Trend Forecasting as Derivative Control

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ABSTRACT

This paper revisits the use of trend forecasting in driving policy in social systems by comparing it with derivative control in classical control theory. While both processes involve use of trend to determine policies for achieving reliable performance, the outcomes of the former have considerable variability while those of the later can create improvement in performance with certainty. The similarities and differences between the two processes are discussed and guidelines suggested for improving the efficacy of trend-forecasting in policy design in social systems.

Key Words: *Trend forecasting, planning, policy, system dynamics, dynamical systems, supply chains, control theory, PID control*

Introduction

The use of trend forecasting in policy design has been viewed with reservation in system dynamics. Jay Forrester pointed to the amplification of changing conditions created by the use of trend forecasting in policy as far back as 1961 in Appendix L of his seminal volume Industrial Dynamics (Forrester 1961). The self-fulfilling nature of forecasts was explored by James Lyneis (1975) and he reported the destabilizing impact of use of forecasts in decision making in further detail in Lyneis (1982). Barry Richmond supplemented this work by developing a trend macro for the System Dynamics National Modeling Project at MIT and testing it with cyclical functions, which confirmed earlier propositions by Forrester and Lyneis about forecasting increasing instability while also adding to the evidence against the reliability of use of forecasting in policy (Richmond 1977).

Trend forecasts applied to the determination of ordering policies in supply chain management systems often seems to exacerbate periods of shortage and oversupply. It is not surprising that trend forecasting has largely been viewed in system dynamics community as a source of instability and as a dysfunctional basis for decisions. Trend information has been used on the other hand with great reliability in engineering in controller control. Trend manifests in derivative control, which adds a correction in response to the rate of change of error, thus further speeding up correction when error is rising (Takahashi, et. al. 1972, pp343-350).

Given that both trend forecasting and derivative control use the same basis for policy, it seems anomalous that the former cannot be applied with reliability while the later can be used with complete certainty to improve performance. It should be noted, however, that while forecasting often entails use of derivative in policy without knowing other components of control in place and also without knowing which feedback loops it will create or reinforce, derivative control is fully cognizant of both these processes. It is also observed that, while derivative control invariably uses the trend in a *tracking variable* within the control system as a basis for determining correction, forecasting may often use a *tracked entity* outside of the organization to develop a basis for a remedial policy which may not deliver an appropriate correction to the tracking entity within the organization. Fore example, in a supply chain, demand is often a tracked variable and supply is a tracking variable. The outcomes of forecasting must remain uncertain depending on what control components might accidentally form, while the outcome of derivative control is always certain based on the components deliberately created in designing the corrective regimes.

In this paper, I will first construct a system dynamics model of the classical PID control mechanisms used in engineering systems that involve use of trend of a tracking variable to improve tracking performance. Next, I'll construct a model of a simple supply chain management system focused on adequately meeting shipment needs by ordering to replenish inventory. Such ordering is often driven by a forecast of shipments (a tracked variable), whose use exacerbates instability. Finally, I will formulate classical control mechanisms in a simple supply chain management system using forecasts of the stock of inventory - a tracking variable, to demonstrate how trend forecasting, used as a policy tool in social and economic systems, can lead to improvement in performance. In conclusion, I'll propose that we can indeed get a reliable performance from use of trend forecasting as a policy if we apply it as in derivative control. In particular, the variable forecast must be carefully selected to represent a tracking variable. Also, the feedback relationship it creates with the decision being controlled must be controlling not reinforcing.

Modeling Classical PID Control

The analogue PID control law can be expressed in the following general form:

$$M(t) = P[e(t)] + I[\int_{-\infty}^{t} e(t)dt] + D[de(t)/dt] \qquad (1)$$

where, M is total correction applied, P (proportional control) is the part of total correction that is proportional to the instantaneous error e, I (integral control) is the part of total correction that is proportional to the integration of the instantaneous error e, and D (derivative control) is the part of total correction proportional to the derivative of the instantaneous error e. Instantaneous error e is invariably the discrepancy between the tracking quantity and the tracked quantity. This law controls many servomechanisms we use in every day life. An example is the power steering in automobiles which allows the wheels of a car to exactly correspond to the position of the steering wheel without the two being rigidly coupled. It is required that when the steering wheel is turned, the car wheels turn smoothly and quickly to the new position of the steering wheel without hunting for the new position and without a displacement from the new position called a steady state error. Another example is the volume control on your stereo or TV. When you turn up the volume control, the sound level smoothly adjusts to the exact position of the control, without hunting. While each of the control components of PID control used in isolation cannot achieve a quick and hunting-free movement to the exact new position, the three components acting together can easily accomplish this.

The proportional control component causes a powerful movement to the new position, but has two problems: 1) It cannot speed up correction in response to a rising error; and 2) it cannot keep track of the corrective force lost over the time period over which the instantaneous error is corrected. When a drain is created for example by a frictional force or an exogenous condition such as embankment of the road or wind condition in our steering example, a steady state error will appear.

The integral control component accumulates uncorrected error in its memory, which continues to apply a correction even when the proportional control has waned due to the reduction in the instantaneous error. Thus, it speeds up the correction while also overcoming the steady state error, but at the same time, it can continue to apply correction when the magnitude of the tracking variable has coincided with the tracked variable. This will cause the tracking variable to overshoot the steady state position, creating an integrating negative error in the process that will drive a correction in the opposite direction. Integral control, therefore, is often a source of instability in servomechanisms as it causes hunting for a goal without getting there.

Derivative control can speed up correction when the error grows at a fast rate. When used together with proportional and integral control, it can speed up the movement to the new equilibrium, although, on its own, it can neither create conformance to goal, nor stability in the path to that goal. Forecasting being a derivative-driven policy, the variability experienced in its outcomes should not be surprising when it is applied to variables that might not have an appropriate relationship with the error or in situations where the other types of control are weak or absent.

Figure 1 shows classical PID control rendered into a system dynamics model. The position or the quantity being controlled is represented by the “tracking stock”. In equilibrium, this stock is at its goal which is the “tracked position”. The tracking stock can be changed by a flow named “total corrective change” while an entropic drain possibly caused by frictional and environmental conditions can also affect a change in the tracking stock. Thus, the drain represents the “tracked flow” and the total corrective change - the “tracking flow”. P Wt, I Wt and D Wt respectively represent the weights assigned to proportional, integral and derivative control components of the total corrective change. Each type of control can be deactivated by assigning a weight of zero to it. The tracked flow is set to zero in the initial equilibrium to simplify the case. Annex 1 gives the equations of this model.

An initial equilibrium in this system can be disturbed by changing the tracked goal as in case of moving the steering wheel in an automobile to a new position. It can also be changed by the entropic forces created by disequilibrium (like friction) and environmental factors (like change in the banking of the road or wind condition). Either type of disturbance will create an error in terms of the discrepancy between the tracking position and the goal.

