Understanding Recent Developments in Growth Theory

Lars Weber

Brandenburg University of Technology Cottbus
Faculty Mechanical Engineering, Electrical Engineering and Economical Engineering
Chair of Macroeconomics
P.O. Box 10 13 44
03013 Cottbus / Germany

Phone: +49 355 69-3967 Fax: +49 355 69-3624
Email: lars.weber@tu-cottbus.de

Abstract

The growth theory has, through so-called 'endogenous' or new growth theory taken on decisive impulses. This contribution delivers an overview of the various extensions without going into detail about the mathematical observations and the main focus on supply-side orientated approaches. The main goals of the growth theory are to understand the exponential climb of the population's income or also the per-capita increase and to divert from the extensions for policy makers. The paper uses stock-flow-graphs to visualize the major loops. Because changes tend to be incremental I adopt standard textbook models first. All models are used in economic teaching with additional simulation and extensions. Later on students learn to modify those models.

Key words

endogenous growth theory, system dynamics, model, simulation, visualization

1 Introduction

At first sight, the current economic problems of today seem to have very little to do with growth politics. Numerous times it is emphasized that the economic trend is paralyzed, due to the insignificant demand for consumer goods, or the challenges presented in foreign trade. However, if one examines this more exactly, for example in globalization, demographic change and unemployment have much to do with resilient economic growth. Thus, without **steady and appropriate economic growth**, we will hardly be able to master the challenges of globalization or dismantle the numbers of unemployment on a larger scale.

1.1 Exponential growth is multiply underestimated

It is said again and that growth is not everything. However growth is a decisive size in order to increase the wealth of a nation. An entirely simple example should clarify this: There are two countries. Exponential growth follows at the same time the formula:

(1) $A_t = A_0 \cdot e^{n \cdot t}$, with A_0=Beginning amount and n = Growth rate of country A and

(2) $B_t = B_0 \cdot e^{g \cdot t}$, with B_0=Beginning amount and g = Growth rate of country B.

One country A is exactly twice as "rich" as country B. It counts:

(3) $A_0 = 2 \cdot B_0$

The stronger country A grows at a slower rate than country B: n <g. The difference of both rates amounts to 1%. How long would it take for the poorer country to catch up to the richer country? Answer: circa 70 years. How long does it take if the growth difference amounts to 2%? Now it would only take around 36 years. And at 5% about 15 years until country B becomes equal. It is important to understand the dynamics behind these examples. Over time even only small difference in the growth rate makes a large different.

1.2 *Technical progress is a key growth factor*

For a long time, growth theories have been paid less attention. However, the neoclassical growth model from Robert Solow (1956) changed this. Many economists, also notable noble prizewinners, dedicate themselves to the growth theory. Technical development is now seen as an influential size behind economic growth. Solow's model included an explicitly technical progress, but was only integrated as an exogenous factor into the model. Joan Robinson (1962) noticed correctly, that technical progress does not fall "like Manna" from the sky. Therefore, the newer growth theories try to describe the technical progress **within the models**.

It is the goal of this contribution to deliver an overview of the over the supply-side growth theories, also named endogenous or new growth theory. Nevertheless there are yet further current growth disciplines, for example, the empirical growth research or growth theories with an emphasis on demand-side. These also entail new aspects, thus the concept of newer growth theories can be misleading.

Starting with the model from Robert Solow, we can explain the AK-Model, the Uzawa-Lucas-Model, and the Romer-Model and in conclusion the Jones-Model. The main consideration in the representation lies at the same time on the comparative analysis. The exact proof for this balance can be found in the cited literature.

2 General Structure of Growth (Systems Archetypes)

In general the exponential growth follows the form: A Stock K that changes over time through the sum of the in- and outflows per time t with $\dot{K} = K_t - K_{t-1}$. The growth is exponential if the net change of $\dot{K}$ depends on the stock K and a certain factor multiplied becomes: $\dot{K} = K_{t-1} \cdot n$. A certain share of the stock K leads to an increase if n>0. This basic growth scenario is in figure 1 represented.

