# Closed-Loop Supply Chain Stability under Different Production-Inventory Policies

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#### Abstract

Product returns from market to the manufacturer can happen because of the motivation to capture the value content of the product after use or the obligation from legislation and the environmental concerns. A manufacturing system capable of integrating both the manufacturing and remanufacturing activities can help the recovery of value content in the products after the end of product's use life. Ordering policy designed for a production system with unidirectional flow of materials (i.e., up stream to down stream only), when applied to such an integrated system involving manufacturing remanufacturing can create dynamics in the system. Our investigation into this problem has revealed that these dynamics can be avoided if a well defined productionordering rule that explicitly account for the returned products, is adopted. We do the analysis by modifying the well known automatic pipeline inventory and order based production control system (APVIOBPCS) to achieve this goal.

Keywords: Closed-loop supply chain, remanufacturing, return flows, stability, end - of - life

### 1 Introduction

Closed Loop Supply Chain (CLSC) refers to the forward and reverse supply chain activities for the entire life cycle of the products. The integration of the forward (manufacturing) and the reverse (remanufacturing) supply chains together forms the basics of a CLSC. A CLSC thus consists of the manufacturing facilities, the collection facility for used returned products, system for inspection, and then application of a suitable remanufacturing activity on the accepted-for-reuse items in a manufacturing/remanufacturing facility. The term '*remanufacturing*' stands for the activities done on the returned products in the manufacturing/remanufacturing facility, while '*remanufacture*' stands for one of the options available for the processing of returned products.

Product recovery systems aim to minimize the amount of waste sent to landfills by recovering

materials and parts from returned products by means of suitable remanufacturing activities. The concept of CLSC starts with the recapturing of value content at the end-of-life (EOL) of products by remanufacturing. The motive behind the remanufacturing can be either life extension of the products or recapturing of value at the EOL. Thus, in a remanufacturing environment both forward distribution of products and reverse flow of used products are involved.

### **1.1 Research Focus**

We do the dynamic stability analysis in the manufacturer echelon of a generic closed loop supply chain under different ordering policies. The objective of the analysis is to study the behavior of a supply chain which is robust enough to recognize the changes in the system parameters and has the ability for self control, when subjected for remanufacturing. The research objectives in this study include modifying the policy to adapt to remanufacturing, so that the system behavior become sensitive to the changes in the system parameters and avoid the dynamics otherwise present in it. We make use of three models of a production-inventory system. The first one is the case where there is no remanufacturing involved in it. i.e., the forward flow of materials alone is considered in this case and the policy adopted is 'automatic pipeline inventory and order based production control system' (APVIOBPCS). The second model is for remanufacturing system and the policy adopted is similar to that in the first model. The third model is also for a remanufacturing system and uses the modified policy, which takes into consideration the effects of product returns into the system.

We consider the collection activities in CLSC to be outside the system. The productioninventory system explained by Forrester (1961) is analyzed to study the effects of product return on the dynamic stability of the system. It is assumed that the returned products are collected by some means and are available for remanufacturing activities. The efforts, the network, and the cost associated with the collection of EOL products are not subject matter of this research. Studies on the system revealed that the control policy, APVIOBPCS can lead the system to unstable state and eventually the system can go out of control. We experiment on the effects of returned products in the stability of a CLSC and aim only on redefining production-ordering policies, which can account the effects of return flow of EOL products in to a CLSC, and at the same time will minimize the dynamics in the system. System Dynamics (SD) simulation technique is used to analyze the system models in this study. SD techniques allow easy modeling for policy analysis and experimentation in dynamic system models. We represent the simulation models of the system in SD by stock and flow diagrams (SFD). In stock and flow diagram, the accumulations, the rates, and other variables are represented using levels, flows and auxiliary variables respectively.

# **1.2 Related Literature**

Fleischmann et al. (1997) presented a survey of the practices in remanufacturing and reverse lo-

gistics and proposed a framework for reverse distribution system in a supply chain. According to them, the term 'reverse' involves the physical transportation of used products from the end user back to the producer. The remanufacturing step takes care of the transformation of the returned products into usable products again by the producer. In remanufacturing, they observed that all the reuse activities do not fit into traditional manufacturing setting. Also the inhomogeneity of the returned products create uncertainty and complexity in remanufacturing systems. Gharote et al. (2008) analyzed a reverse logistics system with options for remanufacturing, disposal and stocking. They experimented with different stocking policies in a periodic review system such as; (a) remanufacturing of the returned products on receipt at the facility, (b) stock it for remanufacturing lead time, or (c) dispose it off. By stocking they effectively increased the remanufacturing lead time. They concluded that a stocking based policy for models for reverse logistics is significantly less expensive than the models without using the stocking option.