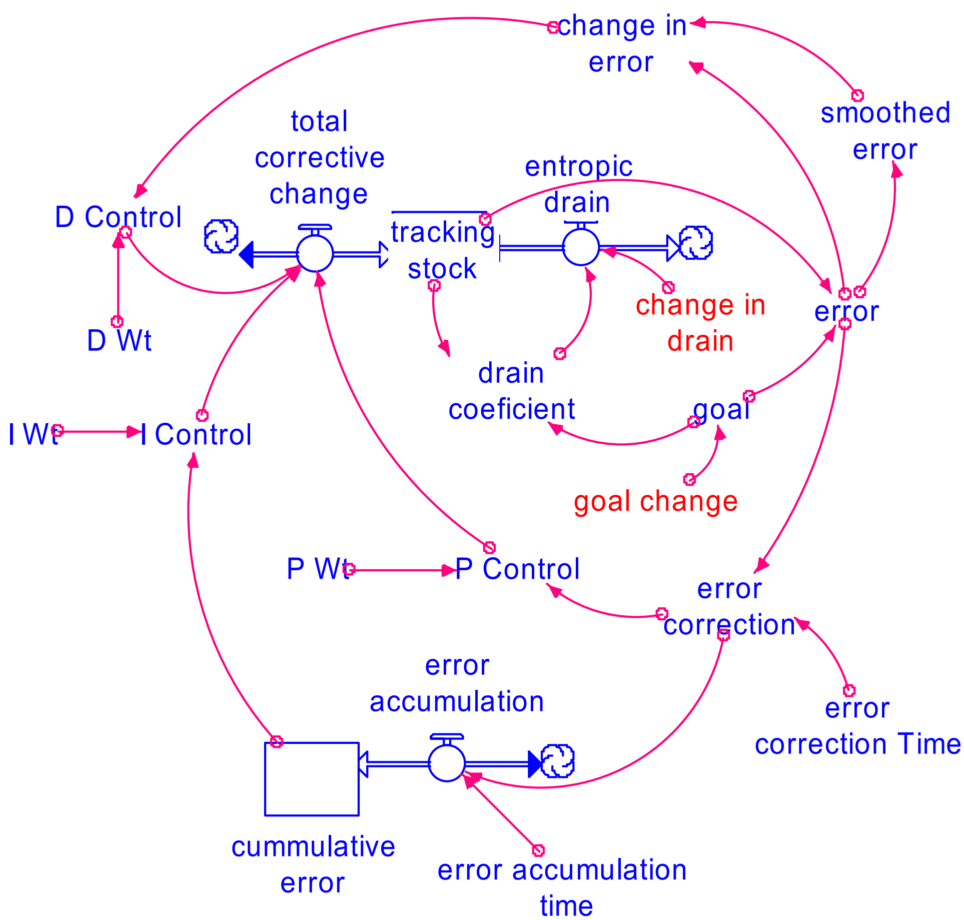

Figure 1 **System dynamics flow diagram of a PID control system**

Figure 2 shows how this error is overcome through the activation of the various types of control mechanisms when the tracked position is changed. Figure 3 shows the respective adjustment paths when the tracked flow is changed. In both cases, only the behavior of the tracking stock is shown.

In the first case, no adjustment occurs in the absence of any type of control. Activation of proportional control starts a powerful adjustment, which becomes sluggish over time. Adding integral control to the process helps to move the tracking stock rapidly to the new goal, but creates an overshoot and oscillation before the goal is reached. Activating derivative control in addition to the proportional and integral components further increases the speed of adjustment, while decreasing overshoot and oscillation.

