Does system dynamics or control theory help you to strike a balance?

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The performance of laymen on tasks testing for knowledge of basic dynamics, such as the bathtub tasks, is consistently weak. There are some reports of beneficial effects of introductory courses in system dynamics or a strong mathematical background. This study investigates the effects of a system dynamics background beyond the introductory level, and strong mathematical background including courses in differential equations and control theory, on the strategies applied to, as well as performance in, the rabbits-and-foxes task. The task objective is to establish equilibrium in a predator-and-prey system. These well-educated participants performed no better than social science students. The strategies applied differed however, and the dynamic systems educated participants did not demonstrate as much misconceptions about the system as social science students have been found to do. The weak performance raises the question if there are better ways to develop a mature concept of equilibrium.

KEY WORDS: basic dynamics, education, balance, equilibrium, predator-prey, modeling, concept development

Since the publication of the bathtub study by Booth Sweeney and Sterman (2000), a number of studies have accumulated that demonstrate laymen's weak grasp of basic dynamics (see e.g., Cronin, Gonzalez, and Sterman, 2006; Jensen and Brehmer, 2003; Moxnes, 2000; Moxnes and Saysel, 2004; Ossimitz, 2002). The pattern of performance in the bathtub (or cash flow) tasks (Booth Sweeney and Sterman, 2000), with the particularly weak performance in the more difficult version with the saw-tooth pattern of the inflow, has been replicated in several studies with various student populations (Armenia, Onori, and Bertini, 2004; Fisher, 2003; Kapmeier, 2004; Kubanek, 2003; Lyneis and Lyneis, 2003; Ossimitz, 2002; Quaden and Ticotsky, 2003; Zaraza, 2003). Mathematical education (calculus) (Fisher, 2003) and introduction to systems dynamics (Kainz and Ossimitz, 2002; Lyneis and Lyneis, 2003; Zaraza, 2003) do enhance performance, albeit without entirely eliminating the errors people make when performing these tasks.

Performance in the department store task (Sterman, 2002), inspired by a task developed by Ossimitz (2002), has also proven consistent over various student populations (Armenia, Onori, and Bertini, 2004; Kapmeier, 2004; Pala and Vennix, 2005) and analogous tasks (Cronin, Gonzalez, and Sterman, 2006; Kainz and Ossimitz, 2002; Ossimitz, 2002). Mathematical talent (Quaden and Ticotsky, 2003), and introductory system dynamics (Lyneis and Lyneis, 2003) benefits performance, but the effects are small. Simplifying the task does little to improve performance (Cronin and Gonzalez, 2007; Cronin, Gonzalez, and Sterman, 2006). About 50% of a group of university students failed to calculate the stock accumulated from a constant inflow and a constant outflow (Cronin, Gonzalez, and Sterman, 2006). Their performance did, however, improve over trials in the original department store task, when asked repeatedly for a new answer when they had answered incorrectly. Even then, they did not all answer all four questions correctly (Cronin, Gonzalez, and Sterman, 2006). It is impossible, from this study, to tell if the participants eventually arrived at the correct answer as a result of reflection, or if they just tried different years when something potentially interesting seemed to be happening in the graph, and thereby happened to chance on the correct one.

While there was a beneficial effect of attending a system dynamics introductory course on the department store task, Pala and Vennix (2005) found no increase in performance on either the manufacturing task (Sterman, 2002), or the global warming CO_2 zero-emission task (Sterman and Booth Sweeney, 2002).

There are, at least, two populations who ought to perform well on these basic tasks: people who have studied system dynamics (quite obviously), and people who have studied control engineering. An introductory course in system dynamics appears to enhance performance on, at least, the less complex tasks, while Armenia, Onori and Bertini (2004) found no better performance by undergraduate engineering students, who had taken a course in control theory, than is generally reported in either the bathtub or cash flow tasks, the department store task, or the manufacturing task.

The task we use in our studies, the rabbits-and-foxes task, requires that the participants bring about and maintain equilibrium in a predator-and-prey ecology (Jensen and Brehmer, 2003). This is a more complex task than the aforementioned ones. It has a lot in common with Moxnes's reindeer management and fishing fleet tasks (Moxnes, 1998a, b, 2000). Prior experience with the rabbits-and-foxes task actually facilitates performance in the reindeer management task (Jensen, 2005).

With the global warming CO_2 zero-emission task (Sterman and Booth Sweeney, 2002) the rabbits-and-foxes task shares the goal of reaching and sustaining a balance. A stronger mathematical background proved somewhat beneficial in a simplified version of this task (Moxnes and Saysel, 2004).

Performance in earlier experiments with the task, with laymen in terms of knowledge about dynamic systems as participants, has been a fairly consistent 50 % success rate. Success has mostly been achieved by trial-and-error. Efforts to provide hints to the solution have met with no success (Jensen and Brehmer, 2008). For the present study, I recruited the participants with the strongest background in system dynamics and control theory, respectively, that I could find in sufficiently large numbers. The system dynamics experienced group consisted of masters students in system dynamics at the University of Bergen. These are students who have continued their studies in system dynamics beyond the introductory level. The control theory experienced group consisted of masters students in engineering at the Royal Institute of Technology in Stockholm, in their third year, or later, of study, and who had taken at least one course in control theory. These students have experience with dynamic systems from their control theory studies, and they have studied differential equations prior to the control theory courses. These two groups were compared to a group of undergraduate students who were laymen in terms of knowledge about dynamic systems and who had no advanced education in mathematics, a group of participants similar to those who have participated in our earlier studies.