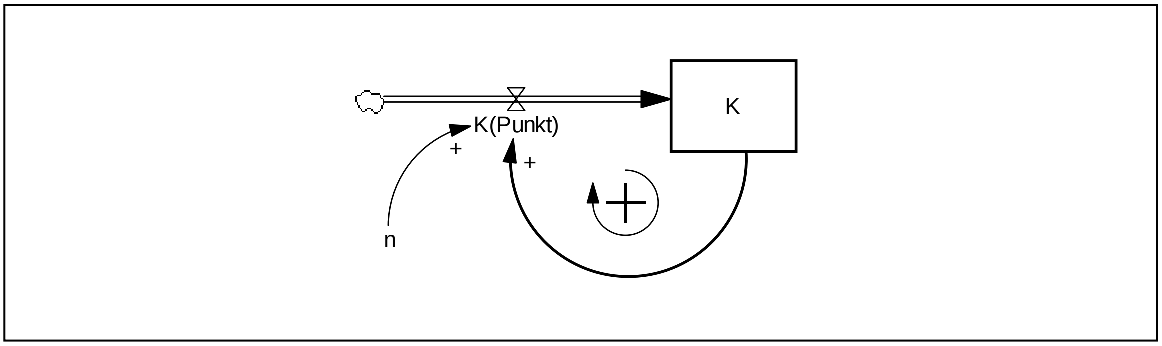

figure 1 – basic scenario of exponential growth

2.1 Stocks and flows

The stock K is represented as a rectangle. K is determined by the changes in K(Punkt), which one can observe on the thick straight arrow. K(Punkt) represents the change in stock K from t to t+1. In addition, there can also be auxiliary variables and constants. Auxiliary variables use the calculation between stocks, flows, constants and other variables. They change in each period t. In the graph, they border on the circular. Constants are independent of the time. They are exogenous. In this example, there are no assisting variables, however a constant: n.

2.2 Arrows create causal links

The arrows show the relation of the variables, stocks and flows to each other. A "+" on the arrow tip means identical direction. In figure 1, an increasing growth rate n would lead K(Punkt) climbing. A "-" sign symbolizes climbs towards the opposed direction, therefore a variable increases, which leads to a decrease in the dependent variable. The strength of this relationship is mathematically determined and cannot be indicated explicitly in this representation. The large advantage of a graph instead of a pure mathematical representation is to be sure that the observer can optically recognize, which relations form the model.

2.3 Exponential growth has a positive total effect on the variable

When the total effect of a loop is reinforcing, represented by a large "+", then the statement of the effect direction is in the middle of the loop. Within the total effect of the loop is negative because of an uneven number of negative polarities amounts to a marking of “-”. Generally the model structure determines the behavior and same structures evoke similar patterns of behavior. Yet we will see that the simple exponential basic structure can be regained in all introduced growth models.

3 Solow-Model

The Solow-model with Harrod-neutral technical progress and population growth consists of three important factors:

- the **capital stock K**,
- the population or also the **labor forces supply L** and finally
- the **technical progress A**.

The population’s income Y is a **Cobb-Douglas-production function** with $Y = K^{\alpha} \cdot (A \cdot L)^{1-\alpha}$ and consists of the three stocks. Alpha is the production elasticity and amounts to about 0.30.

3.1 L and A grow exogenous and exponentially

The labor force grows exogenous and exponentially with the rate n. The technical progress also grows exogenous and exponentially with the rate g. Formally, it is represented by:

(4) $L = L_0 \cdot e^{n \cdot t}$

(5) $A = A_0 \cdot e^{g \cdot t}$

A constant share S of the income Y is saved:

(6) $S = s \cdot Y$; s = Savings.

Investments I are the change of the capital stock over time, therefore $\dot{K}$. Because the **identity I = S in a closed economy** counts, we can formulate the first order equation of the capital stock with:

(7) $\dot{K} = I = s \cdot Y$

The formulation of the capital stock therewith not exogenous is purported, but rather diverted out of the model. Depreciations are thereby considered within the net investments.

The model includes **exponential growth patterns** at three places:

- In the growth of the technical progress A,
- In the growth of the population L and
- In the growth of the capital stock K.

This becomes especially evident if we represent the structure of the Solow-model graphically (figure 2). The picture is comparable with figure 1, however it includes several of these simple growth patterns.

