
16	0.75	1.5	0.75	0.75	1.8	1.5	3

Table 3 - Factorial Effects Aliases (Design Expert output, Stat-Ease, 2004)

7 Factors: A, B, C, D, E, F, G	
Design Matrix Evaluation for Factorial Reduced 3FI Model	
Factorial Effects Aliases	
[Est. Terms] Aliased Terms	
[Intercept] = Intercept	
[A] = A + BCE + BFG + CDG + DEF	[AB] = AB + CE + FG
[B] = B + ACE + AFG + CDF + DEG	[AC] = AC + BE + DG
[C] = C + ABE + ADG + BDF + EFG	[AD] = AD + CG + EF
[D] = D + ACG + AEF + BCF + BEG	[AE] = AE + BC + DF
[E] = E + ABC + ADF + BDG + CFG	[AF] = AF + BG + DE
[F] = F + ABG + ADE + BCD + CEG	[AG] = AG + BF + CD
[G] = G + ABF + ACD + BDE + CEF	[BD] = BD + CF + EG
[ABD] = ABD + ACF + AEG + BCG + BEF + CDE + DFG	

3.1.1. Fractional Factorial Analysis of the First Response Variable: Period of the Price

Firstly, the contribution of the model terms is examined. The factors and their interactions with highest contribution are selected. Factors that have small contribution with respect to other factors / interactions are not selected if hierarchy does not make it necessary. Namely, if one or more of the terms selected for the model are interactions (such as AB consisting of the two parent terms, A and B) both of the parent terms should also be included in the model, even if they do not appear to be significant on their own.

Then the aliases analysis has been done. Since two way interactions are aliased with two way interactions it may be possible that a given two way interaction initially stated significant by the software may not be really significant. It may be possible that one of its aliases may be significant instead. Because of this reason when an interaction is added to the model, the most appropriate interaction between aliases is determined with the aid of factors meanings and their contributions to the model. The ANOVA Table of the reduced model is in Table 4.

Table 4- ANOVA Table of the First Response Variable (Design Expert output, Stat-Ease, 2004)

Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**P-value Prob > F**	
Model	25.04427	6	4.174045	78.89973	< 0.0001	significant
D-Other's Profit Margin	0.010851	1	0.010851	0.205105	0.6614	
E-Cons. Time	8.146267	1	8.146267	153.9845	< 0.0001	
F-Other's Cons. Time	11.53168	1	11.53168	217.9772	< 0.0001	
DE	2.312934	1	2.312934	43.72015	< 0.0001	
DF	2.312934	1	2.312934	43.72015	< 0.0001	
EF	0.729601	1	0.729601	13.79125	0.0048	
Residual	0.476128	9	0.052903			

The Model F-value of 78.90 implies the model is significant. According to P value (Prob>F) there is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are highly significant. In this case E, F, DE, DF, EF are significant model terms. Values greater than 0.1000 would indicate the model terms are not significant.

Also there are some other measures that help the user to fit a good model. R^2 is loosely interpreted as the proportion of the variability in the data "explained" by the analysis of variance model. Clearly values approaching 1 are more desirable. However, adding a variable to the model will always increase R^2, regardless of whether the additional variable is statistically significant or not. Thus while selecting the factors to the model, adjusted R^2 is checked. Adjusted R^2 is a variation of the ordinary R^2 statistics that reflects the number of factors in the model. It can be a useful statistic for complex experiments with several design factors when the impact of increasing or decreasing the number of model terms is evaluated. In general, the adjusted R^2 statistic will not always increase as variables added to the model. In fact, if unnecessary terms are added, the value of adjusted R^2 will often decrease. In the model in order to avoid the decrease in the adjusted R^2 value, the added factors are checked step by step during the analysis. Additionally, the predictive R^2 value, which is a measure of how good the model predicts a response value, is controlled. The adjusted R^2 and predictive R^2 should be within approximately 0.20 of each other to be in "reasonable agreement". If they are not, there may be a problem with either the data or the model (Montgomery, 2001). R^2, adjusted R^2 and predictive R^2 values of this model is 0.9813, 0.9689 and 0.9410 respectively. R^2 value is high enough and the predictive R^2 of 0.9410 is in reasonable agreement with the adjusted R^2 of 0.9689.

As the result of analysis of residual plots (which are not included here for the sake of space saving), it is concluded that the assumptions for error terms are not violated.

Finally, the Pareto Chart (Figure 4) and model graphs (Figure 5) given by Design Expert (Stat-Ease Inc., 2004) software can be interpreted in order to have further information about the effect of significant factors on the period of real estate prices. The one factor graphs show that for high values of construction time (for hypothetical construction firm and / or its competitors) the period of the real estate prices in Istanbul increases. When the interaction graphs are examined, it is seen that construction times' effect changes with others' profit margin and others' construction time. When others' profit margin is low, construction time has a higher effect on the price than high level of others' profit margin. Moreover, the effect of others' construction time is higher when others' profit margin is high. Also, the effect of others' construction time slightly increases when construction time is high.

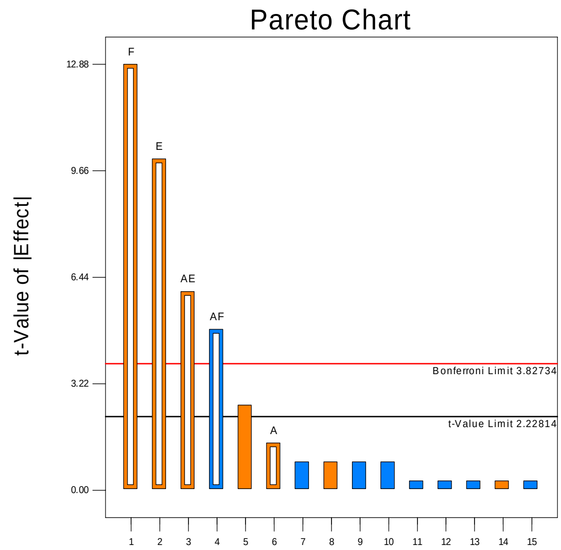

Figure 4- Pareto Chart of the First Response
Vertical axis shows the t-value of the absolute effects. This dimensionless statistic scales the effects in terms of standard deviations. The top line gives the Bonferroni limit, which provides a conservative limit corresponding to multiple pair wise tests. The bottom line is the t-value limit (Design Expert output, Stat-Ease, 2004).