One purpose was to see if an education in system dynamics prepares people for solving the rabbits-and-foxes task. System dynamic students model and simulate systems themselves, which ought to be a useful experience. There was also a wish to see if a more formal approach to dynamic systems, as is taught in control engineering courses, would suffice to solve the task. The question was how these groups of participants would approach the task, and with what success, compared to people with no training in either control theory or system dynamics.

The description of the rabbits-and-foxes system

In the rabbits-and-foxes task (Jensen and Brehmer, 2003), the participants receive a complete description of the system (see Appendix A). The system is a computer-simulated predator-and-prey ecology, where the predators are foxes feeding on rabbits, their prey. The system description consists of four short sentences:

1. Every rabbit produces two offspring a year.
2. Every fox eats 4% of the rabbits a year.
3. For every 180 rabbits consumed a new fox is born.
4. 20% of the fox population dies each year.

Very few of our prior participants have, however, been able to make effective use of it and achieve control. They failed to draw the correct conclusions from the description (Jensen, 2003, 2005; Jensen and Brehmer, 2003, 2008).

The predator-prey system used is described by the Lotka-Volterra equations (Boyce and DiPrima, 1997; Lotka, 1925; Volterra, 1926). The equations are not very intuitive, particularly not the conversion of the rabbits consumed by the foxes into newborn foxes. In addition, the parameters describing, and determining, system behavior, are quite arbitrarily chosen. The sole objective was to achieve a system that was reactive, yet controllable, not reacting too strongly to minor changes, but who has ever heard of a rabbit that produces only two offspring a year, for example? This kind of system is, however, exactly what we wanted. If the simulated system behaves in accordance with people's intuitions, the participants would not need to make use of the system description. They could then solve the task by simply following their intuition, and we would not learn anything about their ability to infer system behavior from system structure. It is when dealing with a non-intuitive system that the support of a good model is needed.

The rabbits-and-foxes task

The task setting is an isolated island, populated exclusively by rabbits and foxes. The participants can order foxes to be shipped to or away from the island once every simulated year (see Appendix A). They receive feedback on their performance in two line graphs, one for the rabbit population and one for the fox population (see Fig. 3).The rabbit population can only be influenced by means of the fox population. The task is to find the population sizes where the system reaches equilibrium. Then both populations will remain constant, and no further shipping of foxes will be needed. Each trial extends over 30 simulated years, and the participants are allowed numerous trials, as described below. The initial populations are 40 foxes and 500 rabbits.

The causal model of the rabbits-and-foxes system

The system consists of two sub-systems, one for each population. From the first sentence in the system description: *Every rabbit produces two offspring a year*, it should be concluded that new rabbits are added to the rabbit population as a consequence of reproduction, and the size of this addition is dependent on the present size of the rabbit population. The more rabbits there are the more new rabbits there will be, i.e. the relation is positive.

From the second sentence: *Every fox eats 4% of the rabbits a year*, one should conclude that in the model, the only cause of death for rabbits is to be eaten by foxes, and that the number of rabbits eaten, and hence removed from the rabbit population, depends on, and is positively related to, *both* the size of the fox population *and* the size of the rabbit population.

Note that there is no direct access to the rabbit population. The participants are not allowed to add or remove rabbits, only to add or remove foxes. They need to conclude that the only way to affect the size of the rabbit population, beyond what is caused by the system's inherent dynamics, is to change the size of the fox population. The participants also have to realize that the goal of a constant rabbit population is reached when equally many rabbits are eaten by foxes as are born.

From the forth sentence: *20% of the fox population dies each year*, it can be concluded that a fixed proportion of the fox population dies every year. This means that the number of foxes removed from the population by death is larger the larger is the population, i.e. the relation is positive. The more numerous the deaths, the smaller the number of foxes, i.e., that relation is negative.

The somewhat peculiar, but at the same time the most important, feature of the system is described in sentence three: *For every 180 rabbits consumed a new fox is born*. The participants need to consider, and understand, the underlying logic: that even though more foxes ought to imply more puppies, the reproduction of foxes also depends on the availability of food. Hence, additions to the fox population depends on both the number of rabbits available and the number of foxes present to hunt them, since these are the factors determining the number of rabbits eaten by foxes. Both these relations are positive.

As mentioned above, a fixed proportion of the foxes die each year, so what is required to keep the fox population constant is to make sure that a corresponding number of new foxes is born. In contrast to the rabbit sub-system, the participants are allowed direct access to the fox system. Once every simulated year they can add or remove any number of foxes.

Figure 1 combines the two sub-models into a complete causal model of the system.