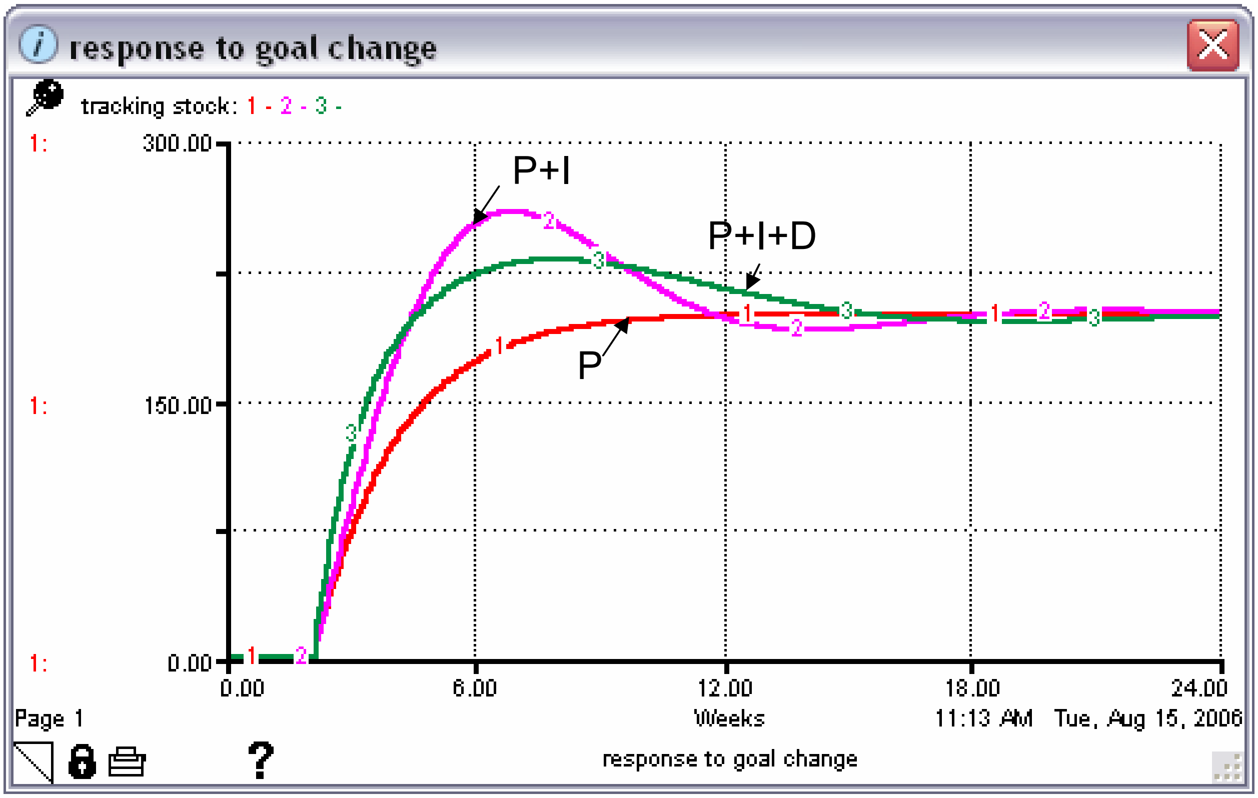

Figure 2 **PID control response to changes in tracked goal.**

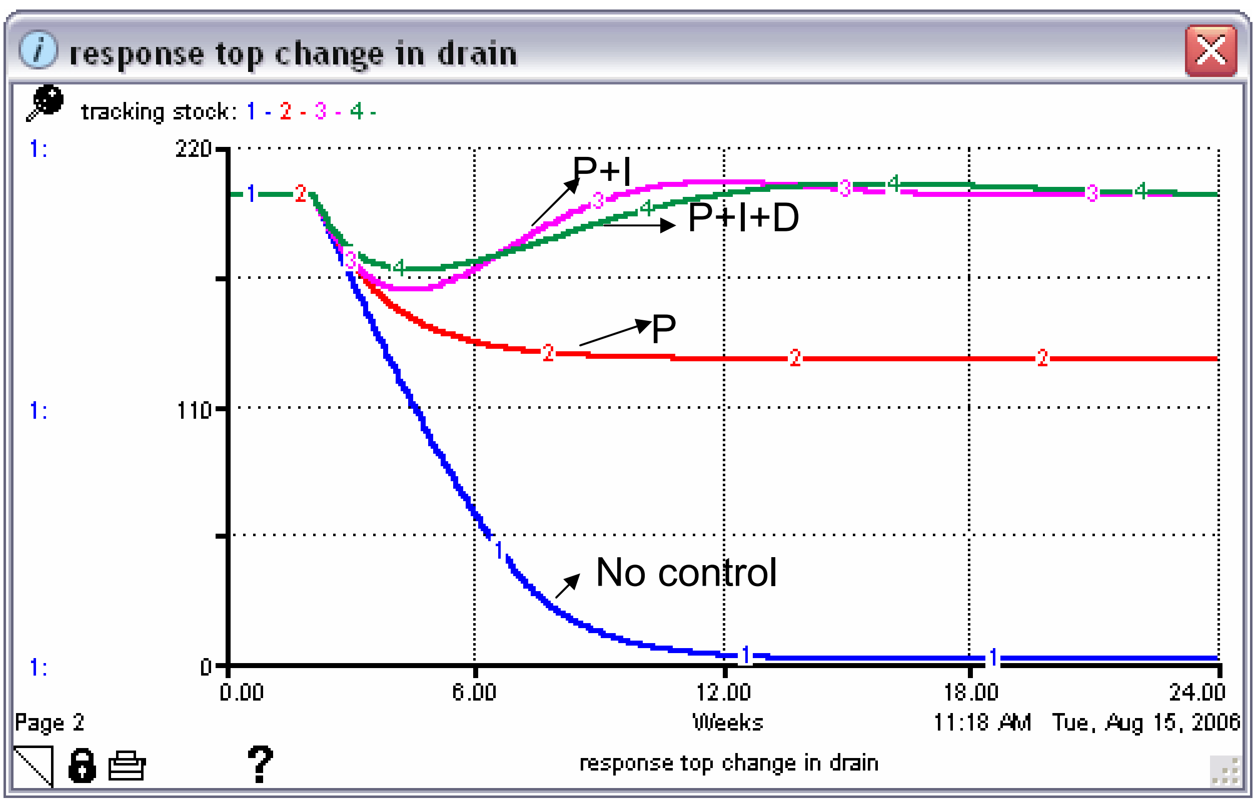

Figure 3 **PID control response to changes in tracked flow.**

In the second case, the tracked flow is autonomously changed. In the absence of any type of control, this change will cause the tracking stock to erode all the way to zero since structure representing the entropic forces creates a first order control on the stock. With the activation of proportional control, this erosion can be contained, but the losses over the course of the correction cannot be compensated. Therefore, the tracking stock comes to equilibrium at a level lower than the goal, thus creating a steady state error. Adding an integral control component to this system overcomes the steady state error, but creates an overshoot and oscillation. Further adding derivative control, increases the speed of adjustment while reducing instability at the same time. Note that PID control involves three elements, one of which is based on trend. This trend actually is a trend of the instantaneous error or the discrepancy between the tracking variable and the tracked goal. All three types of control mechanisms create negative feedback loops and we can control the strength of each feedback loop to achieve desired results. By adjusting the weights of each type of control, it is possible for the tracking stock to track its goal with minimal delay and oscillation. An optimal set of weights can often be found in engineering systems since the other components of the system can be precisely defined, although such weight may be hard to define in social systems where precise measurements are often not possible.

Control processes created by use of tend forecasts in supply chain management

Trend forecasting can be used in supply chain management in many ways. It can entail forecasting shipments or inventory condition for generating information for ordering or planning production. It should be noted that shipments is a tracked variable while inventory represents a tracking variable. I'll discuss below the implications of using forecasts of each of these in an inventory ordering policy in a simple supply chain.

a) Use of forecasts of a tracked variable for determining orders

While a derivative of a tracking variable, residing often in a stock within the system being controlled, is invariably used in engineering control systems, forecasting in social systems might often be applied to a tracked variable that is a flow or a goal condition determined outside the system. Shipments are an example of such a variable. Shipments and inventory discrepancy are widely used as a basis for ordering to replenish inventory. Such an ordering policy creates a correction of the form given in equation 2.

$$M(t) = \mu[G - I] + S \qquad (2)$$

where G is inventory goal, I is instantaneous level of inventory, S is shipments and μ is fraction of inventory discrepancy corrected per unit time.

If we substitute S by its forecast F(S) in equation 2,

$$M(t) = \mu[G - I] + F(S) \qquad (3)$$

$$F(S) = S + FT*(d(S)/dt) \qquad (4)$$

where FT is forecasting time horizon.

Therefore,

$$M(t) = \mu[G - I] + S + FT * d(S)/dt \qquad (5)$$

Figure 4 shows the flow diagram of a model articulating the ordering policy of equation 5. Note that I have used recognized shipments instead of shipments to drive the ordering policy and also to calculate the trend. If shipments are a basis for a conscious decision, only recognized value, which is an average over the observation period can be known. The use of the instantaneous value of shipments, which can never be known, will create a feedback loop without a stock in it which is theoretically incorrect. Fortunately, most software used for system dynamics modeling will not allow the modeler to create a formulation leading to such a feedback process. Please also note that a derivative control based on shipments will create a positive feedback loop coupled with the negative feedback loop of the proportional control in this system.