Linton and Jayaraman (2005) identified different modes of product life extension. The modes of life extension and the products suitable for each mode identified by the authors are: recall - telecommunications, repair - electronic products, preventive maintenance - transportation equipment, predictive maintenance - industrial equipment, upgrade - telecommunications, product reuse - used goods, remanufacturing - industrial products, part reuse - automotive parts, recycle - paper and metals. They observed that product life extension offers economic and environmental benefits in a wide range of applications and industries. They showed that product life extension increases economic value and reduces the environmental impact of a product. Guide et al. (2000) termed remanufacturing systems as recoverable-manufacturing systems which seek to minimize material waste by recovering the maximum content of returned manufactured products. According to Guide et al. (2003), the remanufacturing activities include repair, reuse, part reuse, overhaul and recycling. The authors gave definitions for the terms and explained the different levels of effort and expertise required for these activities. Gungor and Gupta (1999) observed that an environmentally conscious manufacturing and product recovery (ECMPRO) has become an obligation to the environment and to the society. They emphasized the need for design for recycling (DFR), design for environment (DFE) and design for disassembly (DFD) in the manufacturing, so that the waste materials sent to the environment during the different phases of the product life and remanufacturing can be eliminated. They termed such a system with forward and reverse flow of products as Remanufacturing Integrated Manufacturing (RIM).

In CLSC simulation studies, Georgiadis and Vlachos (2004) examined the impact of environmental issues on long-term behavior of a single product supply chain with product recovery. The environmental issues examined were the firm's green image effect on customer demand, the take-back obligation imposed by legislation, and the state campaigns for proper disposal of used products. The authors developed a system dynamics model of a forward-reverse logistics system with remanufacturing and analyzed the behavior of the system. They concluded that the increasing environmental consciousness will force states to introduce new laws which enforce take-back obligations on the manufacturer, which will drive the industry decisions to a new competitive environment. Vlachos et al. (2007) analyzed a SD model of a CLSC, for an efficient capacity planning policy for a single product forward and reverse supply chains with transient flows. They analyzed the influence of different factors such as the green image effect, failure percentage of the collected products, addition of remanufacturing capacity, and addition of collection capacity, on total supply chain profit. Their policies confirmed the feasibility of remanufacturing and collection capacity expansion and gave answers to questions about when and how much to expand. Also, they analyzed the impact of the green image on optimal capacity planning policies. They found that, higher the impact, higher is the factor for the capacity expansion strategy and shorter the review periods. They also found out that less initial reuse ratio had a negative effect on the total supply chain profit. However if the initial capacities are leading, then the reuse ratio will improve and the total supply chain profit also will increase.

In the field of SD modeling of supply chains, Towill (1996) analyzed a production system. In the paper the author explained the methods by which the SD models can be used in modeling supply chains, analyzing the effects of different policies in supply chains, and also the different control policies that can be used in supply chains. The stability of a production inventory model was the research objective in the papers by Riddals and Bennet (2002), Wikner et al. (1991), Warburton et al. (2004), and Venkateswaran and Son (2007). The dynamic structure of the model was well addressed by Sterman (2000). The control theory application to an production-inventory system was discussed in the research by Ortega and Lin (2004). The authors identified some potential research areas in production-inventory problems where control theory can be used as a tool.

In certain research, recovered materials are assumed to feed demands of both manufacturing and remanufacturing operations (Beamon 1999, Gungor and Gupta 1999, Guide et al. 2000, Gharote et al. 2008). We restrict remanufacturing activities only for remanufacture of a returned product. No materials are recovered for use in manufacturing operations. A generic model of RIM consists of product manufacturing and remanufacturing; both feeding to a stock of serviceable inventory. There is no difference between manufactured and remanufactured items as a remanufactured item is assumed to be "as-good-as-new" (van der Laan and Salomon 1997). We also make this assumption. The products at their EOL are to be collected at some collection point, and then to be transported to the remanufacturing facility of the manufacturer or the third party if they are involved in the remanufacturing activities (Krumwiedea and Sheub 2002).

Vast variety of literature is available in the areas of supply chain management, supply chain stability analysis, supply chain optimization, reverse logistics models, reverse logistics network designs and evaluation of alternate inventory policies in remanufacturing. The behavior of the production-inventory system and its dynamic stability are well addressed by many researchers (Sterman 2000, Riddals and Bennet 2002, Wikner et al. 1991, Warburton et al. 2004, Venkateswaran and Son 2007). However in the SD analysis of CLSC only few publications exist (Kumar and Yamaoka 2007, Spengler and Schröter 2003).

In the remaining sections; Section 2 explains a forward flow model (Base Model) of production inventory system which uses the modified APVIOBPCS, and analyze its dynamic behavior. Section 3 explains the RIM model under investigation, the experiments conducted on the model, and the results. In Section 4, the second RIM model, Model II is presented with redefined policies and the experimental results. The RIM models Model I and Model II differs in the ordering policies alone. Model I uses the same ordering policy as that of Base Model while

Model II uses a modified ordering rule. We compare the behavior of these models and in Section 5 the conclusions drawn based on the model study about the CLSC system behavior under different ordering policies are presented along with the directions for future research.

#### **1.3** Notations used

Table 1 gives the notations used throughout this analysis for the base model, Model I, and Model II.