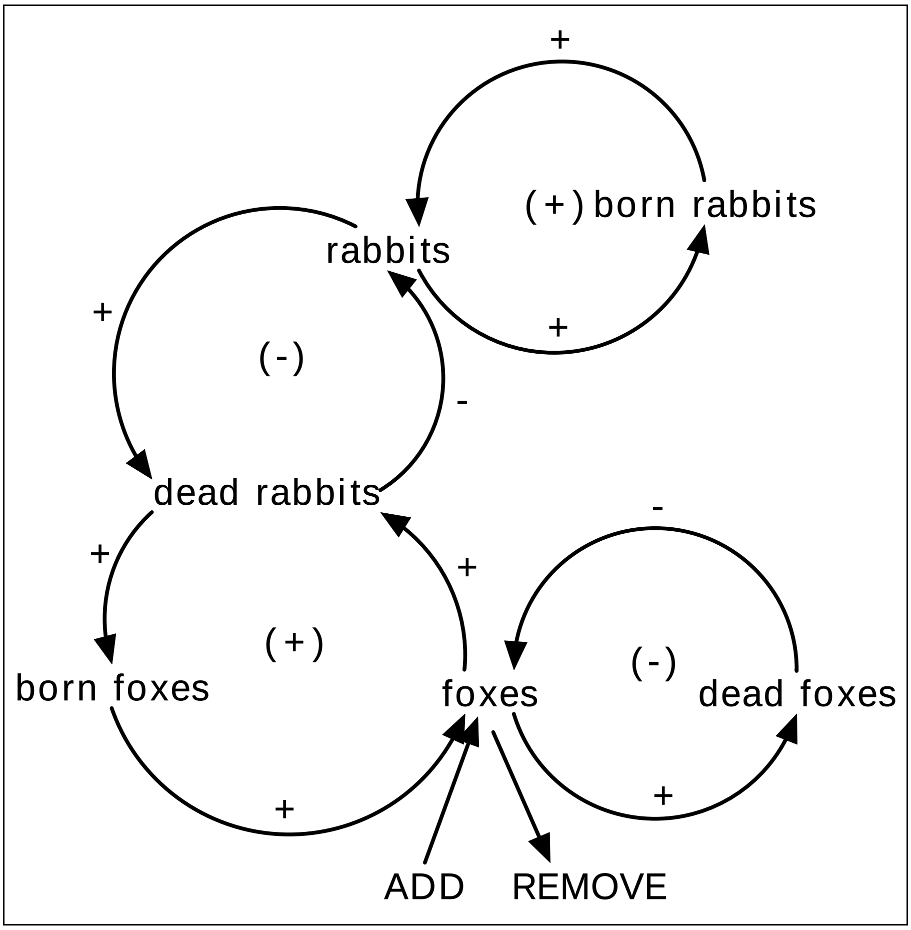

Figure 1. The causal model of the rabbits-and-foxes system

To keep the rabbit population constant, the fox population has to be large enough to eat as many rabbits as there are rabbits born (but no larger). And, to keep the fox population constant, the rabbit population has to be large enough (but no larger) for the foxes to eat enough rabbits so that equally many new foxes are born as the number that dies. Hence, to control the foxes is a question of controlling their food supply, i.e., the number of rabbits.

In essence, to solve the task requires drawing the correct conclusions from the four sentences in the system description, and bringing them together in a representation of the entire system, as in Figure 1, or at least identifying, and representing, the central relation connecting the two sub-systems.

This shows that it is possible to solve the task by logic alone. The numerical information is not necessary, and, hence, no mathematics. The causal loop diagram in Figure 1 is just to illustrate the causal structure of the system. Not one of our prior participants has ever attempted to put the causal relations of the system on paper. Modeling, even in this simple form, does not come naturally to people. Questioning the participants on their reasoning while they performed the task, revealed that several of them applied explanations outside of, and contradicting, the system description. They seemed to refer to their everyday conceptions and not accepting the system description as *the* true and complete description of the system (Jensen and Brehmer, 2008). We have even removed the system description from the task instructions. The participants only received the general story of the rabbits and foxes, and were asked to bring the system to equilibrium. This had no effect on performance. We received the, by now, well-established 50 % success rate achieved by trial-and-error (Jensen and Brehmer, 2008).

Schools, in general, offer their pupils little training in building and using models, even in science classes (Greca and Moreira, 2002). They are taught ready-made models as facts, but have a rather weak grasp of what a model really is and how it can be used, i.e. they lack an established model concept (Schwartz and White, 2005). Alarmingly, this appears to be true of even a fair number of science teachers (Van Driel and Verloop, 1999). Ossimitz (2000) found that ninth-grade students were able to quickly learn to make causal loop diagrams, and that their reasoning about systems benefited from this activity. This did not, however, entirely remove their tendency to seek explanations to events outside of, and in conflict with, their own models.

The social science (mainly psychology) students who have participated in our prior studies, have frequently complained of their low ability in, and dislike of, mathematics. In the present study, a qualitative system description, describing the logical structure of the system devoid of numbers, has been injected in the task description before the original system description with numbers (see Appendix B). This could invite the participants to consider the causal structure of the system.