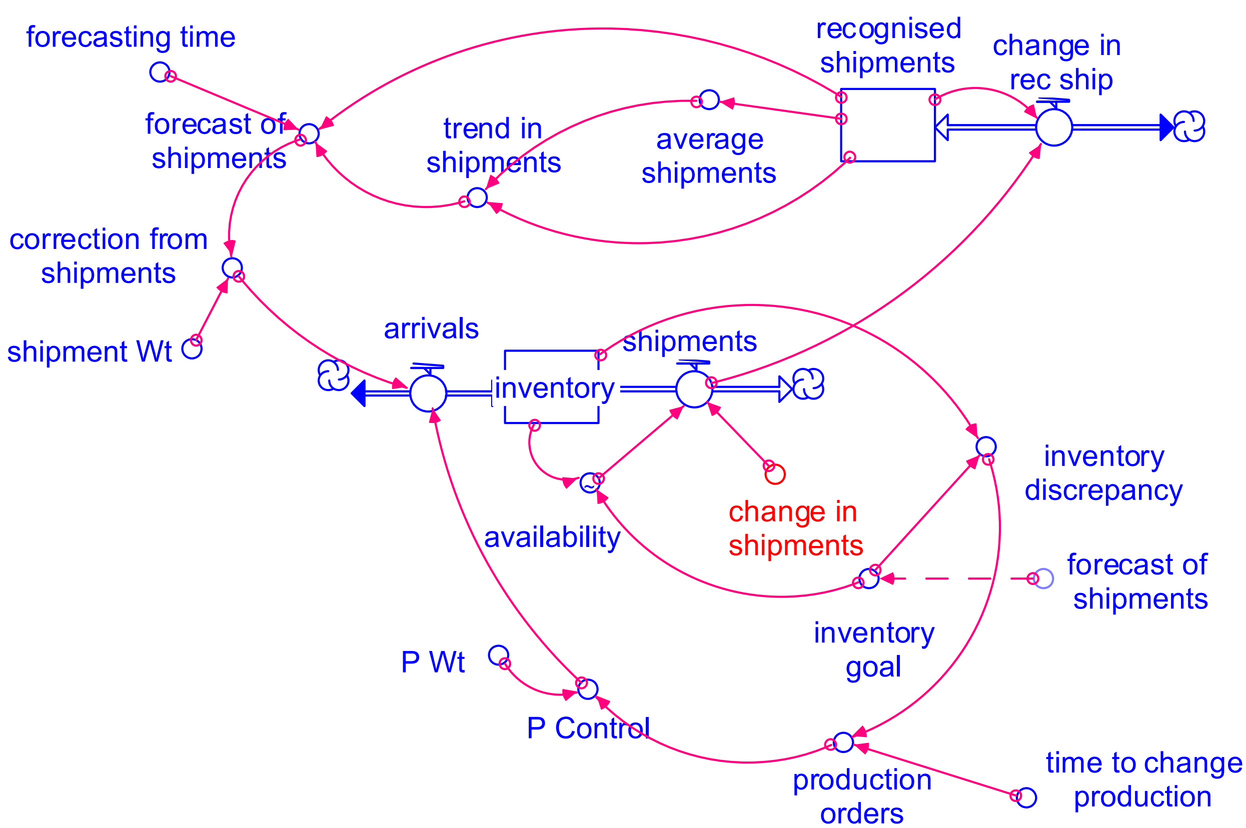

Figure 4 Flow diagram subsuming ordering policy of equations 5 and 7

When a forecast of shipments is applied also to modifying the inventory goal in the system of Figure 4, the correction is further modified as follows:

$$M(t) = \mu[G + F(S) * c - I] + S + FT * d(S)/dt \quad (6)$$

Where c is additional inventory coverage sought to accommodate shipments, and G is constant safety stock. Thus,

$$M(t) = \mu[G + S * c - I] + S + FT (1+\mu*c) * d(S)/dt \quad (7)$$

Annex 2 gives equations for the system of Figure 4. Figure 5 compares adjustments in response to a step change in shipments for four cases:

Policy 1: Orders are based on inventory discrepancy only as given by the expression: $\mu[G - I]$.
Policy 2: Orders are based on inventory discrepancy and shipments as given by the expression: $\mu[G - I] + S$

Policy 3: Orders are based on a forecast of shipments and inventory discrepancy as given by the expression: $\mu[G - I] + S+FT*d(S)/dt$

Policy 4: Orders are based on forecast of shipments and inventory discrepancy with inventory goal being determined by a fixed safety stock, a forecast of shipments and a fixed inventory coverage parameter as given by the expression:
$\mu[G + S * c - I] + S + FT (1+\mu*c) * d(S)/dt$

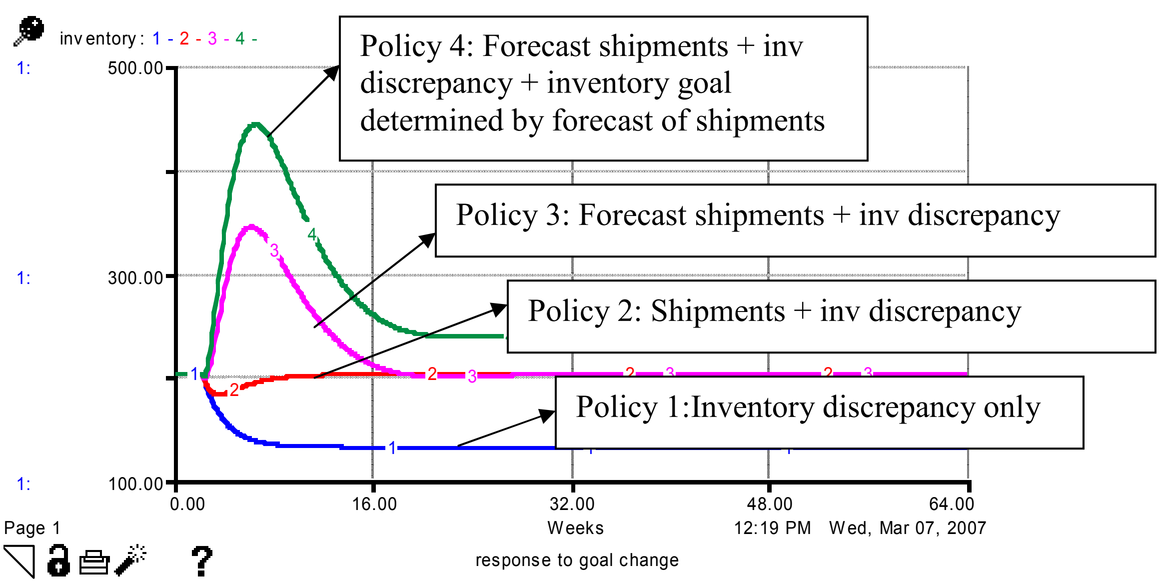

Figure 5 Use of a tracked variable and its forecast in determining ordering policy

Use of an ordering policy proportional to only inventory discrepancy as in policy 1 leads to decay in inventory over the course of adjustment creating a steady state error. An ordering policy based on shipments and inventory discrepancy as in policy 2 will seek a balance between arrivals and shipments that coincides with a zero inventory discrepancy. The use of a forecast of shipments together with inventory discrepancy as a basis for replenishment as in policy 3 creates a considerable overshoot over the course of correction, thus confirming the proposition that use of forecasting is destabilizing. When ordering policy also includes determination of inventory goal on the basis of a forecast of shipments as in policy 4, an even bigger overshoot and an unnecessarily high inventory goal are created. Indeed, it is not surprising that instability is often associated with the use of forecasting determining ordering policies.

If we also attempted to correct production capacity based on a forecast of shipments, as would be commonly expected, we would effectively add an integral control component of the following form:

$$I(t) = \tau * \int [G+F(S)*c-I]dt \qquad (8)$$

where is τ is a weight determined by the time constants of the capacity adjustment process.

$$= \tau * \int [G+S*c-I]dt + \tau * FT*c* \int [d(s)/d(t)]dt \qquad (9)$$

$$= \tau * \int [G+S*c-I]dt + \tau * FT*c*S \qquad (10)$$

When all components of correction are assembled, we get:

$$M(t) = \mu[G + S*c - I] + S + FT\ (1+\mu*c)*d(S)/dt + \tau * \int [G+S*c-I]dt + \tau * FT*c*S \qquad (11)$$

$$= \mu[G + S*c - I] + \tau * \int [G+S*c-I]dt + FT\ (1+\mu*c)*d(S)/dt + S[1+\tau * FT*c] \qquad (12)$$

Figure 6 shows the structure of an ordering policy based on equation 12, which appears to be of the form:

$$M(t) = P\ [e(t)] + I[\int e(t)dt] + D[de(t)/dt] + f(S),$$

but with all control mechanisms linked to shipments, which is a tracked flow.