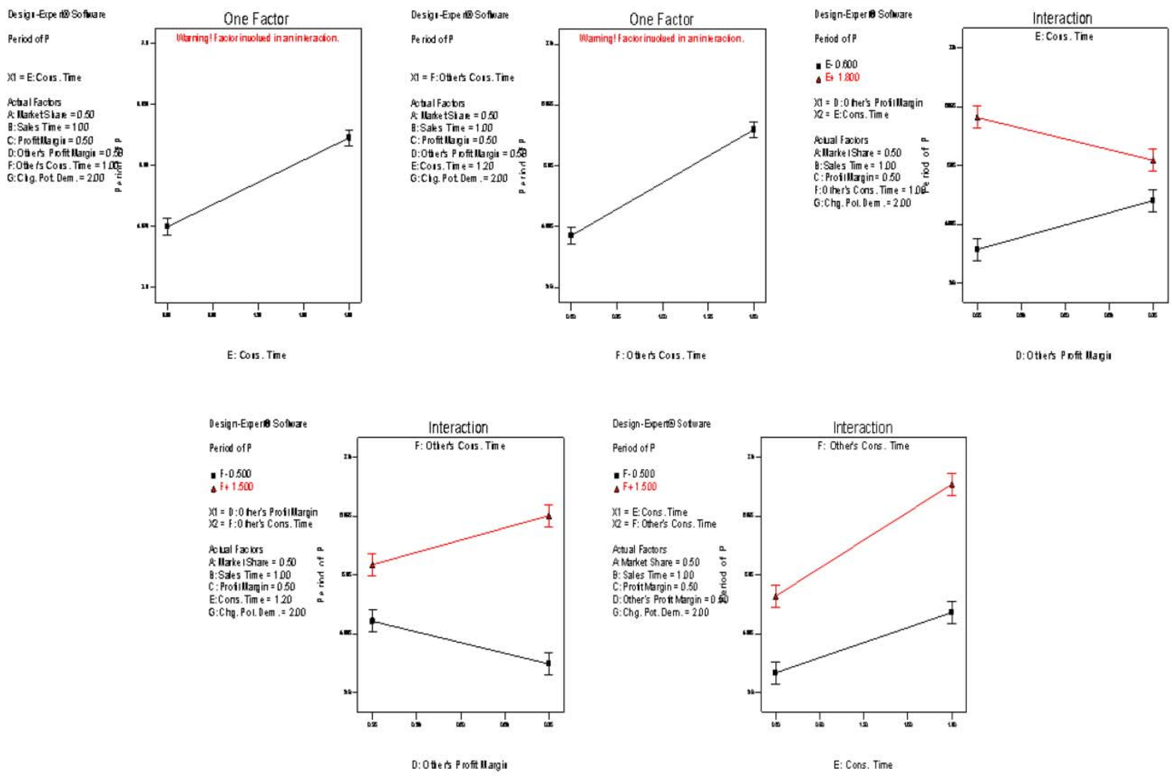

Figure 5- Model Variable and Interactions Graphs of the First Response
(Design Expert output, Stat-Ease, 2004)

3.1.2. Fractional Factorial Analysis of the Second Response Variable: Amplitude of the Price

The same analysis procedure as for the first response is followed for the second response variable. The ANOVA Table of the reduced model is in Table 5.

In this case A, B, F, AD, AE, AF are significant model terms. Also other measures that help the user to fit a good model are checked. R^2, adjusted R^2 and predictive R^2 values of this model is 0.9676, 0.9305 and 0.8306 respectively.

Finally, the Pareto Chart (Figure 6) and model graphs (Figure 7) given by Design Expert software can be interpreted in order to have further information about the effect of significant factors on the amplitude of real estate prices in Istanbul.

Table 5- ANOVA Table of the Second Response (Design Expert output, Stat-Ease, 2004)

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Model	4.53E+09	8	5.66E+08	26.11367	0.0002	significant
A-Market Share	1.84E+08	1	1.84E+08	8.507061	0.0224	
B-Sales Time	2.09E+09	1	2.09E+09	96.37339	< 0.0001	
D-Other's Profit Margin	73203068	1	73203068	3.375703	0.1088	
E-Cons. Time	58319912	1	58319912	2.689377	0.1450	
F-Other's Cons. Time	1.35E+09	1	1.35E+09	62.13271	0.0001	
AD	1.48E+08	1	1.48E+08	6.802703	0.0350	
AE	1.79E+08	1	1.79E+08	8.241749	0.0240	
AF	4.51E+08	1	4.51E+08	20.7867	0.0026	
Residual	1.52E+08	7	21685284			
Cor Total	4.68E+09	15				

The one factor graphs show that, as the market share of the firm and others firms' construction time increases and sales time decreases, the amplitude of the prices also increase. This shows that as empty houses are sold in a long time, the amplitude in price oscillation decreases and as the hypothetical firm which has higher construction time (1.2 years) with respect to its competitors (1 year) has higher market share the amplitude of price oscillation increases. It is similar for other firms' construction time. As their construction time increases, the amplitude of the prices also increases. The hypothetical firm does not have a significant effect on the amplitude because it is assumed that it has lower construction time than other firms in the market. So when the interaction graphs are examined, the interaction of other firms' profit margin and market share is significant. It shows that when market share is at small values, others' profit margin has higher effect on the response. As expected when the hypothetical firm's market share is small, other firms have higher power on the market so they have higher effect on the amplitude of the price especially when they have higher construction time. Also, when the market share is high, that is the hypothetical company is more effective in the market, the construction time of the company can influence the amplitude of prices, otherwise it's ineffective. The same is valid for the competitors.