The mathematical model of the rabbits-and-foxes system

The engineering students participating in this study have, before taking a course in control theory, studied differential equations. They are also quite familiar with systems of equations. The Lotka-Volterra equations are frequently used as an example in textbooks on differential equations (see, for example, Arnol'd, 1992; Boyce and DiPrima, 1997; Braun, 1993). The system is, in this case, described by a mathematical model. The mathematical model of the rabbits-and-foxes system takes on the following form:

$R(t)$: Rabbit population; $F(t)$: Fox population

$$\frac{\partial R(t)}{\partial t} = \alpha R(t) - \beta R(t)F(t);\ \ \alpha = 2;\ \ \beta = 0.04$$
$$\frac{\partial F(t)}{\partial t} = \beta\chi R(t)F(t) - \delta F(t);\ \ \chi = \frac{1}{180};\ \ \delta = 0.2$$

The engineering students were expected to construct the equations above from the system description, since this is the kind of modeling most familiar to them. In equilibrium, there are no changes in the population, i.e. they are constant. If the engineering students thought of this, as I expected them to do, they would be able to perform the following calculations:

$$\frac{\partial R(t)}{\partial t} = 0; \frac{\partial F(t)}{\partial t} = 0 \ \rightarrow\ F = \frac{2}{0.04} = 50;\ \ R = \frac{0.2 \cdot 180}{0.04} = 900\,,$$

and thereby calculate the populations in equilibrium. With this knowledge, steering the system to these population sizes is a fairly straightforward task. This indicates that experience with differential equations should be sufficient to solve the task. By including studies in control theory as a requirement for participation, I hoped to get participants with experience of applying the mathematics to real dynamic systems, and experience with modeling and studying the behavior of these systems.

The stock-and-flow model of the rabbits-and-foxes system

Figure 2 shows the stock-and-flow model of the rabbits-and-foxes system. Stock-and-flow models illustrate the structure of the modeled systems (Forrester, 1961). They also include the quantitative aspects of the task. They keep the equations of the mathematical model (see above) intact, while, at the same time, illustrating the structure.

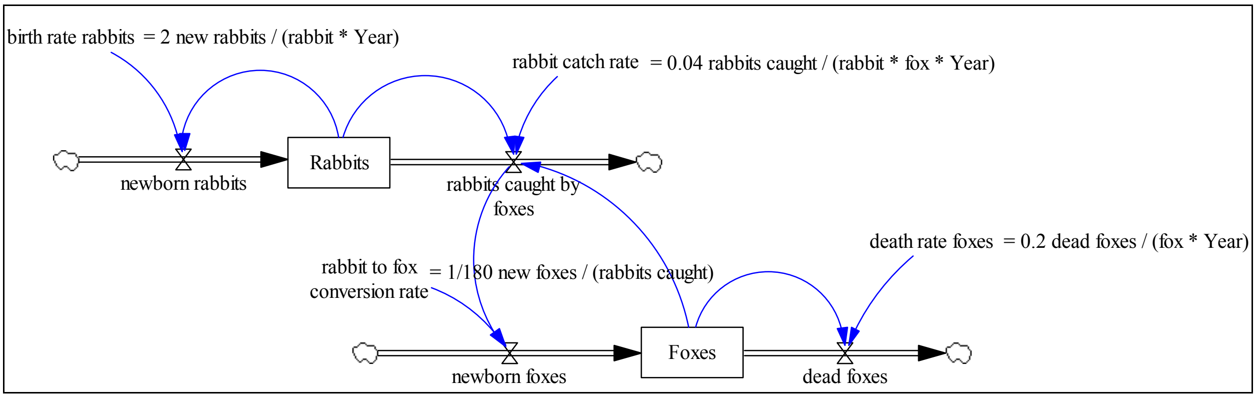

Figure 2. The stock-and-flow model of the rabbits-and-foxes system

The system dynamics students were expected to construct the stock-and-flow model from the system description. Since the stock-and-flow modeling helps the modeler structure the task, this approach was expected to be more helpful than the mathematical modeling approach described above. It ought to be easier to arrive at the correct solution to the rabbits-and-foxes task using stock-and-flow modeling than using mathematical modeling.

If the system dynamics students, when they have constructed the stock-and-flow model, realize that in equilibrium inflows equal outflows, they would be able to perform the following calculations:

Newborn rabbits = Rabbits caught by foxes

$$2 \cdot R = 0.04 \cdot R \cdot F \;\rightarrow\; F = \frac{2}{0.04} = 50$$

Newborn foxes = Dead foxes

$$\frac{0.04 \cdot R \cdot F}{180} = 0.2 \cdot F \;\rightarrow\; R = \frac{0.2 \cdot 180}{0.04} = 900$$

Now, they have the equilibrium populations, and, as mentioned above, there only remains the rather easy task of bringing the system there.

Method

Participants

The first group consisted of 10, first-year students at Uppsala University in Sweden, two male and eight female. They all studied subjects that do not demand or include any sophisticated mathematics, such as, for example, psychology, history and French. Their mean age was 20½ years, ranging from 19 to 22 years. Participation was voluntary and rewarded with two cinema tickets.

The second group consisted of 22 students at the Royal Institute of Technology in Stockholm, Sweden, seventeen male and five female. Their mean age was 25 years, ranging from 22 to 31 years. They were all in their third year, or later, of their studies for a master's degree in physical or electrical engineering, and all of them had completed at least one course in control theory.

The third group consisted of 15 students at the University of Bergen in Norway, ten male and five female. Their mean age was 27 years, ranging from 23 to 33 years. Participation in the experiment was part of the course requirements in a course in laboratory experiments and bounded rationality, and the participants were rewarded with two cinema tickets. The course, which is an international course taught in English, is normally part of a master's degree in system dynamics. A few participants were other system dynamics students who volunteered to participate in the study. All the participants included in the study had taken prior courses in system dynamics modeling.