_α	Fractional rate of adjustment for WIP	β	Fractional rate of adjustment for INV
CD	Customer demand	DPSR	Desired production start rate
EWIP	Effective WIP	FD	Forecasted demand
INVADJ	Inventory adjustment	PCR	Production completion rate
PINV	Inventory in hand	PLT	Total production lead time
PS	No. of production stages	PSR	Production start rate
RCR	Reprocessing completion rate	RFF	Refurbishing fraction
RFR	Refurbishing rate	RINV	Remanufactured products inventory
RLT	Total reprocessing lead time	RPF	Reprocessing fraction
RPR	Reprocessing start rate	RPSR	Remanufactured products sales rate
RR	Return rate	RS	No. of remanufacturing stages
SALES	Fresh products sales rate	VWIP	Virtual WIP
WIP	Work in progress inventory	WIPADJ	WIP adjustment

Table	1:	Notations	used

#### 1.4 Models of production-inventory system in CLSC

The production-inventory system considered is an integrated forward and reverse flow system. The system employs a general replenishment rule. A remanufactured item is assumed to be *as - good - as - new*, which ensures that there is no secondary markets for the remanufactured items, and no price discounts. The product is also assumed to be a remanufacturable one. After inspection, the returned products get separated for refurbishing, reprocessing, and those not suitable for any of these two remanufacturing activities are disposed off.

We present here, a forward flow model and two models of a RIM system. Models of forward flow type of system was studied by many authors (Towill 1982, John et al. 1994, Sterman 2000, Dejonckheere et al. 2002, Disney and Towill 2002, 2003, 2005, Venkateswaran and Son 2007) with different number of manufacturing stages and different control policies. The forward flow model assumes a modified form of Automatic Pipeline Inventory Order Based Production Control System (APVIOBPCS) (Dejonckheere et al. 2003). The first RIM model, "Model I"

represents a system when the returned products are coming into the forward flow system for remanufacturing and the production ordering policy adopted is modified APVIOBPCS. The model, "Model II" represents the same system, but it improves over the modified APVIOBPCS. In both the RIM models, customer demand (CD) is partly met by the remanufactured items. The remaining portion of demand is met by products manufactured from virgin raw materials.

# 2 Forward flow model of manufacturing system

The production-inventory system, considered here, consists of a forward supply chain and a reverse supply chain in the manufacturer echelon. The forward chain consists of the manufacturing facility where the actual manufacturing operations are taking place, and the market where the actual customer demand get generated. All the other tiers of participants in the supply chain are omitted in the model. The finished products from the manufacturing facility get accumulated as serviceable products inventory (PINV). This inventory is depleted by the demand faced from the market, and get replenished by the production completion rate (PCR). The activities in the manufacturing facility depends on the production orders released to the facility.

The forward flow model assumed in the system consists of a number of sequential production stages represented as PS. The finished products inventory (PINV) caters the customer demand. The production order releases are based on the error in the inventory, the error in the work-in-progress (WIP), and the perceived demand (FD) for the period. The error in the inventory is observed by comparing the desired and actual inventories for the period and is adjusted by a fractional adjustment rate (Eqn. 5). The error in WIP also is adjusted by a fractional adjustment rate (Eqn. 7). The desired WIP (DWIP) depends on the total production lead time (PLT), and calculated as;  $PLT \times FD$ .

The production lead time is divided between the PS number of production stages as effective lead time (ELT) by assuming each manufacturing stage require equal amount of time for the processing. The value of PCR is taken as the rate of production completion from the last stage of production. It is the rate at which the inventory gets replenished each period. The customer demand consumes the inventory at a rate of CD per period. The system is assumed to be having infinite capacity of production and storage, and backlogs allowed. The simulation time step  $\Delta t$ is assumed as 1. In demand forecasting, the exponential smoothing constant  $\rho$  is taken as 1. This forward flow model is called the 'Base Model' of our analysis.

Figure. 1 shows the stock and flow diagram of the SD simulation model developed using Vensim® for the Base Model. The interdependency of the system variables can be expressed by the following linear difference equations, Equations  $(1 \dots 9)$ , in time 't'. The update of levels is made as: LEVEL(t+1)=LEVEL(t)+NET\_FLOW\_RATE(t).

The rates and auxiliary variables are evaluated as: RATE(t)=LEVEL(t)×Const. and,  $AUX_i(t)=f(LEVEL(t) \text{ and/or } RATE(t) \text{ and/or } AUX_i(t))$ 



Figure 1: Simulation model of Base Model

$$PINV(t+1) = PINV(t) + PCR(t) - SALES(t)$$
(1)

$$SALES(t) = CD(t) \tag{2}$$

$$FD(t+1) = FD(t) + C_{-}IN_{-}ECD(t) \times \rho$$
(3)

$$C_{IN} ECD(t) = CD(t) - FD(t)$$
(4)

$$INVADJ(t) = [DINV(t) - PINV(t)] \times \beta$$
(5)

$$DWIP(t) = FD(t) \times PLT \tag{6}$$

$$WIPADJ(t) = [DWIP(t) - EWIP(t)] \times \alpha$$
<sup>(7)</sup>

$$DPSR(t) = FD(t) + WIPADJ(t) + INVADJ(t)$$
(8)

$$PSR(t) = DPSR(t) \tag{9}$$

By adopting the assumptions made above, Eqn. (3) becomes Eqn. (10), as given below.