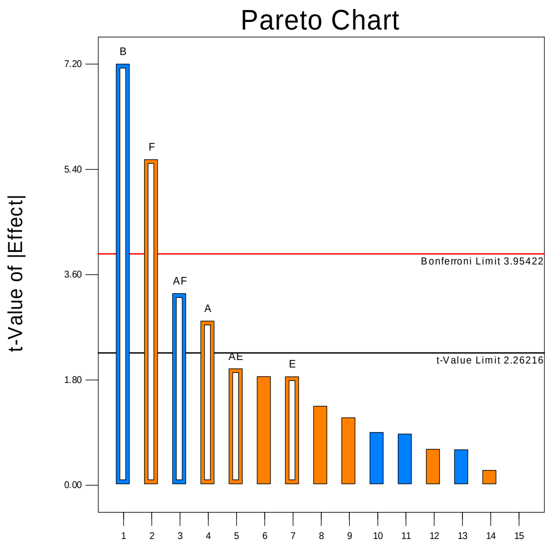

Figure 6- Pareto Chart of the Second Response (Design Expert output, Stat-Ease, 2004)

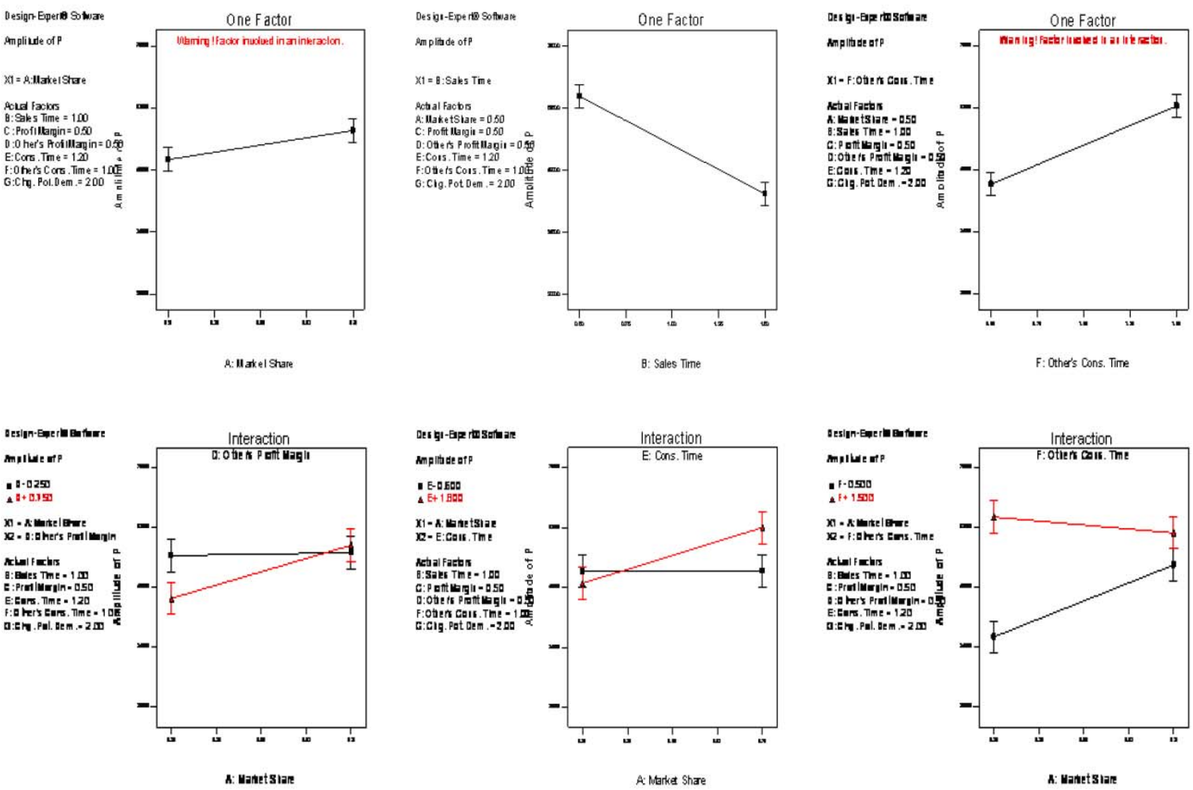

Figure 7- Model Graph of the Second Response (Design Expert output, Stat-Ease, 2004)

3.1.3. Fractional Factorial Analysis of the Third Response Variable: Mean of the Price

The ANOVA Table of the reduced model is given in Table 6. The Model F-value of 11.78 implies the model is significant. In this case C, D, E, F, DE, DF, EF are significant model terms.

Table 6- ANOVA Table of the Third Response (Design Expert output, Stat-Ease, 2004)

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Model	4.28E+08	7	61103101	11.78428	0.0012	significant
C-Profit Margin	84591581	1	84591581	16.31425	0.0037	
D-Other's Profit Margin	64095133	1	64095133	12.36133	0.0079	
E-Cons. Time	48459480	1	48459480	9.345848	0.0157	
F-Other's Cons. Time	1.17E+08	1	1.17E+08	22.4776	0.0015	
DE	40911395	1	40911395	7.890132	0.0229	
DF	41290396	1	41290396	7.963226	0.0224	
EF	31824330	1	31824330	6.137609	0.0383	
Residual	41481075	8	5185134			
Cor Total	4.69E+08	15				

Also other measures that help the user to fit a good model are checked. R^2, adjusted R^2 and predictive R^2 values of this model is 0.9116, 0.8342 and 0.6464 respectively.

The Pareto Chart (Figure 8) and model graphs (Figure 9) given by Design Expert software can be interpreted in order to have further information about the effect of significant factors on the mean of real estate prices in Istanbul.

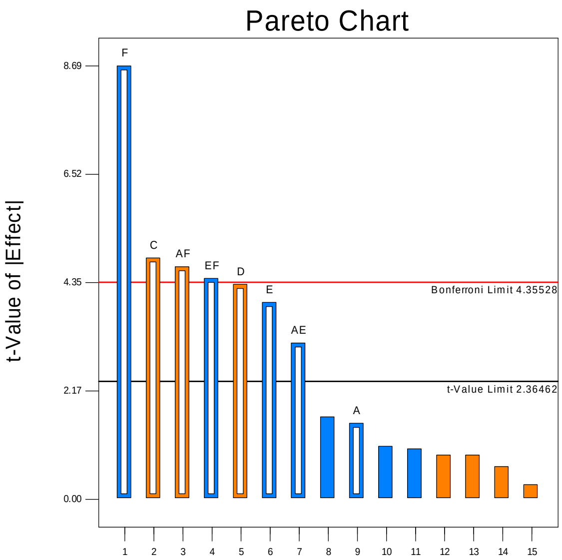

Figure 8- Pareto Chart of the Third Response (Design Expert output, Stat-Ease, 2004)

The one factor graphs show that as the profit margin (for hypothetical construction firm and / or its competitors) increases, the mean of the price also shows an increase while for high values of construction time (for hypothetical construction firm and / or its competitors) the mean of the real estate prices in Istanbul decreases. When the interaction graphs are examined, it is concluded that others' profit margin's effect changes with construction time and other's construction time. When construction time of the hypothetical firm is low, others' profit margin has a minor influence on the mean of the price. On the other hand, for low values of other's construction time, others' profit margin has a high effect. Also, the effect of others' construction time increases when construction time is high.