Furthermore, an additional component also linked to this tracked flow f(S) is added to the correction. Annex 3 gives the equations for the system in Figure 6. Figure 7 compares performance of an ordering policy subsuming proportional and integral corrections and shipments driving ordering policy with and without forecasting. Four ordering policies are compared:

<u>Policy 1</u>: An ordering policy based on inventory discrepancy from a fixed inventory goal, and production capacity created by cumulative inventory discrepancy as given by the expression: $\mu*(G - I) + \tau * \int (G - I)dt$

Policy 2: An ordering policy based on inventory discrepancy from a fixed inventory goal, production capacity created by cumulative inventory discrepancy, and shipments as given by the expression: $\mu*(G - I) + \tau* \int(G - I)dt + S$

Policy 3: An ordering policy based on inventory discrepancy from a variable inventory goal determined by safety a stock and coverage for shipments, production capacity created by cumulative inventory discrepancy, and shipments as given by the expression:
$\mu*(G + c*S - I) + \tau* \int(G + c*S - I)dt + S$

Policy 4: An ordering policy based on inventory discrepancy from a variable inventory goal determined by a safety stock and coverage for forecast shipments, production capacity created by cumulative inventory discrepancy, and forecast of shipments as given by the expression: $\mu*[G + c*F(S) – I] + \tau* \int[G + c*F(S) – I]dt + F(S)$

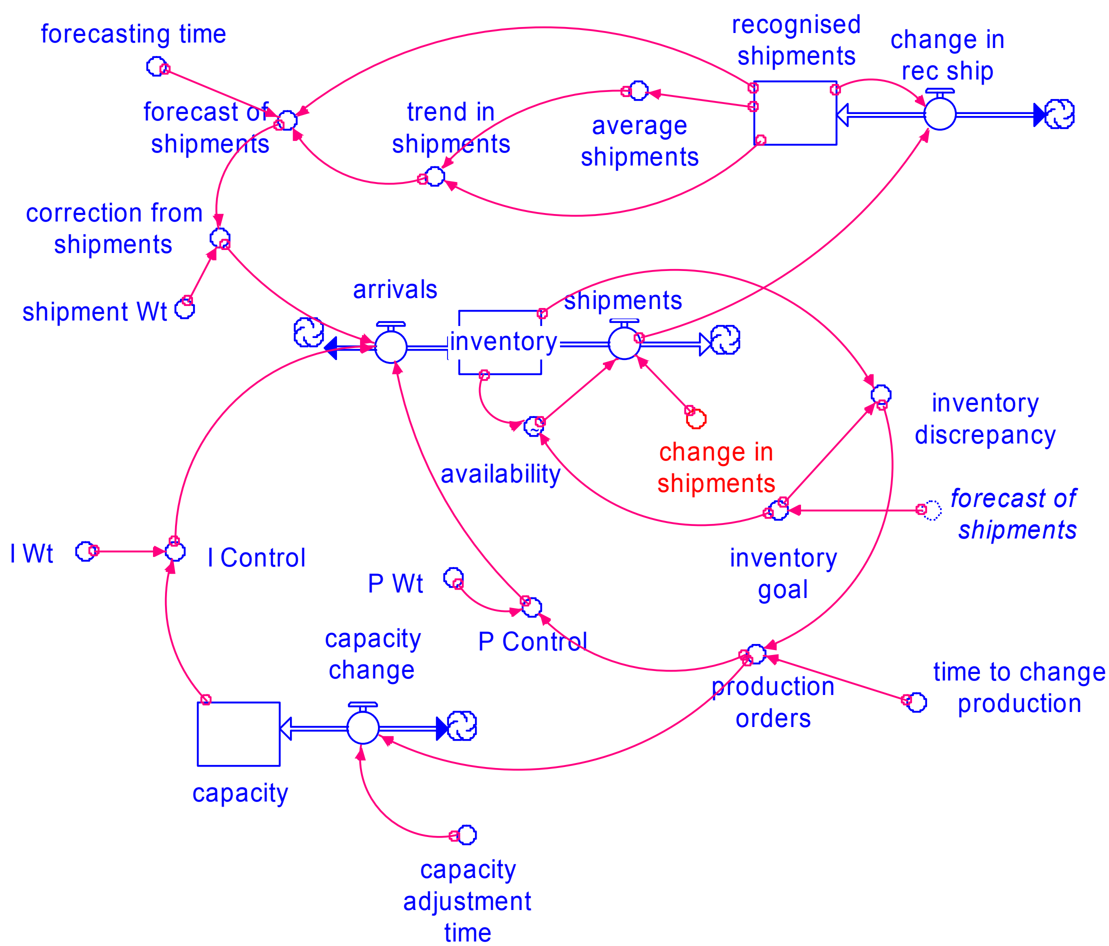

Figure 6 **Structure of the ordering policy based on forecast of a tracked variable (shipment)**

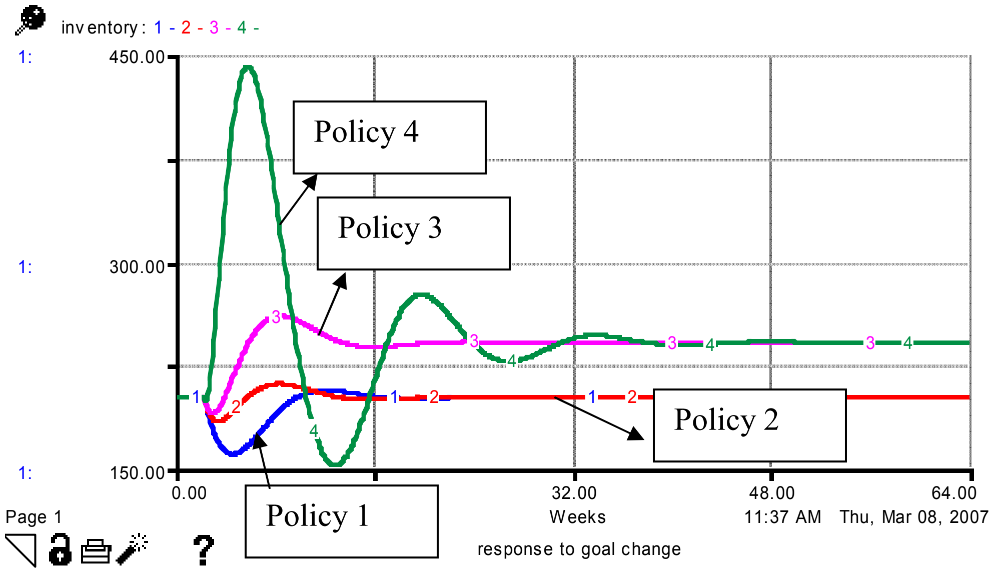

Figure 7 **Performance of ordering policy driven by a tracked variable (shipments) and its forecast**

Policy 1 subsumes a combination of proportional and integral control components. It creates damped adjustment towards a fixed goal representing only safety stock since integral control is able to off-set inventory decay created over the course of adjustment. Policy 2 adds an adjustment proportional to shipments, which further prevents inventory from falling. It does not change the equilibrium since arrivals and shipments still must equate when there is no inventory discrepancy. Policy 3 changes inventory goal beyond the safety stock by also considering inventory coverage needed for shipments. It also leads to a larger overshoot since integral control adjustment occurs in response to a larger discrepancy between inventory and its goal, although damping is still achieved quite quickly. Policy 4 adds a shipment trend based element to all components of correction related to shipments. It increases correction due to shipments as well as the inventory goal. It adds a derivative based component based on shipments which creates a positive feedback loop between shipments and ordering. The result is an increased instability as reported in Forrester (1961), Lyneis (1982) and otherwise widely attributed to use of trend forecasting in policy.