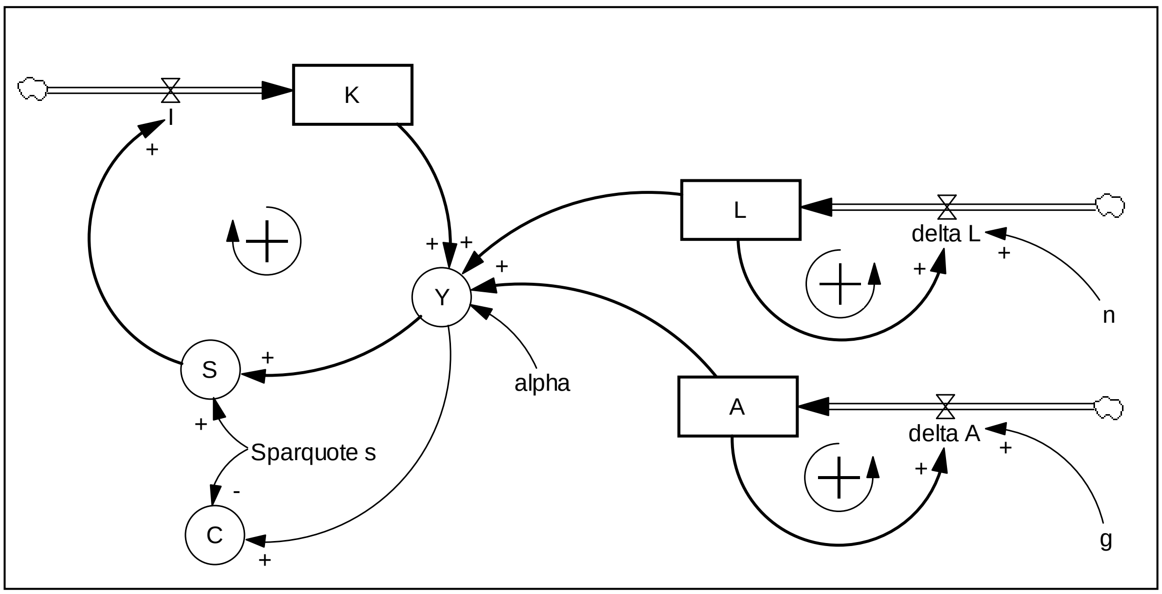

figure 2 – Solow-Model with technical progress and population growth

J. Robinson expressed criticism that in this model the technical progress is not fully explained, stating that one can recognize this because the technical progress surly influences the population's income Y, and yet no variable except the growth rate g changes the progress. Therefore, it is only explained exogenously.

The model aims at a related steady state on $k = \frac{K}{A \cdot L}$ (with k = capital intensity in efficiency units). The variables still grow exponentially, however the ratio $k = \frac{K}{A \cdot L}$ remains constant k*. One can show that the equilibrium Y and K with the growth rates of the technical progress and population growth grows: **gY=gK=gL+gA**. Interesting is that the per-capita income at the steady state are still growing with the same rate as the technical progress. The statement is clear – steady wealth increasing growth is doable only through technical progress.

3.2 Convergence thesis

If one compares two countries with different initial values of the capital stock than follows, according to the Solow-model, that the capital stocks will approach each other over time. When the growth rate of the population and the technical progress are equally large and the saving ratio is identical than steady state will be the same. This is designated as an **absolute convergence**. When the saving ratios or the growth rates of the population or of technical progress differ, then they simply approach each other. This is designated as a **relative convergence**. Therefore, it is only a matter of time before the wealth of these countries adjusts itself. This is due to the decreasing boundary productivities in the production factors. In 1992 Mankiw/Romer/Weil presented a famous study, which empirically examined this approach and within the approach confirmed the core statements of the Solow-model.

4 AK-Model

4.1 Overcoming the decreasing marginal productivities

The Rebelo model (1991) also called AK-Model goes a step further and builds on the Solow-model. The **decreasing marginal productivities** of the input factors should be overcome. Thereby, adjustments of these countries would be explicable, and also a durable divergence or also overtaking would be possible.

4.2 Divergence thesis

The drift is designated as a **divergence thesis**. Rebelo undertakes some decisive variations from the Solow basic model. First the technical progress A is no longer Harrod-neutral and laborsaving, but rather Hicks neutral. This means that it no longer functions as productivity factor for work L, but rather as a total productivity factor. Secondly, a human capital factor H is introduced. It takes the place of the technical progress for the Harrod-neutral. Thus, the new production function reads:

(8) $Y = A \cdot K^{\alpha} \cdot (H \cdot L)^{1-\alpha}$

4.3 Introduction of the human capital factor

This is similar to the Solow-model. However, the decisive step is the determination of the human capital factor. It is no longer exogenous, but rather becomes endogenous and is described in the model with:

(9) $H = \frac{K}{L}$.

Verbally this means that the efficiency of the factor work L is determined by factor H. H is larger when the capital intensity grows. For example, through the uses of a computer (stands for K) at a workplace (stands for L) the output of the work can be increased. Use of $H = \frac{K}{L}$ in the new and summarized production function includes:

(10) $Y = A \cdot K^{\alpha}$; with 0 <<1.

Other equations, like the growth of A and L or the movement equation of the capital stock are identical to the Solow-model.