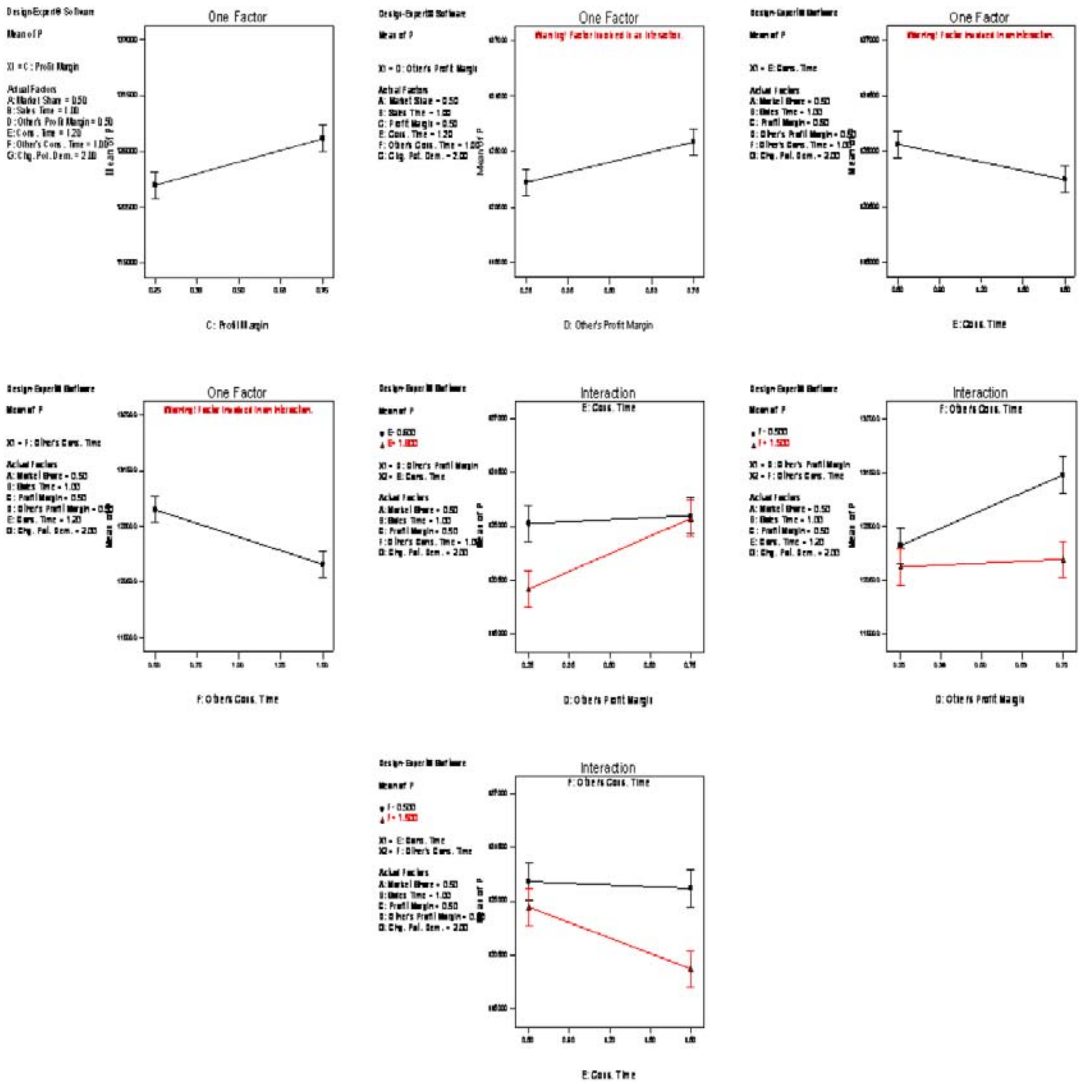

Figure 9- Model Graphs of the Third Response (Design Expert output, Stat-Ease, 2004)

3.2. Latin Hypercube Sampling

The seven factors shown in Table 1 are also used in LHS design. The range between high and low values given in the table is divided into 41 strata. At each run, one of these strata from each factor is randomly selected without replacement. For each combination, three replications are carried out, which makes a total of 123 runs. The experimental design that has been built for LHS is shown in Table 7 below. As in the case of fractional factorial design, the outputs are collected by Stella (isee systems, 2002) and analyzed in BTS II (Barlas et al., 1997).