Ossimitz (2002) found as strong gender effect to the female participants' disadvantage, and Booth Sweeney and Sterman (2000) report a weak effect in the same direction. The proportion of male participants is large in the second and third group, and low in the first. As for the system dynamics group, these were the students that were available. Female engineering students are few, and particularly so in the programs that require study of control theory. I might, of course, have recruited more males to the first group, but earlier testing for gender effects in the rabbits-and-foxes task, have shown no consistent advantage of being either male or female (Jensen, 2003). Nevertheless, if there is an effect of gender, it ought to be to the disadvantage of the first group.

The task

Task instructions. The basic instructions are included in Appendix A, and the qualitative description added to the instructions to the first group, and half of second group (see the Design section below), is found in Appendix B.

Task interface. In both the population line graphs (Fig. 3), the abscissa represents the years passing in the simulation, running from zero (start) to 30 years (end of trial). In the graph to the left of the screen, the ordinate represents the number of existing rabbits (ranging from 0 to 5000). The graph to the right represents the number of existing foxes (ranging from 0 to 150). The actual population sizes are presented numerically in their respective graphs. Initial population sizes are 500 rabbits and 40 foxes.

Population changes are calculated as:

$R_{t+1} = R_t + \Delta R_t$; where $\Delta R_t = (2 * R_t - 0.04 * R_t * F_t) * \Delta t$ for the rabbits, R, and

$F_{t+1} = F_t + \Delta F_t$; where $\Delta F_t = ((0.04 * R_t * F_t) / 180 - 0.20 * F_t) * \Delta t$ for the foxes, F.

The selected time-step, Δt, was one month (1 year / 12).

Close to the fox graph, there is an editing box for changing the actual number of foxes to the number desired. Clicking the step-button close to the editing box makes a year pass in the simulation. The years pass month by moth, at a rate of about two months per second. The participant has to watch twelve months pass before he or she is allowed to make another entry. The number of rabbits born and the number of rabbits eaten by foxes during the year passing, or just passed, are presented separately below the rabbit graph. Similarly, the numbers of foxes born and dying during the preceding or passing year are presented below the fox graph (Fig. 3).

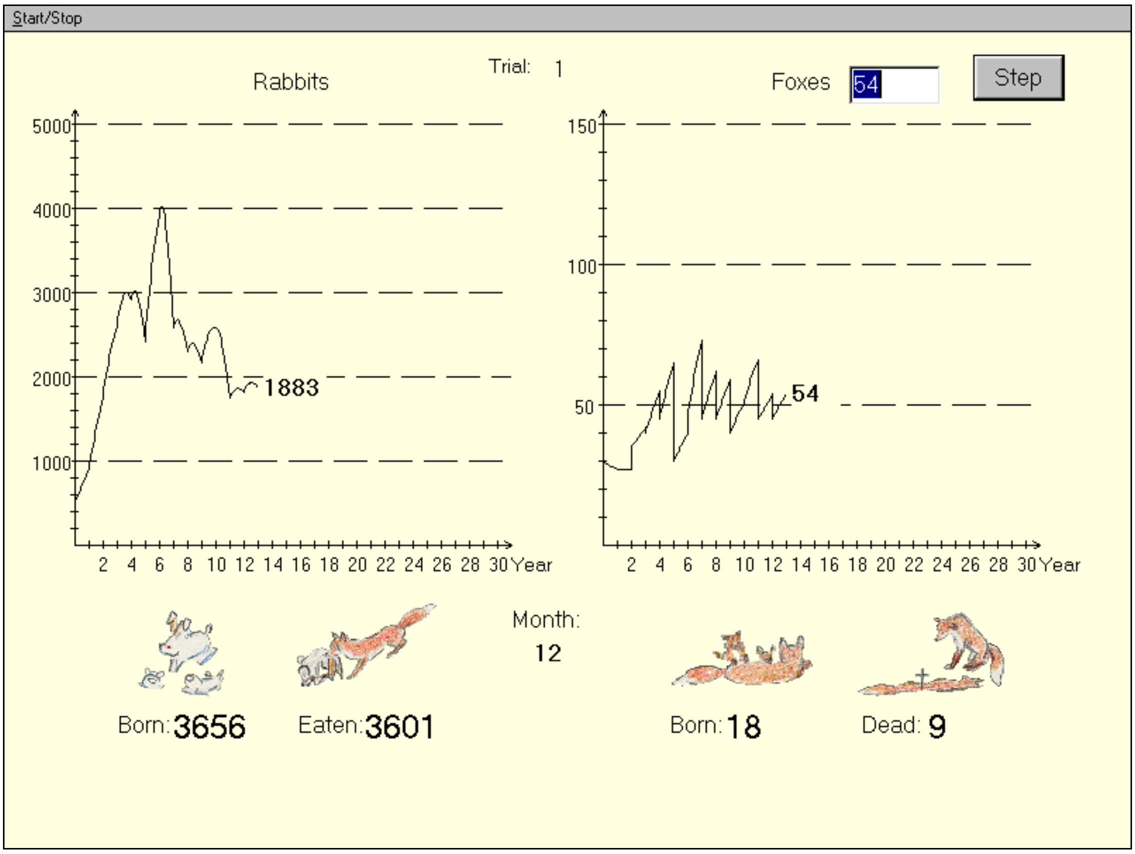

Figure 3. The interface to the rabbits-and-foxes task

Equilibrium is reached when the fox population remains constant at 50, which is the only level at which it ever remains constant. A rabbit population *close to* 900 is necessary to achieve this. It does not have to be *exactly* 900. That would be really difficult for the participants to achieve. When the rabbit population is approximately 900 and the fox population remains constant at 50, the rabbit population steadily approaches and eventually reaches 900, where it remains.