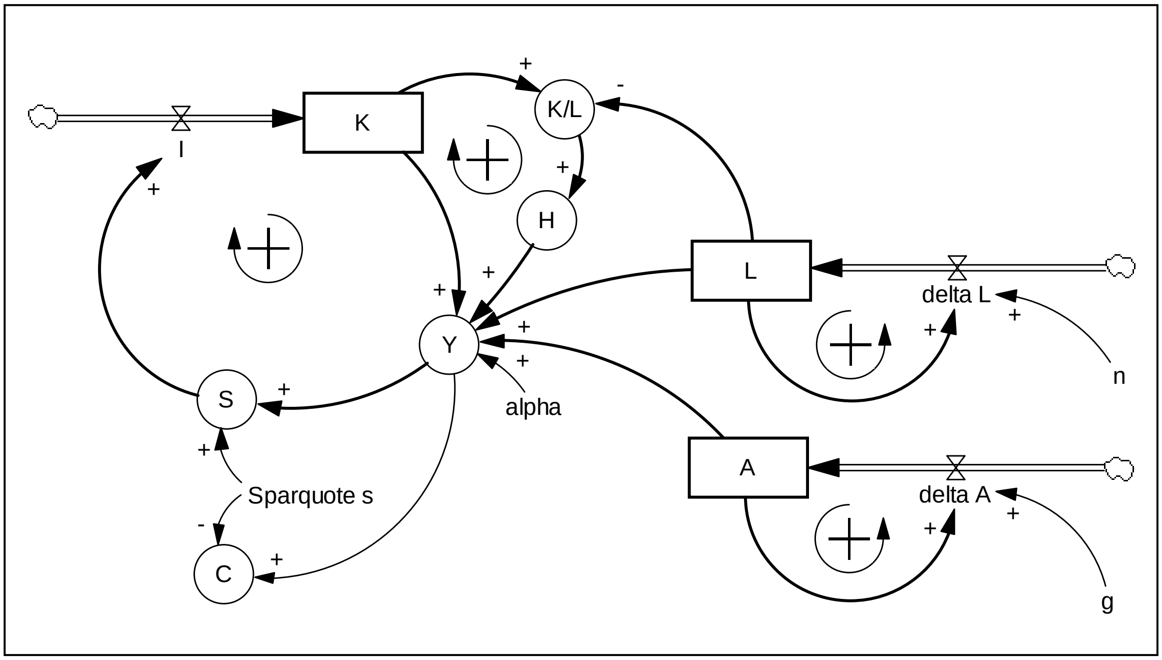

figure 3 – AK-Model

Figure 3 shows the structure of the AK-model. Preceding this, the Solow-model comes, by which a new calculation shows the difference in human capital. The capital stock K is once again **doubly reinforced**. The consequence of this is the description of overcoming the decreasing marginal productivity of the capital. The model no longer shows a steady state. The AK-model is attributed the endogenous growth theory. On the first glance this is remarkable, because the technical progress A is still declared exogenous and reinforces only the accumulation process. However, the AK-model is an attempt to get around the decreasing marginal productivity in capital and for the first time the meaning of the human capital is underlined. Although the human capital cannot yet be calculated, it nevertheless shows that a connection exits with the capital stock. This is explained therewith that the capital stock is composed of real capital and human capital.

5 Uzawa-Lucas-Model

The Uzawa-Lucas model does not try to integrate the human capital as one of two stocks K or L, but rather it models it directly as an own stock. The weakness of the AK-Model is that the human capital is not directly accumulative. Uzawa presented this idea in 1965.

Lucas took up this model and developed it further. Usually it is represented as an Uzawa-Lucas-Model.

5.1 Education as a stock

The capital stock increases through net investments. However, how does one increase human capital? The answer is short: through investments in education. The labor force supply L is replaced by the human capital. The labor force supply is described quantitatively and is no longer qualitative. Therefore, it is no longer an absolute quantity for labor forces, but rather a quality. Macro economically the human capital H is the labor force supply weighted with the average qualification level. By doing this the working hours u of an average worker can now be calculated into the production sector Y and in (1-u) for the advanced education divide. Thus, the amount of education enlarges the human capital stock. Thus, the human capital appears:

(11) $H = u \cdot H + (1 - u) \cdot H$

The variation of the human capital stock results from:

(12) $\dot{H} = (1 - u) \cdot H_{t-1} \cdot B$; B = technology parameter of the education sector.

A high productivity B yields a faster increase in human capital H. The original model also included the depreciation of the human capital, for example through the retirement from professional life or outward migration from the economic system. This not explicitly observed in this contribution, however, but $\dot{H}$ is rather understood as a net increase. The human capital stock grows consequently exponentially with:

(13) $H_t = H_0 \cdot e^{B \cdot (1-u) \cdot t}$

Next to the productivity B, yet the spent time (1-u) and the initial value H_0, the human capital also is important.