Table 7- Latin Hypercube Sampling Experimental Design

Exp. No	Market Share	Sales Time	Profit Margin	Other's Prof. Mrg.	Construction Time	Other's Cons. Time	Coeff. of CPD
Run 0	0.675	1.3	0.675	0.475	0.63	0.7	2.85
Run 1	0.75	0.9	0.65	0.725	0.72	0.8	1.7
Run 2	0.5375	0.625	0.725	0.3875	0.6	1.375	2.6
Run 3	0.6125	1.5	0.35	0.3125	0.99	0.65	2.25
Run 4	0.3375	0.575	0.575	0.6375	1.02	1.325	2.3
Run 5	0.6	0.825	0.6	0.3375	1.11	0.9	1.3
Run 6	0.3875	0.5	0.4625	0.425	1.08	0.6	1.65
Run 7	0.475	1.025	0.7	0.7375	1.17	1.2	2.2
Run 8	0.5875	0.725	0.3375	0.5	1.77	0.875	1.35
Run 9	0.525	1.475	0.6125	0.5625	1.2	0.5	2.35
Run 10	0.7125	1.375	0.3	0.35	1.5	0.75	1.4
Run 11	0.325	1.2	0.5375	0.575	1.23	0.95	2.4
Run 12	0.6375	1.35	0.6375	0.3	1.74	1.45	2
Run 13	0.275	1.45	0.7125	0.65	1.59	1.25	1.55
Run 14	0.375	0.95	0.4125	0.4875	1.56	1.3	2.55
Run 15	0.5	0.7	0.625	0.5125	1.32	1.425	2.05
Run 16	0.625	1	0.3125	0.275	0.78	1.225	1.95
Run 17	0.6625	1.275	0.75	0.2875	1.71	1.475	2.5
Run 18	0.725	0.85	0.6625	0.325	0.75	1.4	1
Run 19	0.5625	1.4	0.3875	0.55	0.81	1.275	1.6
Run 20	0.575	1.125	0.45	0.25	1.47	1.5	2.95
Run 21	0.55	0.75	0.5625	0.525	1.44	0.725	1.45
Run 22	0.5125	0.65	0.325	0.4375	1.68	1.025	2.75
Run 23	0.2875	0.975	0.4	0.3625	0.84	0.575	2.9
Run 24	0.4875	1.15	0.525	0.6125	0.66	1.175	1.2
Run 25	0.65	0.925	0.5125	0.5375	1.41	0.975	1.15
Run 26	0.7375	1.425	0.4875	0.4125	1.38	1.1	1.05
Run 27	0.2625	0.55	0.55	0.6	1.35	1.125	2.1
Run 28	0.4625	1.175	0.2875	0.675	1.26	0.525	1.5
Run 29	0.4	1.05	0.475	0.375	0.69	0.625	2.65
Run 30	0.25	0.775	0.375	0.4	1.29	0.775	1.25
Run 31	0.4125	1.225	0.275	0.45	0.93	0.825	2.7
Run 32	0.6875	0.875	0.5	0.75	1.65	1.075	3
Run 33	0.425	1.075	0.25	0.7	1.8	1	1.75
Run 34	0.35	0.6	0.6875	0.4625	0.9	0.85	1.1
Run 35	0.7	1.25	0.7375	0.6625	0.87	0.675	1.9
Run 36	0.3	1.1	0.425	0.2625	0.96	1.15	2.15
Run 37	0.4375	0.525	0.4375	0.625	1.05	0.925	1.8
Run 38	0.45	0.675	0.5875	0.7125	1.53	1.35	1.85
Run 39	0.3125	0.8	0.3625	0.6875	1.14	0.55	2.45
Run 40	0.3625	1.325	0.2625	0.5875	1.62	1.05	2.8

3.2.1. Latin Hypercube Analysis of the First Response Variable: Period of the Price

Similar to the analysis explained in the previous section, the factors having high contribution to the output are determined. Unlike the fractional factorial design, two-way interactions are not aliased with each other. In this way, interaction effects can be clearly distinguished. Regarding main factors and two-way interactions, the ones having high contributions are selected and analysis of variance is carried out. The ANOVA Table of the reduced model is in Table 8.

Table 8- ANOVA Table of the First Response (Design Expert output, Stat-Ease, 2004)

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Model	28.42766	7	4.061094	89.37309	< 0.0001	significant
A-Market Share	0.242966	1	0.242966	5.346978	0.0271	
D-Other's Profit Margin	0.001865	1	0.001865	0.041035	0.8407	
E-Cons. Time	7.149895	1	7.149895	157.3488	< 0.0001	
F-Other's Cons. Time	8.855501	1	8.855501	194.8843	< 0.0001	
AD	0.030922	1	0.030922	0.680495	0.4153	
AE	0.947639	1	0.947639	20.85484	< 0.0001	
AF	0.432575	1	0.432575	9.519742	0.0041	
Residual	1.499513	33	0.04544			
Cor Total	29.92717	40				

The Model F-value of 89.37 implies the model is significant. According to P value (Prob>F) there is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. Values of "Prob > F" less than 0.0500 would indicate model terms are significant. In this case A, E, F, AE, AF are significant model terms. Values greater than 0.1000 would indicate the model terms are not significant.

R^2, adjusted R^2 and predictive R^2 values of this model is 0.9499, 0.9393 and 0.9336 respectively. R^2 value is high enough and the predictive R^2 of 0.9336 is in reasonable agreement with the adjusted R^2 of 0.9393. As the result of analysis of residual plots, it is concluded that the assumptions for error terms are not violated.

Finally, Pareto Charts (Figure 10) and model graphs given by Design Expert software (Figure 11) can be interpreted in order to have further information about the effect of significant factors on the period of real estate prices. Market share, construction time and others' construction time have positive effect on the period of the price oscillations. When market share is high the construction time of the hypothetical company has effect on the period, on the other hand when market share is low, the competitor companies' construction time effects the price oscillation periods.

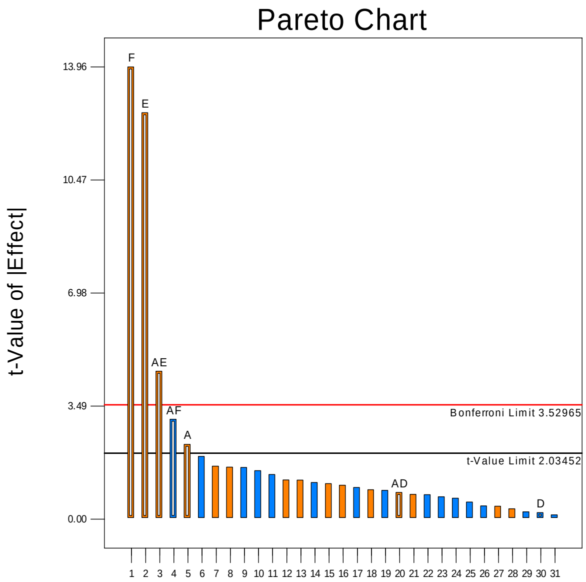

Figure 10- Pareto Chart of the First Response (Design Expert output, Stat-Ease, 2004)