Design

As mentioned above, the first group, with no education about dynamic systems, received the instructions (Appendix A) with the qualitative description (Appendix B) included. In the second group, the engineering students, half of them were randomly assigned to receiving the combined instruction (Appendix A with Appendix B included) as the first group, while the other half received the original instructions without the qualitative description (Appendix A). This was done in order to assess the effect of adding the qualitative description. These two groups received the instruction in Swedish. The third group, the system dynamics students, were administered the original instructions (Appendix A) in English. They were expected to find it natural to look for the causal structure.

Procedure

In group 1, the participants without sophisticated math knowledge, all of the participants were tested individually. In group 2, the engineering students, the task was group administered in four sessions with three to eight participants in each session. In group 3, the system dynamics students, the task was group administered to all the participants in one session.

All the participants were equipped with writing material, paper and pencil, if they wished to make notes of any kind. These notes were collected afterwards by the experimenter.

The participants were introduced to the interface and received the task description in writing, which they were allowed to keep during the whole session. There was no time pressure. The participants decided themselves when to let a new year pass. There was no limit on the number of trials. The participants were allowed to continue until they had learned how to accomplish the task. After 45 minutes, they were allowed to decide whether they wanted to give up or to continue. If they decided to go on, they could continue for another 15 minutes. After that they had to stop.

Throughout the experiment, the experimenter remained in the room to answer direct questions only, to decide when the participants had reached the goal, and, if needed, to provide encouragement to continue. The task was considered completed when the participants had figured out how to reach equilibrium and were able to repeat the performance on request.

In group 2, the participants were asked, after they had completed the test session, to describe, in writing, their approach to the task, what strategies they had applied, successfully or unsuccessfully, to the task.

Analysis

The participant's performance was registered, whether they succeeded at the task or not. In addition, the strategies they applied to the task were analyzed, from the notes they made while performing the task and from the logs of their decisions. For the engineering students, this analysis was based on their own reports on how they approached the task as well.

Results

Students without education about dynamic systems

Table 1 summarizes the performance and strategies applied to the task by the students without education about dynamic systems. The first column describes whether the participants made any notes, and what kind of strategies the notes revealed. The second column describes the number of participants who applied the different strategies, and, within parenthesis, how many of them succeeded at the task.

Table 1. Performance and strategies applied by group 1, students without education about dynamic systems (the number of participants who succeeded at the task in parentheses)

Notes	N (successful)
Unsuccessful equilibrium equation	1 (1)
Calculating forwards	3 (0)
Bookkeeping - detailed	2 (1)
Bookkeeping – less detailed	2 (1)
None	2 (2)
Sum	**10 (5)**

Eight of the ten participants made notes while performing the task. One made some attempt at constructing one of the balance equations, the one for the rabbit population. He did not get it quite right, and did not know how to proceed, but he did solve the task. Of the remaining seven, three tried to calculate what will happen, and the other four performed some bookkeeping. Two of them noted carefully the input (demanded fox populations) and the result, recording births and deaths of both populations, one noted only population sizes, and one noted assessments of the results from her inputs, such as: "Less than 70; 40 pretty good; less than 55?."

Students with education in control theory

Table 2 summarizes the performance and strategies applied to the task by the engineering students. The first column describes whether the participants made any notes, and what kind of strategies the notes (and the participants' descriptions of the applied strategy) revealed. The second and third column describes the number of participants who applied the different strategies, for the respective instructional conditions, and, within parenthesis, how many of them succeeded at the task. The fourth column shows the results of the engineering students in total.

Table 2. Performance and strategies applied by group 2, the engineering students, divided on the respective conditions: with or without the qualitative description (the number of participants who succeeded at the task in parentheses)

Notes	Original instruction	Qualitative description	Sum
Correct equilibrium equations	1 (1)	1 (1)	2 (2)
Incorrect or only partly correct equilibrium equations	4 (2)	2 (1)	6 (3)
Equations, or calculations, with unclear purpose	3 (1)	4 (3)	7 (4)
Calculate forwards	1 (0)		1 (0)
Bookkeeping		3 (0)	3 (0)
No notes	2 (2)	1 (0)	3 (2)
Sum	**11 (6)**	**11 (5)**	**22 (11)**

Only one of the participants made any attempt at constructing any model other than a mathematical one; this participant tried to construct a block diagram of the system[1]. Sixteen of the twenty-two participants performed some calculations. Eight of them tried to construct and solve the equilibrium equations; two were successful, two were partly successful (they were able to calculate the fox population in equilibrium). One of the participants who got the equations wrong, and who failed at the task, did, however, notice the trend switches occurring for the rabbits when the fox population passes 50, and for the foxes when the rabbits are around 1000, and concludes that the equilibrium population has to be close to these numbers.