$$FD(t+1) = FD(t) + C_I N_E CD(t)$$
<sup>(10)</sup>

### 2.1 Production-ordering policy structure in Base Model

The ordering policy assumed in the Base Model is adapted form APVIOBPCS (Dejonckheere et al. 2003) and similar to the one used by Venkateswaran and Son (2007). APVIOBPCS uses the SALES information to generate a perceived demand (forecasted demand - FD), by smoothing the SALES figures with the previous period's perceived demand, with a smoothing factor:  $\rho$  (Eqn. 3). This perceived demand is used to set targets on inventory (DINV) and work-in-progress (DWIP). The actual value of work-in-progress (effective WIP) is the sum

of the individual values of WIP in the three manufacturing stages. The discrepancy in the desired and actual values of the WIP (DWIP - EWIP) is adjusted at a fractional rate of adjustment for WIP: ' $\alpha$ '. The target inventory (DINV) according to the perceived demand is evaluated as the perceived demand times the inventory coverage. It is then compared with the available inventory (PINV). The correction for the inventory error is applied at a fractional rate of adjustment for inventory: ' $\beta$ '. According to APVIOBPCS the desired production start rate (DPSR) is the sum of all the three adjustments required; Pipeline (WIPADJ), Inventory (INVADJ), and Order Adjustment i.e., FD (Eqn. 8). Since the system in this analysis uses a number of stages of production and each stages are assumed to have first order exponential delay, the 'Pipeline' concept in the APVIOBPCS get modified to accommodate any type of delay involved in the system. Even if the value of ELT is unity, the effect of more number of stages in production never give a pipeline delay in the system.

#### 2.2 Base model behavior

Many authors have investigated the stability of manufacturing systems (Towill 1982, John et al. 1994, Sterman 2000, Dejonckheere et al. 2002, Disney and Towill 2002, 2003, 2005). Venkateswaran and Son (2007) give a stability region for the parameters fractional rate of adjustment for WIP -  $\alpha$  and fractional rate of adjustment for inventory -  $\beta$  with infinite inventory coverage. They derived the system transfer function:

$$\frac{PREL[z]}{SALES[z]} = \frac{\left[\delta(L(z-1)+Q\delta)^Q\left((z-1)(1+L\alpha)\rho+\beta(-1+Z-\rho+z\rho+\delta\rho)\right)\right]}{\left[(z-1+\delta\rho)\left\{\begin{array}{l} -L^Q(z-1)^Qz\alpha\delta((-1)^Q(Q\delta/(L-Lz))^Q\\ -((L(z-1)+Q\delta)/(L(z-1)))^Q\\ +(z-1)(Q^Q\beta\delta^Q+z(L(z-1)+Q\delta)^Q\end{array}\right\}\right]}$$

where PREL[z] is the Z-transform of the system parameter production release (*PREL*) which is equivalent to *PSR* in this analysis and *SALES*[z] is the Z-transform of the parameter *SALES*. For the simulation and analysis purpose, they took the values Q = 3, L = 3,  $\rho=1$ , and  $\delta=1$  and found out the system stability conditions in terms of  $\alpha$  and  $\beta$  using Jury's test (Jury 1964). The stability region is plotted on the parameter plane as in the Figure 2.

In our analysis the number of production stages and the total lead time are assumed to be 3. i.e, (PS = 3 and PLT = 3) The exponential smoothing parameter for forecasting ( $\rho$ ) and the update frequency of the system are assumed as 1. These are similar to the parameter setting in their model. Therefore both the systems analyzed by them and by us have the same stock management structure and use the same production ordering policy in the manufacturing flow, however here the production delay is not pipeline. The model is tested to see if the model follows the same stability boundaries for parameters  $\alpha$  and  $\beta$ .

In this analysis, the production start rate (PSR) is set as the system performance parameter in all three models. To get the Base Model behavior, four points in the parameter plane of  $\alpha$  and  $\beta$  are selected. One point is selected from the stable equilibrium state represented by;  $\alpha = 1$ 



Figure 2: Stable and unstable regions of  $\alpha$  and  $\beta$  (Source: Venkateswaran and Son (2007))

&  $\beta = 1$ . Two points are selected from the critically stable state represented by;  $\alpha = 0.382$  &  $\beta = 1$ , and  $\alpha = 0$  &  $\beta = 0.445$ . One point refers to the unstable state;  $\alpha = 1.55$  &  $\beta = 1$ . The model parameters were initialized in such a way that the system starts at stable equilibrium state. The model was subjected to a step change of 20units in the demand at period 50 weeks.