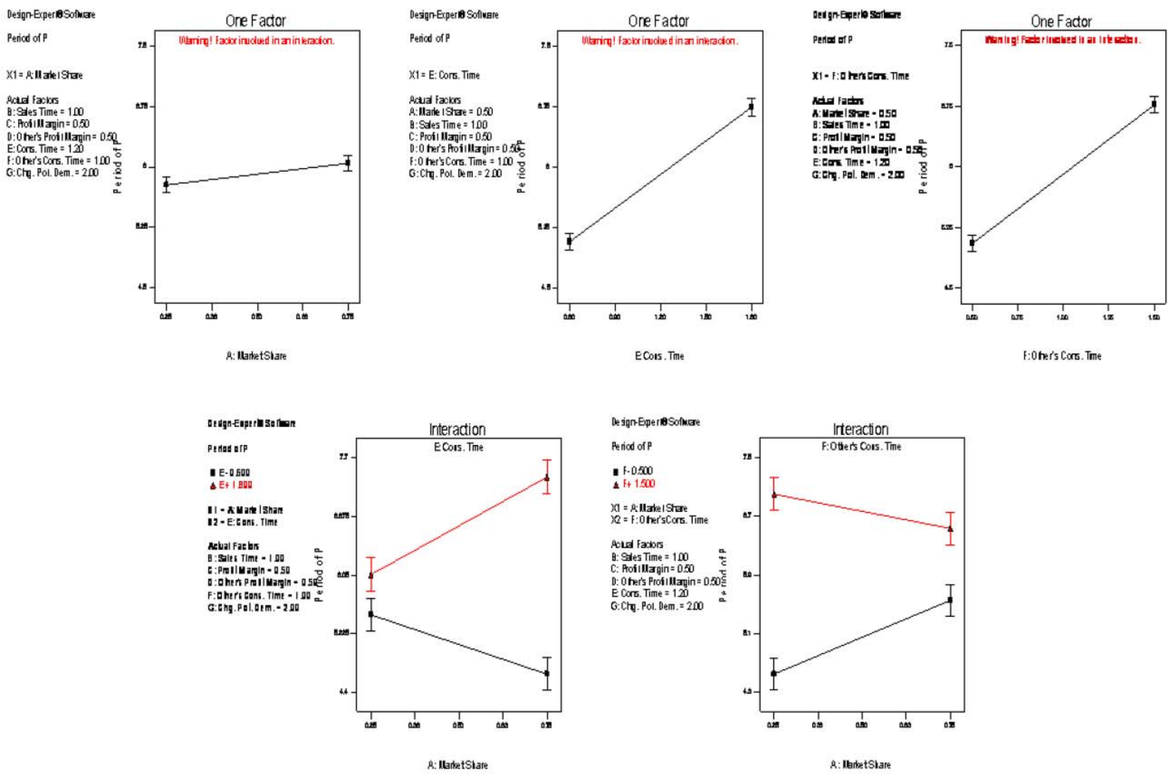

Figure 11- Model Graphs of the First Response (Design Expert output, Stat-Ease Inc., 2004)

3.2.2. Latin Hypercube Analysis of the Second Response Variable: Amplitude of the Price

The ANOVA Table of the reduced model is in Table 9.

Table 9- ANOVA Table of the Second Response (Design Expert output, Stat-Ease, 2004)

Source	Sum of Squares	df	Mean Square	F Value	P-value Prob > F	
Model	3.17E+09	9	3.52E+08	19.17983	< 0.0001	significant
A-Market Share	9756840	1	9756840	0.531025	0.4716	
B-Sales Time	1.09E+09	1	1.09E+09	59.55386	< 0.0001	
C-Profit Margin	13228321	1	13228321	0.719964	0.4027	
E-Cons. Time	4.01E+08	1	4.01E+08	21.79814	< 0.0001	
F-Other's Cons. Time	3.84E+08	1	3.84E+08	20.91945	< 0.0001	
G-Chg. Pot. Dem.	318796.1	1	318796.1	0.017351	0.8961	
AC	1.05E+08	1	1.05E+08	5.714139	0.0231	
AG	89246671	1	89246671	4.857335	0.0351	
BF	97534973	1	97534973	5.308434	0.0281	
Residual	5.7E+08	31	18373586			
Cor Total	3.74E+09	40				

In this case B, E, F, AC, AG, BF are significant model terms.

R^2, adjusted R^2 and predictive R^2 values of this model is 0.8478, 0.8036 and 0.7059 respectively.

As the result of analysis of residual plots, it is concluded that the assumptions for error terms are not violated.

The Pareto Chart and model graphs are given in Figure 12 and 13, respectively. One-factor model graphs show that the amplitude of the price oscillations is sensitive to the sales time. As the sales time increases, the amplitude decreases while it increases with increasing construction times. The interaction plots explain that when the market share of the hypothetical construction company is high, the profit margin of the hypothetical company is more effective on the amplitude of the prices. When the market share is low (i.e. competitors' market share is high) the change in potential customer demand becomes more effective on the amplitude of the price oscillations. Also when the sales time increases, the construction time of the competitors becomes more effective in the model.

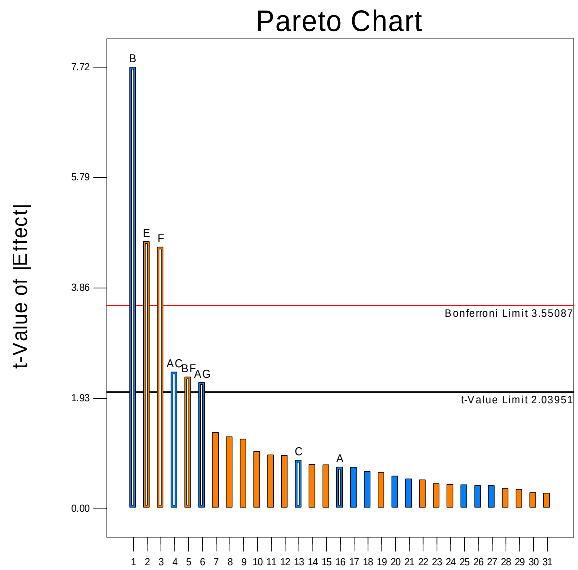

Figure 12- Pareto Chart of the Second Response (Design Expert output, Stat-Ease, 2004)