b) Use of forecasts of a tracking variable (inventory) in determining ordering policy

Let us now consider a case where the ordering policy depends on inventory discrepancy, which is determined by the difference between the inventory goal and a *forecast* of inventory, while the integral control is absent. The total correction in such a case is given as follows:

$$M(t) = \mu[G - F(I)] \quad (13)$$

$$F(I) = I + (d(I)/dt)*FT \quad (14)$$

Thus, $M(t) = \mu[G - (I + d(I)/dt)*FT)]$ (15)

$= \mu[G - I] - \mu *FT * [d(I)/dt)]$ (16)

$= \mu[G - I] + \mu *FT * [d(G-I)/dt)]$ (17)

A system dynamics model incorporating an ordering policy outlining above control mechanisms is shown in Figure 8. It will be noted that this model is very similar in structure to the one in Figure 1 (without the integral control part), although it has different variable names. The tracking stock becomes inventory, the tracked goal is inventory goal, drain coefficient is availability (of inventory), the tracked flow becomes shipment, the tracking flow becomes arrivals and the error is the inventory discrepancy, while the rate of change of error is really an inventory discrepancy trend.

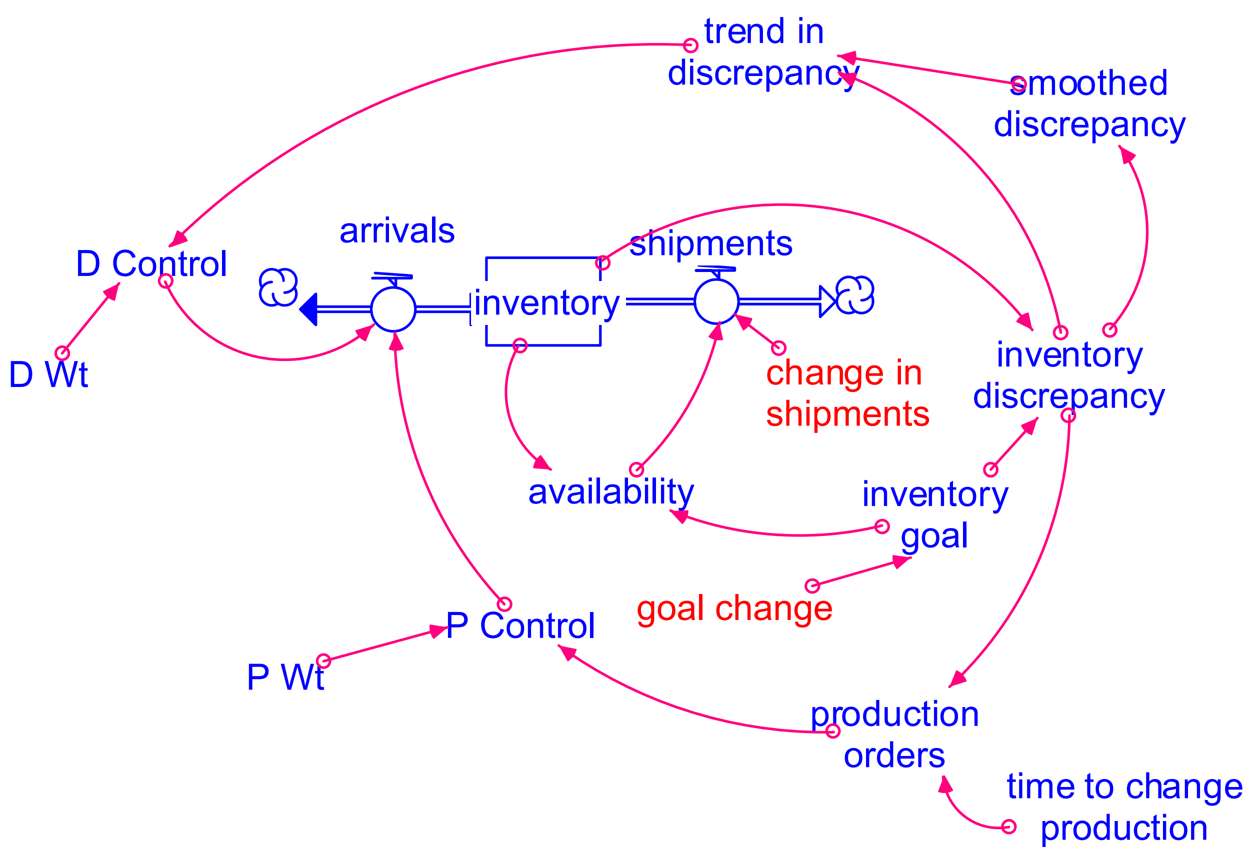

Figure 8 **Proportional and derivative control mechanisms created by linking ordering to forecast of inventory discrepancy with fixed inventory goal**

The proportional control is driven by the production orders determined by inventory discrepancy. Assuming that these can be handled by changing an overtime/undertime factor represented by the proportional control weight P wt. The proportional control can vary arrivals within the limits of the existing capacity. The trend in inventory discrepancy can be used to further modify arrivals using a weight D wt., speeding it up when this discrepancy is rising. Thus, the Proportional and Derivative control components are incorporated into the production planning policy. When compared with the proportional and derivative components of equation 1, equation 17 reduces to:

$$M(t) = P\,[e(t)] + D[de(t)/dt]$$

When we add an integral control based on a forecast of inventory discrepancy to the ordering policy in equation 17, we get:

$$M(t) = \mu[G - I] + \mu * FT * [d(G\text{-}I)/dt)] + \tau * \int (G\text{-}F(I))dt \qquad (18)$$

$$= \mu[G - I] + \mu * FT * [d(G\text{-}I)/dt)] + \tau * \int [G\text{-}I]dt + FT * \tau * \int [d(G\text{-}I)/dt)]\, dt \qquad (19)$$

$$= [\mu + FT * \tau] * [G - I] + \tau * \int [G\text{-}I]dt + \mu * FT * [d(G\text{-}I)/dt)] \qquad (20)$$

Figure 9 shows a system dynamics model of the production planning policy represented in equation 20. In addition to the system of Figure 5, we now also have the inventory discrepancy driving capacity change and integration of this change into the stock of capacity, which drives the integral control. Annex 4 gives the equations for this system.

It will be noted that the integral control components added to the system render a structure that is similar to PID control process illustrated in Figure 1. Equation 20 is similar to the classical control equation 1, with weights of the control components determined by the parameters representing aggressiveness with which the inventory discrepancy is adjusted and the forecasting time.