The following figures show the Base Model behavior under modified APVIOBPCS. The behav-



Figure 3: Base Model behavior

ior of the model shows that the system follows the same stability boundary as given in Figure 2. Taking this forward flow structure as the basic model, we move on to the first model of the remanufacturing integrated system. The objective in giving the return flow of materials in to the system was, to see how the system will behave in the presence of the return flow while adopting modified APVIOBPCS as the control policy.

# 3 Model I

Model I is the first RIM model. In this model, we assume that the return flow of used products happen into manufacturing facility for remanufacturing in the Base Model at a rate RR. The products in the market are collected at their end-of-life (EOL) and brought to the remanufacturing facility for remanufacturing. The model then changes to the one shown in Figure 4. In this model, remanufacturing is assumed to be separate from the manufacturing activities in the plant. The serviceable inventory include the fresh manufactured products (*PINV*) and the remanufactured products (*RINV*). This model is named as "Model I", and employs the modified APVIOBPCS.



Figure 4: Production-Inventory System with Remanufacturing (RIM) - Model I

#### 3.1 Stock management structure in Model I

The remanufacturing options considered in this analysis are; (a) Refurbishing, and (b) Reprocessing. So the returned products after inspection get sorted to refurbishable inventory (RFINV) and reprocessable inventory (RPINV). In refurbishing, the returned products undergo refurbishing operations in a single stage and accumulate to the remanufactured products inventory. In reprocessing, the returned products undergo complete or partial disassembly and remanufactured into new products through a number of remanufacturing stages. In the present model structure it is assumed that, the products that are selected for reprocessing will go through three sequential reprocessing stages, which are assumed to be necessary and inevitable. After

the final stage of reprocessing the products accumulate to the remanufactured products inventory (RINV), which is part of the serviceable inventory. The total remanufacturing lead time (RLT) is divided equally between RS number of remanufacturing stages as 'STAGEDEL'. In our analysis the value of RLT and RS are taken as 3. So the value of 'STAGEDEL' becomes 1.

The return of products from the consumers at the EOL of the products is assumed to happen at the rate RR. For ease of analysis it is assumed that RR happens; (a) independent of the manufacturing system, (b) the average life of products in the market, and (c) the useful life of the product. Throughout this analysis RR is assumed as an exogenous variable. For studying the effect of product returns on the system, some changes in its value is also considered. The returned products are inspected and separated for refurbishing, remanufacturing, and the remaining for safe disposition. The system is assumed to have infinite capacity for manufacturing, remanufacturing, and inventory holding. Also it is assumed that the manufacturing and remanufacturing facilities are separate in the model.

The value of customer demand is always assumed to be greater than the value of return rate of used products (RR). This assumption is based on the fact that there is a chance that a product at the EOL is not returned (Beamon 1999) or the value content in all the returned products is not fully recovered. This forces the system to produce some quantity of fresh products every period to meet the customer demand. The stock and flow diagram of the simulation model of Model I is shown in the Figure. 5.



Figure 5: Stock flow diagram of Model I

It is to be noted that the level variables PINV and RINV are shown separately than a single level variable for '*serviceable inventory*', only for ease of analysis. In reality the RINV can be excluded from the model and the rates RFR and RCR can be directly be added to PINVor another level equivalent to '*serviceable inventory*'. The effect of remanufacturing will be the same on the system parameter in that configuration also. In that case the rate RPSR is to be eliminated and the analysis of the system is to be made based on the rates RCR and RFR.

The linear difference equations of the system in the forward flow are not altered as there is no change in the control policy. Because of the addition of the remanufacturing activities, Equations (11) & (12) are added. By applying the assumptions made in Section 1.4, Eqn. (11) changes to Eqn. (13). Since the value of CD is always assumed to be greater than the value of RR, the remanufactured products, RINV(t) is always less than CD(t). This restricts RPSRequal to RINV. Therefore, Eqn. (12) changes to Eqn. (14) and modifies Eqn. (13) to (15).

$$RINV(t+1) = RINV(t) + \{RCR(t) + RFR(t) - RPSR(t)\} \times \Delta t$$
(11)

$$RPSR(t) = MIN[RINV(t), CD(t)]$$
(12)

$$RINV(t+1) = RINV(t) + RCR(t) + RFR(t) - RPSR(t)$$
(13)

$$RPSR(t) = RINV(t) \tag{14}$$

$$RINV(t+1) = RCR(t) + RFR(t)$$
(15)

The set of equations; Eqns.  $(1 \dots 9)$  along with Eqns. (14) and (15) gives the complete set of equations that are used in the Model I.

The production ordering policy assumed in the system is same as that in the Base Model. It uses the demand forecast information, the inventory status information and its adjustments, and the work-in-progress (WIP) information and its adjustments as explained in Section. 2.1.