The remaining time u of the human capital flows, as already indicated, towards the production sector. The people’s income Y can be described again in a similar form to the Cobb-Douglas-production function:

(14) $Y = A \cdot K^{\alpha} \cdot (u \cdot H)^{1-\alpha}$

A constant part s of the people's income directs over the identity I = S to increase the capital stock with

(15) $\dot{K} = I = s \cdot Y$

The technical progress A is Hicks-neutral and grows exponentially in accordance with:

(16) $A_t = A_0 \cdot e^{g \cdot t}$

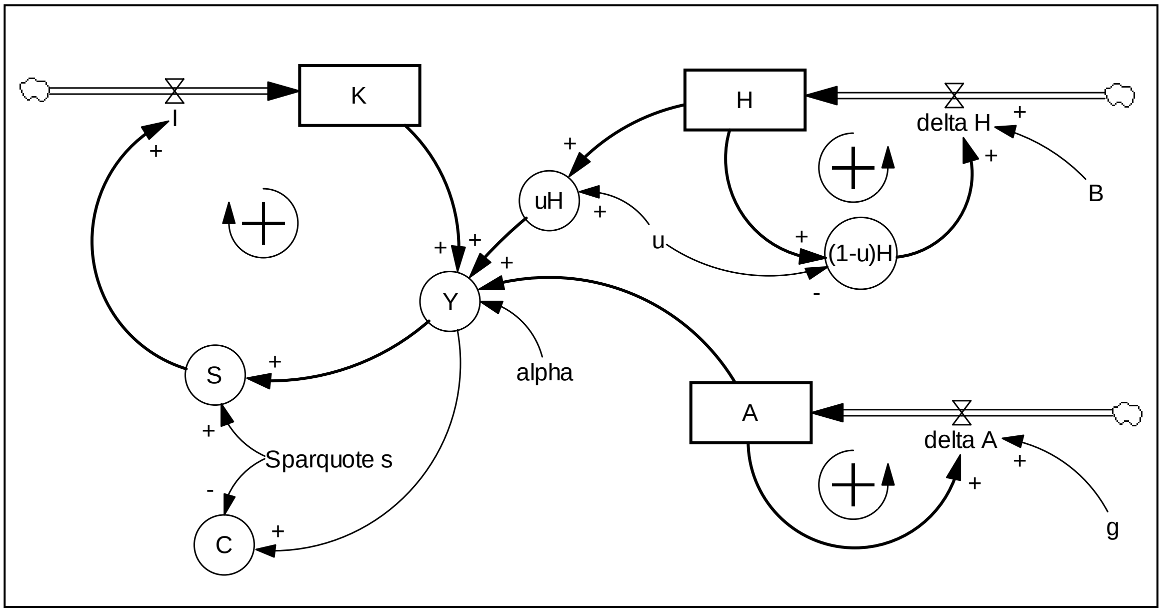

figure 4 – Uzawa-Lucas-Model

Figure 4 represents the Uzawa-Lucas-Model graphically. Again the basic resemblance to the previous models is evident also here – three stocks K, H and A lead to the growth of Y. All three grow exponentially. The model includes a development sector and a final production sector. An important difference is in the growth of the human capital. This is determined through the size of (1-u) and is changeable, thus, u is still exogenous. However, u becomes "endogenous" through the amount of time spent. Each economy can determine how much time is designated to education. Lastly, the entire human capital is the power that determines the per-capita-wealth Y/H.

5.2 Humane capital stock can explain missing convergences

With the Uzawa-Lucas-model the absence of a convergence of the countries can also be explained. The reason for this can be that the **human capital stock is too slight** or also the growth of H is too little. This new knowledge comes vis-à-vis the AK-model. There the absence of convergence is due to the savings.

6 Romer-Model

6.1 R&D-Models in general

All previously introduced models are similar, in that they offer a good explanation for growth pattern, however the technical progress A is always exogenous. The work of Grossman-Helpman changed this. They incorporated an intermediate goods sector, borrowed from Dixit and Stiglitz. The model from Grossman-Helpman is not explicitly introduced here because Romer published a further development, which connects the ideas from Grossman-Helpmans with the extension of the Solow model. Thus, the Romer-model is introduced. In addition to this, there are other models that can be summarized by the "R&D-Model". The basic idea behind was that research and development sets stimuli for economic growth. In the following, the origins of these theories are also clarified.

6.2 Schumpeter's ideas are the basis for the R&D-Model

This group of models is constructed on the basis of Schumpeter's ideas. Already in the 1920's, Schumpeter explained that competition represents a sequence of innovation and transfer processes. Firms support research and development in order to secure themselves by creating a **monopoly over time**. Through a **transfer process**, they encourage customers from other companies to try their new product. By doing such they enable the chance for disbursed research costs and development costs and have a pioneer advantage and competitive advantage in comparison to other companies. Other firms also have a reason to use R&D because they can **generate the innovation** for a positive transfer processes.

6.3 The goods "knowledge" cannot be excluded

Human capital and **knowledge** (from now onward synonymous for technical progress) are different, in that human capital is tied to the person. Here one can assume that people decide about themselves. Knowledge, however, manifests itself in the Schumpeterian understanding for new products and is registered through the patents in abstract form. Thus, the **non-excludability** no longer works automatically, but rather becomes accessible for a larger group. Knowledge is therefore marked with **external effects** (spill-over). It also deals with the free-rider-problem.