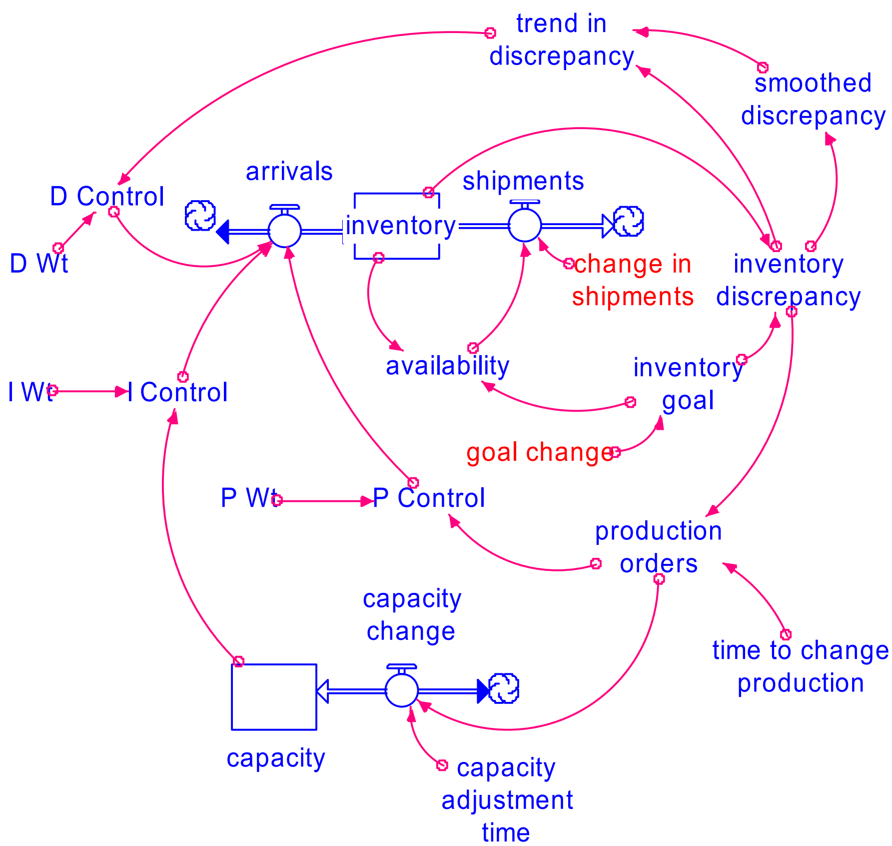

Figure 9 **PID control structure created by using forecast of Inventory in driving proportional and integral control components of ordering policy**

Figure 10 compares simulations with various combinations of PID control residing in equations 17 and 20. These simulations address the following 4 policy options:

Policy 1: An ordering policy based on inventory discrepancy as in proportional control, given by the expression: $\mu[G - I]$.

Policy 2: An ordering policy based on forecast inventory discrepancy leading to a combination of proportional and derivative controls given by the expression in equation 17: $\mu[G - I] + \mu * FT * [d(G-I)/dt)]$

Policy 3: An ordering policy based on a forecast of inventory discrepancy driving integral control in addition to policy 2, given by the expression in equation 20:

$$[\mu+FT*\tau] *[G - I] + \tau*\int [G-I]dt + \mu *FT * [d(G-I)/dt)]$$

Policy 4: An ordering policy similar to policy 3, by with a higher weight assigned to proportional control.

As in case of classical PID control, Policy 1 creates a quick adjustment to a new equilibrium which is lower than the inventory goal. This outcome is similar to the steady state error created by using only proportional control. Policy 2 represents a combination of proportional and derivative controls. It can speed up adjustment over the early period of decay when inventory discrepancy is expanding, thus slowing down the rate of inventory decay over that period, but it cannot change the steady state error. Policy 3 subsumes a combination of proportional, integral and derivative control processes as given in classical control equation 1. It yields an adjustment to an equilibrium which is the same as the inventory goal, but the path to the new equilibrium exhibits some hunting, which can be correcting by increasing the weight of correction directly proportional to the inventory discrepancy as shown by the performance of policy 4.

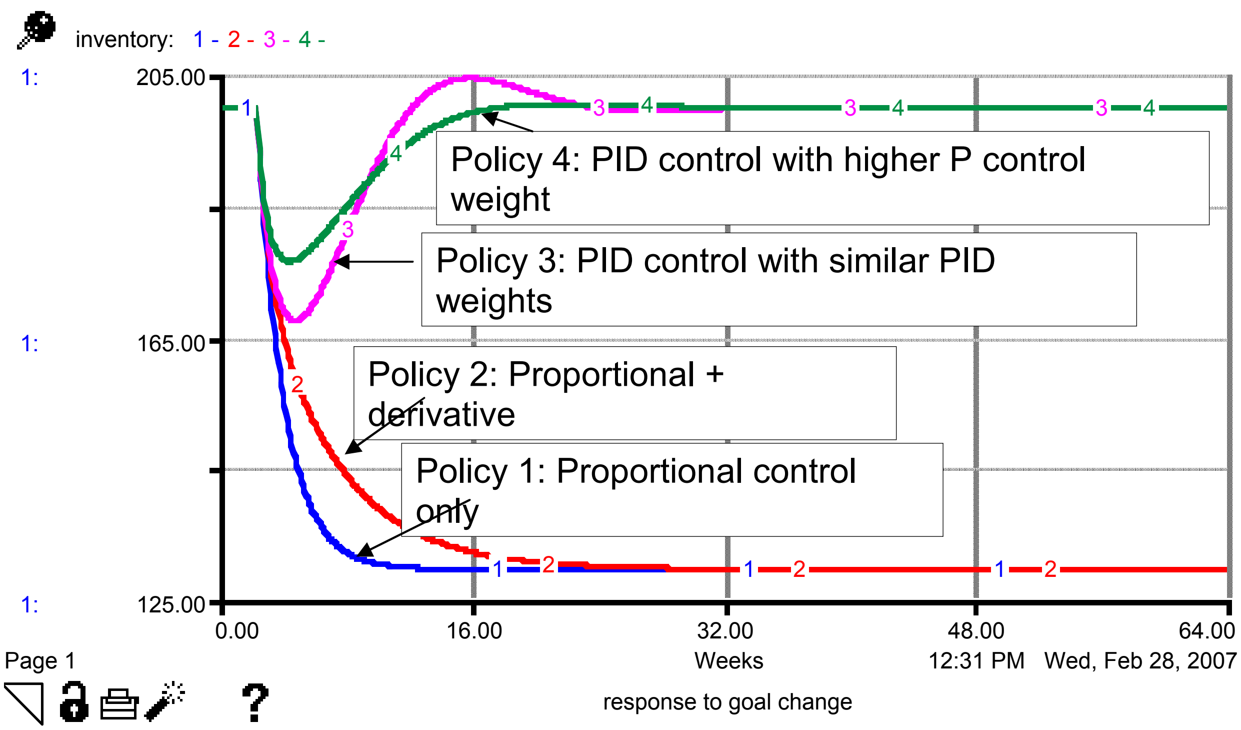

Figure 10 **A comparison of PID control using forecast value of inventory compared with current value**

Equation 20 implies that a combination of proportional and integral controls is applied if we tried to forecast inventory (a tracking variable) and subsumed this forecast in a replenishment policy focused on ordering for inventory discrepancy only. This policy will create a negative feedback loop in the path of the trend-based correction as in case of classical PID control. Thus, forecasting used in this way will improve stability. Unfortunately, it is rare to see a forecast of a *tracking variable* used in this way in supply chain management. More often, a *tracked variable* like the shipments or another measure of demand is forecast for constructing a supply regime.