Of the remaining eight who performed calculations, some might have had equilibrium equations in mind, but neither their calculations nor their written descriptions of their solution efforts made this clear. Participants were only considered calculating equilibrium *equations* if they wrote down *two* equations, one for the rabbits and one for the foxes, and thereby clearly demonstrated that they acknowledged the need for more than one equation. Several of the participants who made other kinds of calculations mentioned search for *one* relation, a misunderstanding that is frequent among social science students. One participant tried to calculate forwards in order to find out what would happen next, an approach frequently attempted by social science students. One participant, who did not construct the equilibrium equations, did, anyhow, calculate repeatedly the number of foxes needed to consume the number of rabbits born, and found that it always turned out to be 50.

Three of the six participants who did not produce calculations, performed book-keeping, including births and deaths (two of them) or trends (one), and the remaining three did not make any notes at all.

Both the participants who solved the equilibrium equations also solved the task. The remaining participants are fairly evenly divided among the various approaches to the task. It seems as if you need to get the full and complete solution to be able to reliably benefit from it. Getting the general idea seems not to be sufficient to solve task. That is also what participants reported when they described their approaches to the task. When the calculations failed to produce the solution, they “played” with the simulation and tried to solve the task by trial-and-error.

Students with education in system dynamics

Table 3 summarizes the performance and strategies applied to the task by the system dynamics students. The first column describes whether the participants made a stock-and-flow model of the system, or not, and whether, if the attempt was made, it was correct or not. The second column is divided into three sub-columns, and describes whether the participants tried to solve the equilibrium equations, or not, and whether, if they did, they were successful or not. The numbers of participants who fit into the combined categories are noted, and, within parenthesis, how many of them succeeded at the task.

[1] A block diagram depicts the input and output signals, disturbances, the comparison of output feedback with desired results, and control signals to bring about desired changes. The system is treated as a box (a block) with its components hidden inside the box (see, for example, Leigh, 2004; Ogata, 2002).

Table 3. Performance and strategies applied by group 3, the system dynamics students (the number of participants who succeeded at the task in parentheses)

Stock-and-flow model	**Solution of equilibrium equations**			**Sum**
	Correct	**Unsuccessful attempt**	**No attempt**	
Correct	4 (3)	2 (1)	1 (0)	7 (4)
Incomplete or incorrect	0	1 (0)	4 (2)	5 (2)
None	1 (1)	0	2 (1)	3 (2)
Sum	**5 (4)**	**3 (1)**	**7 (3)**	**15 (8)**

Seven of the 15 participants constructed a correct stock-and-flow diagram of the system. Four of these also succeeded in correctly calculating the equilibrium populations by solving the equilibrium equations, and three of them solved the task. One participant solved the equilibrium equations directly without constructing a stock-and-flow diagram, and she also solved the task. This means that four, or 27%, of the system dynamics students solved the task after having solved the equilibrium equations. This is slightly more than among the students with no education about dynamic system, where the number was zero ($\chi^2 = 3.17$, $p = .075$).

Of the remaining three who constructed a correct stock-and-flow diagram, two made unsuccessful attempts at solving the equilibrium equations. One got the equations right, but was only able to calculate the fox population in equilibrium. He failed at the task, but the other, who understands the principle of the equilibrium equations but fails to get them right, solves the task. The last of the three, who finally made a correct stock-and-flow diagram after several attempts, and who did not try to calculate the equilibrium populations, failed at the task.

Of the seven participants, who either made incomplete and/or unsuccessful attempts at modeling, or who did not even try to model the system, only one tried to calculate the equilibrium populations. He came pretty far in both his modeling and calculations, and he also came close to solving the task, but did not quite make it. Of the remaining six, three solved the task by trial-and-error.

It is surprising that one participant who both constructed a correct stock-and-flow diagram *and* correctly solved the equilibrium equations, still failed at the task. Since the task was group administered to all the participants in one single sitting, and the group large, the control was not as good as in the other groups studied. It is possible that the participant made these notes after the time allowed for the task had elapsed, but before the notes were collected. He may also have made the notes while discussing the solution with a successful friend, forgetting that the notes were to be collected afterwards. It may, however, also be the case that he actually fully grasped the system structure, and equilibrium calculations, without being able to steer the system to the goal, although it seems unlikely.

Gender

There was no effect of gender on performance. Nine of the 18 participating women, and 15 of the 29 participating men, succeeded at the task. They were fairly evenly distributed on the various strategies applied.

Discussion

If we look only at the success rates, whether the participants solve the task or not, neither a background in system dynamics nor a background in control theory seems to be of any use when dealing with the rabbits-and-foxes task. All the three groups of participants perform at the generally achieved 50 % success rate. Moreover, the qualitative description (Appendix B) appears to have been of no use to the ones who received it.

The inspection of the strategies used, however, reveals a more interesting picture. It appears that the groups succeed for somewhat different reasons. The lengthy and numerous trials allowed result in several, or rather most, participants succeeding from merely trial-and-error. We are not particularly interested in this group; we are interested in people who succeed by reasoning.

Only one of the ten participants without education about dynamic systems attempted (albeit unsuccessfully) to construct the equilibrium equations. This corresponds nicely with our prior results. About one tenth of the social science students who have participated in various versions of this task (with a number of unsuccessful hints added) demonstrate reasoning along the correct lines. What we have found is instead a range of possible misconceptions (Jensen, 2003; Jensen and Brehmer, 2003, 2008).