6.4 The good "knowledge" has no rivalry

The marginal costs of knowledge are not dependable and that knowledge can be ended by patent protection, the feature of knowledge has **no rival**. Comprehensively, according to Schumpeter's characteristics, knowledge also serves a public good.

6.5 Structure of the Romer-model

The Romer model is divided into three sectors:

- a research and **education sector A**,
- a **intermediate goods sector x** and
- a **consumer goods sector Y**.

For simplification, a constant is used as an input for work L. It is seen $L=L_0$. The work can be used in two sectors. Either in the R&D-sector (A) or in the final product sector (Y): $L = L_Y + L_A$. At the same time, the variable z determines the share of work in the R&D-sector and (1-z) shows the remaining share in work for the consumer goods sector free:

(17) $L_A = z \cdot L$

(18) $L_Y = (1 - z) \cdot L$

In the R&D-sector, knowledge is accumulated through the use of work. The variation of the knowledge is given:

(19) $\dot{A} = A_{t-1} \cdot L_A \cdot \left(\frac{1}{g} \right)$

The productivity parameter is g. The growth of the knowledge follows therewith, again the well-known exponential growth scheme:

(20) $A_t = A_0 \cdot e^{\left(\frac{L_a}{g}\right)\cdot t}$

6.6 Knowledge increases the amount of intermediate goods

The decisive step in Romers model is the linking of knowledge to the consumption goods sector. It is supposed that with knowledge, procedure innovations are carried out, i.e. the manufacture of a product occurs through the specialization of singular intermediate good. The amount of the intermediate goods x(j) is identical to the knowledge A. Thereby, the capital K is divided amongst all intermediate goods with $x = \frac{K}{A}$. One can describe the intermediate goods sector x then as follows:

(21) $x^{\alpha} = \int_0^A x(j)^{\alpha}\, dj = A \cdot \left(\frac{K}{A}\right)^{\alpha} = K^{\alpha} \cdot A^{1-\alpha}$

The intermediate goods sector moves toward the production function:

(22) $Y = x^{\alpha} \cdot L_Y^{1-\alpha}$

(23) $Y = K^{\alpha} \cdot A^{1-\alpha} \cdot L_Y^{1-\alpha} = K^{\alpha} \cdot (A \cdot L_Y)^{1-\alpha}$

The last equation is almost identical with the well-known production function from the Solow-model. The difference exists, however, in the share of work, which flows towards the consumption goods sector. Again it applies, that the share s from the people's income is saved and that identity of the capital stock S=I increases.

(24) $\dot{K} = I = s \cdot Y$

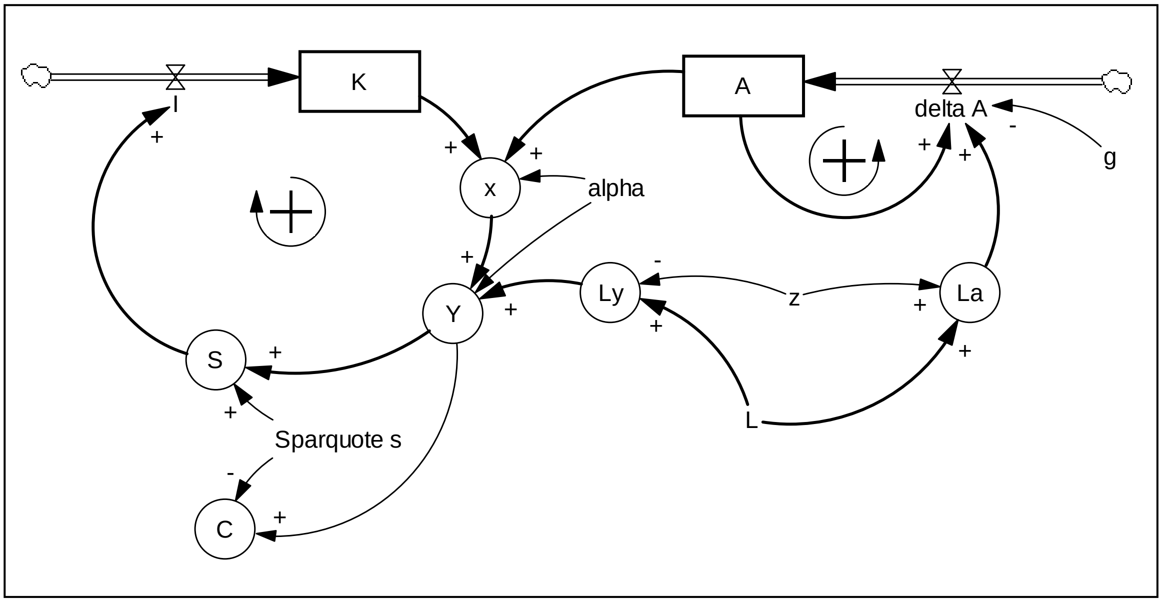

figure 5 – Romer-Model

Figure 5 shows the total Romer-model as a stock-flow-diagram. Because the labor forces offer L as a constant and exogenous, the Romer-model consists of only two stocks, the knowledge A and the capital stock K. With x, the intermediate goods sector is marked, and leads directly into the consumption good sector. Again the exponential growth pattern of the two stocks is clear. The behavior remains, therefore, comparable to the other models, but nevertheless the explanation for exponential growth is unrelated. It is readily conceivable that the labor force L grows at an exogenous rate n (such as in the Solow model). If this was the case, the structure would be very similar to the Solow-model, however other information can be diverted out of the Romer-model.