### 3.2 Experimentation and system behavior

As in the Base Model, the Model I also started at stable state. Then the return rate is assumed to be a step function defined as STEP [20, 10]. i.e., 20 units are assumed to come to the system at time equal to 10 periods. The demand is also changed as a step function as STEP [20, 50]. During the period 10 to 50, the changes that can be observed in the system will only be due to the remanufacturing activities. But from period 50 onwards, it will be the combined effect of RR and CD. By giving the changes in RR and CD at different time horizons, it become easy to see the system behavior at different parameter setting of the system. Especially when the management has to decide upon the control policies, parameter settings, and the time horizons of different types of behavior of the system, such experimentation are required. The results of the SD experiments will give a visualization of '*what-if*' scenarios of different policies. Here the policy is kept constant as Base Model policy, but the parameter settings are changed to see whether the system behaves as if the Base Model. The input parameters to the system model are CD and RR, and the system performance parameter is PSR as in the Base Model. The experiments are conducted at stable equilibrium state, critically stable equilibrium state, and unstable equilibrium state as in the previous case. The stable equilibrium states considered are  $\alpha = 1$  &  $\beta = 1$ , and  $\alpha = 1$  &  $\beta = 1.35$ . The critical stability states selected are  $\alpha = 0$  &  $\beta = 0.445$ , and  $\alpha = 0.382$  &  $\beta = 1$ . The unstable state selected is  $\alpha = 1.55$  &  $\beta = 1$ .

The following figures show the behavior of the Base Model and Model I in comparison, at stable equilibrium and critically stable equilibrium states.



Figure 6: Model I performance in stable equilibrium state



Figure 7: Model I performance in critically stable equilibrium states

#### 3.3 Observations

From the model behavior it can be seen that except in the Figure 6(a), the system response to change in the RR is same as the response to change in the CD. This is expected because

change in RR will be reflected in system as a change in inventory. So, in the stable region of the parameters the control policy will bring back the system to stable equilibrium state. Though in the second stable equilibrium state (Fig. 6(b)) the system stabilizes at around 30 periods, the initial fluctuations in the system behavior is just similar to the change in CD. From Figures 7(a) and 7(b) it is evident that the change in the RR is recognized as change in CD by the system and start to oscillate.

In the adopted policy, APVIOBPCS, the control of the production orders (PSR) is made based on the adjustment for the WIP, inventory, and the demand. Any change in the return rate will cause the inventory to change because of the remanufacturing activity. This will lead to changes in the adjustment for inventory (INVADJ) and will try to bring back the system to stable equilibrium state by the goal seeking behavior of the feed back loop in adjustment for inventory. Though the policy can adjust to the changes in the inventory levels, when the level changes happens due to the addition of inventory by remanufacturing, the other controlling variables WIPADJ and forecasted demand FD will not adjust the production orders in the present ordering policy. In adjustment for inventory, the effect of remanufacturing will be reflected as a change in the inventory level, PINV, and the system will adjust the error in the desired and actual inventory values by INVADJ.

When return rate increases, the inventory value start to increase as the remanufactured products sales increases and INVADJ will try to minimize the error in inventory by adjusting itself at an adjustment rate ' $\beta$ '. The value of production start rate will come to a lower value than that with the same customer demand in Base Model. Because of the three stage delay in the manufacturing, this change in start rate (PSR) will take effect in serviceable inventory only after the total production delay (PLT). As PSR starts to reduce, effective WIP reduces. Since the desired WIP (DWIP) is based only on the perceived demand and production lead time, the value of of DWIP remain unchanged and cause an error in the WIP. So WIPADJ will try to compensate this error by adjusting itself at a fractional rate of ' $\alpha$ '. This happens even before the serviceable inventory get adjusted to another stable level. Depending on the value of  $\alpha$  and  $\beta$  selected, the system will exhibit the behavior similar to the base model. At the critically stable values of the parameters the system will exhibit oscillatory behavior, and at unstable values of the parameters, the system will start to oscillate and become unstable (Fig. 2).

The difference in the behavior of the system to changes in return rate and customer demand can be as follows. When demand changes, the effect will be there in the forecasted demand, and in turn INVADJ and WIPADJ. If the demand increases, the value of inventory start reducing and at the same time forecasted demand will increase. This will cause the desired inventory and thus INVADJ to increase more than that when remanufacturing happens. Also, desired WIP and thus WIPADJ increases which do not happen in the case of remanufacturing. So the amplitude of the variable PSR will be more with change in demand than that with change in return rate.

# 3.4 Managerial perspective and compensation scheme

From the above discussion it is clear that the changes in return rate can cause dynamics in the system and the system behavior can be similar to that with change in demand. If demand and returns are changing together, the amplitude of the parameter PSR can increase further. These dynamics are to be avoided in the system, because there is no change in the system setting. This result will help the decision maker in a situation when returns are received into a system, like the one in discussion. The managerial implication in this situation is that the controller has to allow the system to oscillate as if the demand is changing. So to keep the system stable against the changes in the returns rate, a better policy has to be developed, which will recognize the change in inventory, but do not allow the system to sense it as a change in demand.