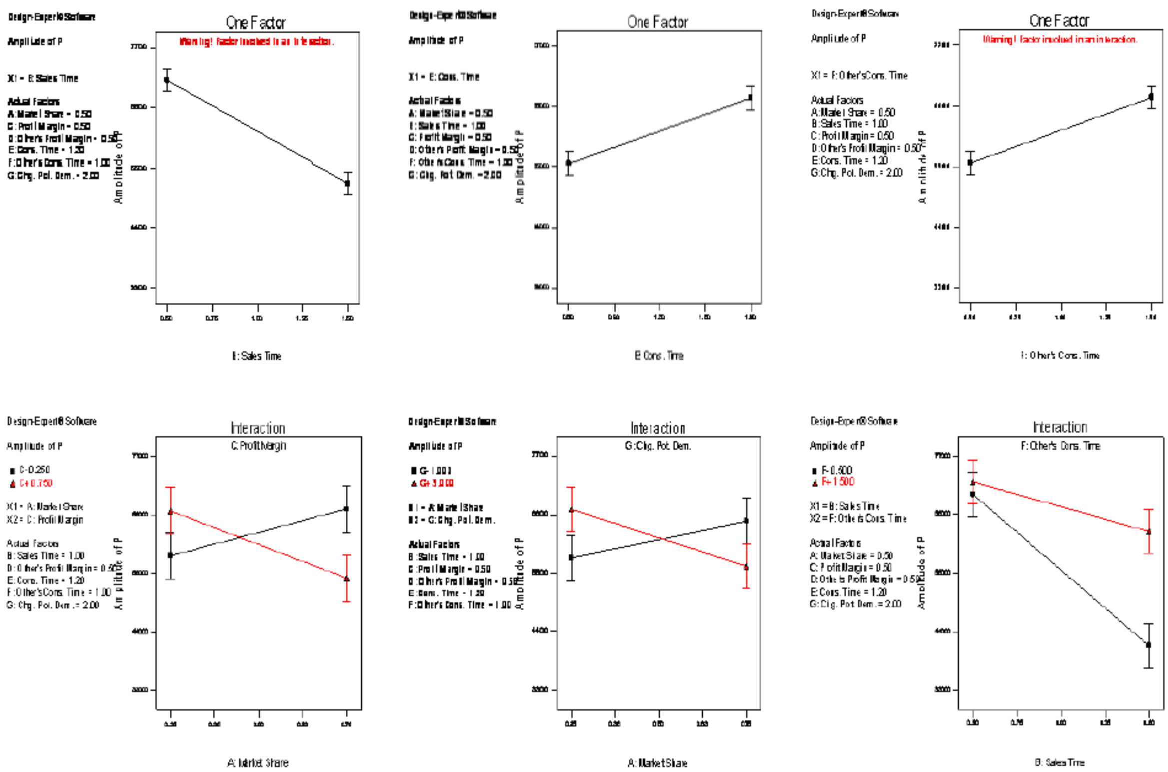

Figure 13- Model Graphs of the Second Response (Design Expert output, Stat-Ease, 2004)

3.2.3. Latin Hypercube Analysis of the Third Response Variable: Mean of the Price

The ANOVA Table of the reduced model is given in Table 10.

Table 10 - ANOVA Table of the Third Response (Design Expert output, Stat-Ease, 2004)

Source	**Sum of Squares**	**df**	**Mean Square**	**F Value**	**P-value Prob > F**	
Model	2.72E+08	7	38787603	25.51542	< 0.0001	significant
A-Market Share	41858.2	1	41858.2	0.027535	0.8692	
C-Profit Margin	25682717	1	25682717	16.89471	0.0002	
D-Other's Profit Margin	59505929	1	59505929	39.14444	< 0.0001	
E-Cons. Time	45258943	1	45258943	29.77243	< 0.0001	
F-Other's Cons. Time	61079744	1	61079744	40.17973	< 0.0001	
AC	14562522	1	14562522	9.579579	0.0040	
AD	18135125	1	18135125	11.92972	0.0015	
Residual	50165378	33	1520163			
Cor Total	3.22E+08	40				

In this case C, D, E, F, AC, AD are significant model terms. R^2, adjusted R^2 and predictive R^2 values of this model is 0.8441, 0.8110 and 0.7845 respectively. R^2 value is high enough and the predictive R^2 of 0.7845 is in reasonable agreement with the adjusted R^2 of 0.8110. As the result of analysis of residual plots, it is concluded that the assumptions for error terms are not violated.

Finally, Pareto Chart and model graphs given by Design Expert software (Figure 14 and 15, respectively) can be interpreted in order to have further information about the effect of significant factors on the mean of real estate prices. The one factor graphs show that for high values of profit margin and competitors' profit margin and low values of construction time and competitors' construction time the mean real estate prices in Istanbul increases. The interaction plots explain that when the market share of the hypothetical construction company is high, the profit margin of the hypothetical company is more effective on the mean of the prices. The profit margin increases the cost of houses and thus the prices. When the market share is low (i.e. competitors' market share is high) the profit margin of the competitors becomes effective.

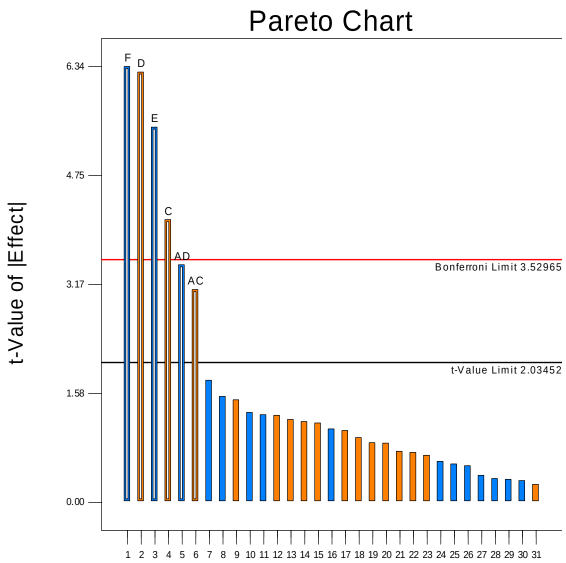

Figure 14- Pareto Chart of the Third Response (Design Expert output, Stat-Ease, 2004)