The growth rate of the R&D-sector is determined through the supply of labor forces (i.e. engineers). The consumption goods sector Y and the R&D-sector A compete for the labor forces L. For the first time, the Romer-model explains how innovations can influence an economic system by a productivity increase. The innovation process is explained in the model, even though the Schumpeterian transfer process is not yet implemented, because older technologies are not squeezed out of the market. It should not be unexpected that in the model the acceptance of an imperfect competition and the existence of monopolistic competition are possible. This represents an inconsistency with the neo-classical acceptance of **perfect competition**.

7 Jones-Model

7.1 Theory and empiricism do not always agree

In spite of the brilliance of Romer's theoretical extensions, it is difficult to find empirical proof for the validity of the theory. Between the growth of the knowledge and the number employed in the R&D-sector, a connection would have to exist after Romer's idea. In 1995 Jones criticized that that unfortunately this cannot happen. Arnold (1997, S. 222) presents the basic problem very appropriately:

> State of the things is, that we have an implausible model with appropriate empirical implications (the Uzawa-Lucas-model with growth through human accumulation) and a plausible model with doubtful empirical implications (the Grossman-Helpman-Romer-model with growth through R&D).[1]

Moreover the problem existed that in the R&D-sector constant return on scales were present. Usually neo-classical models assume decreasing return of scales. In 1995 Jones presented a model of a semi-endogenous growth, which combined the Romer-model with the Uzawa-Lucas-model.

7.2 Integration of the education sector in the Romer-Model

The previously presented Romer model integrates an education sector. Instead of the constant labor force supply L, the **human capital sector H** is introduced. The quantity in the foreground therefore no longer exists, but rather the quality of the labor forces. **Education increases the humane capital stock**. Beside it is the population growth, which grows at the rate n. The human capital stock can consist of three areas:

- Education enlarges the supply of human capital
- Activities in the R&D-sector increase the amount of knowledge or
- Work in the consumption goods sector supplies the people's income Y.

This is formally written:

[1] Translated from German into English

(25) $H = H_W + H_A + H_Y$ with

(26) $H_W = w \cdot H$, with w = share in education

(27) $H_A = (1-w) \cdot z \cdot H$,with z = share in the R&D-sector

(28) $H_Y = (1-w) \cdot (1-z) \cdot H$

The human capital grows exponentially with:

(29) $H_t = H_0 \cdot e^{w \cdot n \cdot t}$

The R&D-sector remains basically identical with the Romer-extension, however Jones' allows scales effects of the human capital and the already available knowledge. This is expressed through the exponents φ and χ. The change over time of knowledge is therefore:

(30) $\dot{A} = H_A^{\varphi} \cdot A_{t-1}^{\chi} \cdot \frac{1}{g}$

Through the exponents, one can see the exponential effect on the stock of knowledge, which can be over or under proportional.