The increase in inventory per period when remanufacturing happens is represented by the variable RPSR. But this increase is felt only in the inventory and this forces the INVADJ to change to a lower value. This is required according to the present ordering policy. Two things are to be observed here; (i) RPSR is similar to production completion which adds to the inventory value. Therefore production start rate (PSR) can be reduced to a lower value than the desired production start rate (DPSR) by an amount equal to RPSR. i.e., the definition of PSR to be changed as PSR(t) = DPSR(t) - RPSR(t). (ii) Reduction in the value of INVADJ and increase in the value of WIPADJ because of the RPSR is to be avoided. Because the effect of RPSR is similar to creating a virtual work-in-progress inventory, and also reducing the target inventory (DINV). The effect of RPSR propagates in the system to the production completion rate through three manufacturing stages. During this time, the virtual WIP and virtual inventory start increasing. To avoid the anomalies in the control of inventory error, and adjustment for WIP.

Now a second remanufacturing integrated manufacturing model is prepared keeping these points in mind. The control policy also changed from that of Model I, to accommodate the effects of RPSR.

# 4 Model II

The schematic diagram for the Model II is shown in Fig. 8. The change made in the figure is the additional information flow from the remanufacturing to the production ordering. Model I is modified to accommodate the effects of returns as in the stock flow diagram of the simulation model in Fig. 9. We name this model as "Model II".



Figure 8: Production-Inventory System with Remanufacturing (RIM) - Model II



Figure 9: Stock flow diagram of Model II

# 4.1 Production-ordering policy structure in Model II

V

In the manufacturing system, when remanufactured products in the serviceable inventory increases, it acts as an addition to the production from fresh raw materials. This increase in the production rate can be adjusted in the production start rate. So the value of PSR is taken with a correction on desired production start rate. This modifies the equation of PSR to Equation (23). However this correction will take a delay equivalent to the production delay to effect in production completion rate. By this time, the system can generate dynamics if the following corrections in the adjustment for inventory (INVADJ) and adjustment for WIP (WIPADJ) are not taken.

The definition of INVADJ is modified to account the effect of the fraction of sales from remanufactured products (RPSR). As RPSR starts to increase, the net flow rate into the serviceable inventory increases and so the value of level 'PINV' starts to increase. To compensate this effect on inventory, the definition of INVADJ is modified as in Equation (21). The additional term used is the virtual WIP (VWIP) which is an accumulation of rate of remanufacturing in the system, and act as an additional WIP in the system. So sufficient correction is made in the definition of WIPADJ as in Equation (22) also.

To capture the value of VWIP, the following scheme is used in addition. The virtual WIP buildup in the system is through the PS number of production stages. The rate of remanufacturing completion is given as the input to the first stage. Each stage has first order exponential delay of with a delay period of effective lead time, as in the production stages. The three level variables VWIP1, VWIP2, and VWIP3 in the model are the virtual WIP buildup in each stage. The total of these three gives the value of VWIP. Equations (16...20) represent this modification.

In Model II, Equation (5) changes to Equation (21), Equation (7) changes to Equation (22), and Equation (9) changes to Equation (23). The other equations in Model I remain same in both models. However, to accommodate the effect of returns Equations (16), (17), (18), (19) and (20) are used in addition. Where R1(t)=RPSR(t), and R2(t) is equivalent to PCR(t), but with different value dependent on R1(t). The equations in the modified form considering the assumptions in the system are:

$$R2(t) = VWIP3(t)/ELT$$
(16)

$$VWIP(t) = VWIP1(t) + VWIP2(t) + VWIP3(t)$$
(17)

$$VWIP1(t+1) = VWIP1(t) + RPSR(t) - VWIP1(t)/ELT$$
(18)

$$WIP2(t+1) = VWIP2(t) + \{VWIP1(t) - VWIP2(t)\}/ELT$$
(19)

$$VWIP3(t+1) = VWIP3(t) + VWIP2(t)/ELT - R2(t)$$
 (20)

$$INVADJ(t) = [DINV(t) + VWIP(t) - PINV(t)] \times \beta$$
(21)

$$WIPADJ(t) = [DWIP(t) - VWIP(t) - EWIP(t)] \times \alpha$$
(22)

$$PSR(t) = DPSR(t) - RPSR(t)$$
(23)

### 4.2 Behavior of Model II

In Model II, the control policy is modified by accounting the effects of returns. To compare the impact of the policy change in system behavior, the same points in the stability region of  $\alpha$ , and  $\beta$  in the previous experiments are selected in the Model II experiments also. The RR and CD are assumed to change as in the previous models. Figures 10 and 11 shows the system behavior with different ordering policies in comparison. Figure 12 shows the system performance in the unstable states;  $\alpha = 1.55$  &  $\beta = 1$  and  $\alpha = 0.5$ , &  $\beta = 1.5$ 



Figure 10: Model II performance in stable equilibrium



Figure 11: Model II performance in critically stable equilibrium

The Figures 6(b) and 10(b) have observable difference between them. The Model II behaves to the return flows cautiously, while Model I assumes there is demand change.