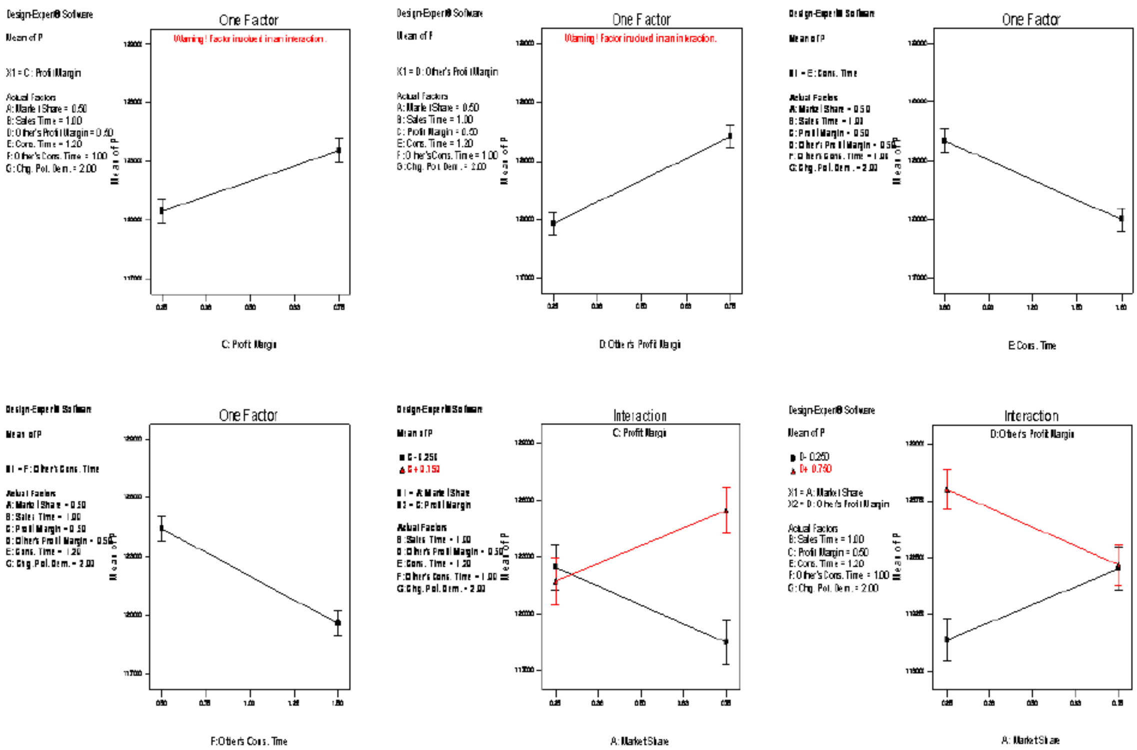

Figure 15- Model Graphs of the Third Response (Design Expert output, Stat-Ease, 2004)

3.3. Comparison of Two Designs

In order to determine the variables to which the model is sensitive, two experimental designs; fractional factorial design and Latin hypercube sampling are used. Table 11 summarizes the significant factors found as a result of analysis of variance. As it is seen, the significant factors for different responses can differ. For example factors C (profit margin) and D (others' profit margin) are effective only on the mean of the prices. These factors change the level of the profit that construction companies gain and thus the price. However, they do not have significant influence on oscillation behavior pattern. Moreover, factor B (sales time) has an effect on the amplitude of the price oscillations but not on the period and the mean. It is not very obvious why it can affect the amplitude but not the others. The power of system dynamics models is that, they can discover the relations that may not be found by static interpretation.

Table 11- Significant factors for three responses found using two different designs

	Fractional	**LHS**
Period	E, F, DE, DF, *EF*	*A* , E, F, AE, AF
Amplitude	*A* , B, F, *AD, AE, AF*	B, *E* , F, *AC, AG, BF*
Mean	C, D, E, F, *DE* , EF, *DF*	C, D, E, F, *AC* , AD

Table 11 also shows that the significant factors are different for different experimental designs. Fractional factorial design used in this study aliases two-way interactions with each other. In the analysis of fractional factorial design, we tried to select the interactions that seem to be most reasonable. However, due to the fact that unexpected relationships between variables can exist, this is not a straightforward task. The interactions whose aliases have turned out to be significant in the other design are shown in regular font and the ones that are not are in italic font. For example, as it is seen in Table 3, DE interaction is aliased with AF interaction and DF is aliased with AE. Although DE and DF seemed more reasonable interactions in fractional factorial design analysis, the results obtained by LHS design give further information for interpretation of interactions. This indicates the difficulty of interpretation problem of the fractional factorial design. It is advised to use the factors and interactions that are common in both designs.

4. Conclusion and Further Studies

In this study a system dynamics model of real estate prices in Istanbul (Barlas et al., 2007) is used as a platform to understand the respond of real estate price oscillations to changes in different factors. In system dynamics methodology, it would be useful to determine the variables to which the model is sensitive, by using a more formal experimental design analysis. A complete sensitivity analysis can uncover the factors that should be controlled or monitored more carefully to obtain desired behavior.

In this research we do a comparative analysis of two different design methods for sensitivity analysis: factorial designs and Latin hypercube sampling. One of the main conclusions of this study is that the significant factors affecting the selected responses can turn out to be different in the different experimental designs. It is safe to choose the variables that are significant in both designs as the variables to which the model is sensitive. For the real estate price model considered in this study, the important variables influential on the period of price oscillations are construction time of the company and the sector; the variable influential on the amplitude is sales time and variables influential on the level of price are profit margins. Another important observation is the subjectivity in selecting reasonable interactions among the aliases in fractional factorial design. Therefore, fractional factorial design should be used with caution when the interpretation of interactions is not obvious.

Another purpose of this study was to investigate if there are differences between the performances of the methods used in sensitivity analysis. Their pros and cons are observed and discussed such as: LHS assumes that the input parameters are independent of each other so if interactions are important fractional factorial design is better. However fractional factorial design output is sometimes hard to interpret. Moreover, since LHS is not a fractional design, it can cover the whole sample space and can provide this information with less number of experiments.

In further studies, the model may be expanded to include a more sophisticated model for the demand creation, the effect of the real estate market on the economy, feedback mechanisms to change the market share and the cost, limitations of availability of resources on the constructions, the effect of interest rates and other investment opportunities on the supply side. Also, other sensitivity analysis methods, such as random sampling, full factorial design and Taguchi method can be applied to this model. In this way, the variables to which the model is sensitive can be determined with higher confidence and comparisons between different experimental designs can be stronger. Also different design methods can be tested on other models to reach more reliable conclusions.

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