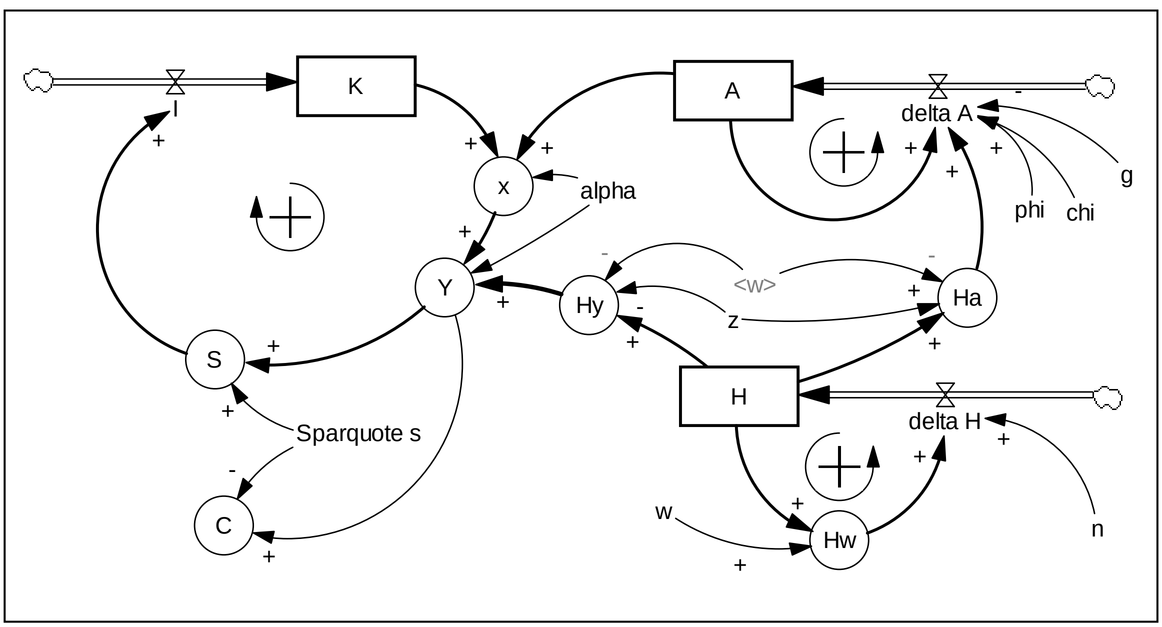

figure 6 – Jones-model of the semi-endogenous growth

The intermediate goods sector and the consumption goods sector are similar to the Romer-model. From that the stock-flow-diagram in figure 6 arises the three stocks K, H and A evidently. All three grow exponentially. However, in the Jones-model human capital growth and accumulation of technical progress (knowledge) are simultaneously possible. The growth of the human capital, such as in the Uzawa-Lucas-model, is dependent on the economic system and how much time will be devoted to development. The Jones-model can present empirical observations sooner and harmoniously (see criticism Romer-model) by using decreasing return on scales in the R&D-sector.

8 Summary

With the graphic representations, it is possible to recognize the uniform exponential growth patterns. The behavior of the model is similar to all of the extensions; however the statements on the growth are very different. In a further step the teaching would include also simulations on those models because there are essential for effective systems thinking (see also Sterman 2002, 525). But the curriculum often does not offer time for these extensions. So we have to deal with this time constraints. The challenge would be to shift the amount near future of the scientific time-scale. This means we have to reconsider what is necessary for students to emancipate them for their future challenges.

The goal of this contribution was to represent different extensions of the endogenous growth theory by means of comparing them. Constructions of the Solow-model, the AK-model and the Uzawa-Lucas-model were used and introduced as representations of the human capital approaches. Following the Romer-model was employed as an example for the R&D-models. The Jones-model represents a type synthesis of the two streams in the new growth theory. Ideas of the human capital theories and the R&D-models were connected. The overview stood in the foreground, thus mathematical representations were carried out in a limited fashion. For further information consultation in the recommended literature would be suggested. Importantly it is to be shown that after the Solow-model, which forms the point of departure in the economic growth education, yet many further interesting extensions are available. With the graphic representations, it is possible to recognize the uniform exponential growth patterns. The behavior of the model is similar to all of the extensions; however the statements on the growth are very different.

9 References

Aghion, P., Howitt, P., & García-Peúnalosa, C. (1999). *Endogenous Growth Theory*. Cambridge, Mass.: MIT Press.

Arnold, L. (1997). *Wachstumstheorie.* Munich: Vahlen.

Barro, R. J., & Sala-i-Martin, X. (2004). *Economic growth.* Cambridge Mass.: MIT Press.

Frenkel, M. R., & Hemmer, H.-R. (1999). *Grundlagen der Wachstumstheorie.* Vahlens Handbücher der Wirtschafts- und Sozialwissenschaften. Munich: Vahlen.

Grossman, G. M., & Helpman, E. (1995). *Innovation and Growth in the Global Economy* (5th ed.). Cambridge Mass.: MIT Press.

Jones, C. I. (2001). *Introduction to Economic Growth.* New York: W.W.Norton & Company Ltd.

Kleinewefers, H., & Jans, A. (1983). *Einführung in die volkswirtschaftliche und wirtschaftspolitische Modellbildung.* Vahlens Handbücher der Wirtschafts- und Sozialwissenschaften. Munich: Vahlen.

Mankiw, N. G., Romer, D., & Weil, D. N. (1992). A contribution to the empirics of economic growth. *QJE, 107*(2), 407–437. Retrieved Jan 01, 2007, from http://links.jstor.org/sici?sici=0033-5533%28199205%29107%3A2%3C407%3AACTTEO%3E2.O.CO%3B2.

Maussner, A., & Klump, R. (1996). *Wachstumstheorie.* Berlin u.a.: Springer.

Sterman. J. D. (2000). *Business Dynamics Systems With CD Rom:* McGraw-Hill Education - Europe.

Sterman, J. D. (2002). All models are wrong: reflections on becoming a systems scientist. *SDR*, *18*(10), 501–531.