In this study, however, eight of the twenty-two engineering students clearly constructed equilibrium equations, and another seven wrote down equations that may, or may not, have been attempts at equilibrium equations. Only two of them got the equations right, however, and they also solved the task.

Of the fifteen system dynamics students, twelve tried to make a stock-and-flow model of the rabbits-and-foxes system; seven of them constructed the correct model. Of the twelve who made stock-and-flow models, seven tried to construct the equilibrium equations, six of those who got the model right and one of the participants who had got the model wrong. Of these seven, only four with the correct stock-and-flow model got the equations correct. Three of them also solved the task, while I suspect, as I mentioned in the Results section, that the fourth wrote down his model and calculations after the session was completed. One of the system dynamics students went directly for the equilibrium equations, as was expected of the engineering students. She solved them correctly and also succeeded at the task.

The participants, who calculated the equilibrium values correctly, were highly likely, almost guaranteed, to achieve equilibrium in the simulation. None of the participants without education about dynamic systems were able to compute the equilibrium, 10% of the engineering students could, and 33% of the system dynamics students, but only 27% of the system dynamics students solved the task after having solved the equilibrium equations. These results provide some, if weak, evidence of the usefulness of system dynamics education when confronted with a balancing task, while the usefulness of control theory studies is less clear.

In addition, many of the engineering students and the system dynamics students correctly classified the problem. They understood what they were expected to do, what strategy to apply, even if they were not able to do so correctly. Moreover, the misconceptions frequent among social science students were rare, even if some of them occurred, in these populations.

People use their knowledge to recognize relevant information and solutions to problems, but also to quickly recognize and ignore the irrelevant, the incorrect, and the impossible. They thereby constrain both the problem and the solution spaces (Vicente and Wang, 1998). This did, however, not help the engineering or system dynamics students to perform the rabbits-and-foxes task.

If the participants were unsuccessful in calculating equilibrium, then only about 50% were able to achieve success, regardless of background. In fact, when forced to rely on trial-and-error, the participants without education about dynamic systems got a 50% success rate, whereas the engineering students got 45% and the system dynamics students got 40%. This looks as if the more education you receive about dynamic systems, the worse you get at trial-and-error solutions to dynamic tasks. The differences are not statistically significant, but the system dynamics students spent more time thinking before they started to interact with the system. They therefore completed slightly less trials, on average 3.6, within the allowed time, whereas the students without education about dynamic systems and the engineering students used 4.1 and 4.3 trials on average.

The weak performance of both the engineering students and the system dynamics students is puzzling. Were not our participants well enough educated? This seems an unlikely explanation. Both groups definitely had prior experience, on more than one occasion, of solving balancing problems within their respective domains. Could it be a question of what Piaget calls décalage in the participant's development of the equilibrium concept? The concept of décalage was coined by Piaget and refers to when an individual has mastered a concept to a stage when the concept is, still, only recognized and applied in the contexts where it has been formerly encountered (Flavell, 1963, pp. 21-23).

So, does system dynamics or control theory help you strike a balance? Yes, it seems like they both do, and systems dynamics perhaps a little more so, although, however, not as easily as we might have expected. Are there other, better ways of helping people develop a mature and robust equilibrium concept? This is a question in need of future study.

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Appendix A

Task Description

In a lake in the Northern parts of Sweden, there is an island. The island is covered by a mixture of grassland and forest. Initially, there is no animal life on the island. A group of biologists wishes to investigate how the vegetation is affected by grazing animals. Therefore they transport **500 rabbits** to the island and set them free on the island to graze and reproduce. As the biologists do not want the rabbits to become too numerous (rabbits are fertile creatures), **40 foxes** are also transported to the island. All the foxes are equipped with radio transmitters, in order to allow the biologists to locate them and catch them if necessary. Your task is to **establish a balance between the rabbits and the foxes**, i.e., to bring the system to equilibrium. You are allowed to transport foxes to or from the island once every year. Your task is to, eventually, be able keep the rabbit population at a constant level. Your task is **also** to reach a situation where the fox population also remains constant. **The goal** is to achieve a situation where the rabbits and foxes can be left to care for themselves, where both populations remain constant without further intervention. The biologists keep track on the population sizes, and reports them to you. Once every year you may decide on **how many foxes you wish there to be on the island**.

[When the text in Appendix B is added, it is inserted here]

Transports will then be arranged according to your wishes. There is always sufficient food for the rabbits, they never stave. If, however, the rabbit population exceeds 5000 rabbits, the island is considered “overrabbited” and the game is over. The game will also be over if the rabbits are extinguished by the foxes. The rabbits only die if they are caught and eaten by the foxes. The rabbits-and-foxes ecology on the island is fully described by the following four sentences:

- A rabbit produces 2 offspring a year.
- A fox eats 4 % of the existing rabbits a year.
- For every 180 rabbits eaten by the foxes, a new fox is born.
- Every year 20 % of the fox population dies.

Appendix B

The qualitative system description:

- Every rabbit produces new rabbits at the same rate, regardless of what else is happening.
- Every fox eats a specified share of the rabbit population. If there is a lot of rabbits, this share is large (many rabbits), and if there is few rabbits, this share is small (few rabbits).
- Whenever the foxes have eaten a specified number of rabbits, a new for is born.
- A specific share of the fox population dies every year from age or diseases.