It is clear that the returns does not cause any dynamics in the system with this modified productionordering policy. The PSR seem to be shifted downwards by an amount equivalent to the increase



Figure 12: Model I & II performance - Unstable equilibrium states

in the inflow rate in the system due to the returns. This is true in the region of change in CD also (i.e., from period 50 weeks onwards). The system performance at the parameter setting at unstable equilibrium state against a change in RR alone is shown in the Figure 12. The models Model I and Model II behavior are plotted together for comparison. In this setting also the Model II is not developing any dynamics as compared to Model I.

#### 4.3 Observations

The following observations are made from the model analysis.

# *a)* The anchor and adjustment policy (APVIOBPCS) adopted for a closed loop supply chain can lead to dynamics in the system.

From the analysis made in the previous sections, it can be inferred that the APVIOBPCS policy when applied to a closed loop supply chain, the system response against changes in return rate is similar to that against changes in customer demand. Though the inventory on hand changes because of RR, the forecasting and consequently the desired inventory and desired WIP do not account these changes in the system. Because, demand forecasting is made based only on the customer demand. When returns come to the system, the serviceable inventory start to increase and let the inventory adjustment to take place. The value of change in order adjustments remain zero at this time however, the adjustment for WIP starts increasing. Thus even small changes in the inventory level alone can lead to dynamics in the system. From the above discussion it is clear that the system can exhibit similar behavior against changes in demand and changes in return flow of products. It is clear from the discussion about Model 1 behavior in Section 3.3.

By the modified policy (Model II), the system does not take any action in the form of adjustment for inventory and adjustment for WIP, against the change in inventory due to remanufacturing activities. In other words, the system try to keep value of adjustment for inventory and WIP always zero, or, the variables INVADJ and WIPADJ are made insensitive to the changes in the return rate. In the modified policy every change in inventory level due to remanufacturing, is accounted as virtual WIP (VWIP). This auxiliary variable is used to make the INVADJ to be zero by adding it to the INVADJ definition or subtracting it from the inventory value. Also it is subtracted in the WIPADJ definition or added to the effective WIP to cancel the effect of inventory buildup in the adjustment for WIP. Once the value of VWIP stabilizes, the production start rate, and inventory also become stabilized. Any adjustment coming out of adjustment for WIP and adjustment for inventory will be only because of changes in demand through the perceived demand. By redefining PSR as DPSR - RPSR, the system accounts for the remanufactured products coming to the serviceable inventory.

# b) The stability boundary of the manufacturing system in terms of $\alpha$ and $\beta$ remain unaltered even when the return flow of products start in the system

From the behavior of the Model I, it is seen that the changes in RR makes the system to behave as if there is demand changes (Section 3.3. Also the behavior of the system at different points in the stability region of  $\alpha$  and  $\beta$  remain the same. This lead to unwanted dynamics to buildup in the system.

# *c) Policy differences can be well observed in a critical equilibrium state or unstable state than in a stable equilibrium state of the system.*

In the discussion made in Section 2.2, 3.2, and 4.2, it has been observed that if the system is in a stable equilibrium state, the changes on the system behavior due to the inclusion of the return flow can not be observed fully. Specifically if the parameters  $\alpha$ , and  $\beta$  are set at 1 each, the change is not at all observable. However, when the system is set at critically stable state, the change in the system behavior due to the policy change is evident (Fig. 11 & 12). The system with the modified production ordering policy (Model II) performs better than the system with Base Model policy (Model I) against the changes in RR.

### 5 Concluding remarks and scope for future studies

The preceding analysis clearly explained how dynamics can be generated when return flow of products starts to happen or changes in a closed-loop supply chain even in the presence of a well defined control policy. We discussed and explained how this dynamics can be eliminated if the effect of return flows are accounted. The production controller has to keep track of the accumulation effect of flow of remanufactured products into the serviceable inventory.

We have explained how the modified policy improves the Base Model policy. The modified control policy is applicable to any number of production stages, any type of delay and delay period involved, and with any number of manufacturing and remanufacturing stages. The modifications in Base Model policy to adapt to a closed-loop supply chain make the control a responsible one with discrimination to the changes in input variables RR and CD.

The orders in the lower level become the demand for the immediate higher level in a multi player supply chain. As seen form this analysis, the orders to a higher level player in a supply chain can vary even if the lower level player receives returned products at constant rate. Future research can address the behavior and stability of a multi player CLSC as a whole when the return flow of used products happen at some lower levels or intermediate levels or both.

In the analysis we have assumed the remanufacturing facility to be separate from the manufacturing facility. This was to acknowledge the fact that the remanufacturing can be offered by a third party. Even though production process requiring sophisticated machining operations will retract third parties from doing the remanufacturing operations, they may engage refurbishing alone. The assumptions can be relaxed to use the manufacturing facility itself for remanufacturing. To enable remanufacturing activities also in the manufacturing facility, the system should have the flexibility to absorb the uncertainties associated with product returns at end-of-life. Also the product designs should include the principles of 'design for recycling' and 'design for disassembly' to enable remanufacturing activities easier. Another direction of research can be to explicitly establish the stability boundary for the parameters  $\alpha$  and  $\beta$  under different ordering policies and varied production delays.

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