It should be noted that it is not the use of trend *per se* that leads to instability, but the way it is applied determines whether it will increase or decrease instability. Use of trend of a tracking variable residing within the system being controlled to apply a correction that creates an additional negative feedback component in the policy process can increase stability. Using trend of a tracked variable lying outside of the system being controlled may often create a positive feedback loop in the policy process which when coupled with other negative feedback loops already in place will increase instability.

Another point to note is that the information about a tracking variable might be more accurate that for the tracked variable since the former is available within the system being controlled, while the later resides outside of it. Hence, the forecast of a tracking variable might also be more reliable that that of the tracked variable. Unfortunately, more often than not, trend forecasting is tied to tracked variables in social systems. Since it often leads to instability, hence, it is often seen as a destabilizing policy, which amounts to throwing the baby out with the bath water.

Conclusion

There seem to be similarities in policies created by use of derivative control and forecasting. Both use trend information about the variable in the system to drive an error correction process. However, while the former explicitly creates a control process driven by the trend of error in a tracking stock, the later may often use trends in tracked variables to create remedial processes with often dysfunctional consequences, since their underlying feedback structure may counter their intended goals. Using complex forecasts of many tracked variables may further complicate the process creating further unforeseen consequences. It is also observed that while derivative control is cognizant of the feedback process it creates, forecasting in social systems may only accidentally create dysfunctional feedback structure since forecasting is done without an intent to create a control process.

There is, however, no reason why forecast-related information cannot be used to improve performance of a policy since derivative control can use similar information to improve performance with great reliability. To accomplish this, we need to carefully identify the structure of the policy, the feedback loops it creates and the variables forecast. Tracking variables seem to be better candidates for forecasting than widely used tracked variables. Complex forecasts involving many variables might only increase the uncertainly of the outcomes. Derivative control in engineering seems to offer a good model for designing policies using trend information. Using this model, we can use forecasting reliably in improving system performance.

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Annex 1

Equations for a system dynamics model of a classical control system shown in Figure 1.

cummulative__error(t) = cummulative__error(t - dt) + (error_accumulation) * dt
INIT cummulative__error = entropic_drain

INFLOWS:
error_accumulation = error_correction/error_accumulation___time
tracking_stock(t) = tracking_stock(t - dt) + (total_corrective_change - entropic_drain) * dt
INIT tracking_stock = 200

INFLOWS:
total_corrective_change = D_Control+P_Control+I_Control
OUTFLOWS:
entropic_drain = (0+STEP(change_in__drain,2))*drain__coeficient
change_in__drain = 0
D_Control = trend_in_error*D_Wt
D_Wt = 0
error = goal-tracking_stock
error_accumulation___time = 2
error_correction = error/error_correction_Time
error_correction_Time = 2
goal = 200+STEP(goal_change,2)
goal_change = 0
I_Control = cummulative__error*I_Wt
I_Wt = 0
P_Control = error_correction*P_Wt
P_Wt = 1
smoothed_error = SMTH1(error,2,0)
trend_in_error = (error-smoothed_error)/2
drain__coeficient = GRAPH(tracking_stock/goal)
(0.00, 0.00), (0.2, 0.558), (0.4, 0.756), (0.6, 0.87), (0.8, 0.942), (1.00, 1.00), (1.20, 1.04), (1.40, 1.06), (1.60, 1.07), (1.80, 1.07), (2.00, 1.08)

Annex 2

Equations for model of Figure 4
inventory(t) = inventory(t - dt) + (arrivals - shipments) * dtINIT inventory = 200
INFLOWS:
arrivals = correction_from_shipments+P_Control
OUTFLOWS:
shipments = (0+STEP(change_in__shipments*availability,2))
recognised_shipments(t) = recognised_shipments(t - dt) + (change_in__rec_ship) * dtINIT recognised_shipments = 0
INFLOWS:
change_in__rec_ship = (shipments-recognised_shipments)/1
average_shipments = SMTH1(recognised_shipments,3)
change_in__shipments = 40*1+1*0*sin(time)+0*NORMAL(0,5,30)
correction_from_shipments = forecast_of__shipments*shipment_Wt
forecasting_time = 9
forecast_of__shipments = recognised_shipments+1*trend_in_shipments*forecasting_time
inventory_discrepancy = inventory_goal-inventory
inventory_goal = 200+1*forecast_of__shipments*1
production_orders = inventory_discrepancy/time_to_change_production
P_Control = production_orders*P_Wt
P_Wt = 1
shipment_Wt = 1
time_to_change_production = 2
trend_in_shipments = ((recognised_shipments-average_shipments)/2)
availability = GRAPH(inventory/inventory_goal)
(0.00, 0.00), (0.2, 0.558), (0.4, 0.756), (0.6, 0.87), (0.8, 0.942), (1.00, 1.00), (1.20, 1.04), (1.40, 1.06), (1.60, 1.07), (1.80, 1.07), (2.00, 1.08)

Annex 3: Equations for model of Figure 6

capacity(t) = capacity(t - dt) + (capacity_change) * dtINIT capacity = shipments
INFLOWS:
capacity_change = production_orders/capacity_adjustment_time
inventory(t) = inventory(t - dt) + (arrivals - shipments) * dtINIT inventory = 200
INFLOWS:
arrivals = correction_from_shipments+P_Control+I_Control
OUTFLOWS:
shipments = (0+STEP(change_in__shipments*availability,2))
recognised_shipments(t) = recognised_shipments(t - dt) + (change_in__rec_ship) * dtINIT recognised_shipments = 0
INFLOWS:
change_in__rec_ship = (shipments-recognised_shipments)/1
average_shipments = SMTH1(recognised_shipments,3)
capacity_adjustment_time = 2
change_in__shipments = 40*1+1*0*sin(time)+0*NORMAL(0,5,30)
correction_from_shipments = forecast_of__shipments*shipment_Wt
forecasting_time = 9
forecast_of__shipments = recognised_shipments+1*trend_in_shipments*forecasting_time
inventory_discrepancy = inventory_goal-inventory
inventory_goal = 200+1*forecast_of__shipments*1
I_Control = capacity*I_Wt
I_Wt = 1
production_orders = inventory_discrepancy/time_to_change_production
P_Control = production_orders*P_Wt
P_Wt = 1
shipment_Wt = 1
time_to_change_production = 2
trend_in_shipments = ((recognised_shipments-average_shipments)/2)
availability = GRAPH(inventory/inventory_goal)
(0.00, 0.00), (0.2, 0.558), (0.4, 0.756), (0.6, 0.87), (0.8, 0.942), (1.00, 1.00), (1.20, 1.04), (1.40, 1.06), (1.60, 1.07), (1.80, 1.07), (2.00, 1.08)

Annex 4: Equations for model of Figure 9:

capacity(t) = capacity(t - dt) + (capacity_change) * dtINIT capacity = shipments
INFLOWS:
capacity_change = production_orders/capacity_adjustment_time
inventory(t) = inventory(t - dt) + (arrivals - shipments) * dtINIT inventory = 200
INFLOWS:
arrivals = D_Control+P_Control+I_Control
OUTFLOWS:
shipments = (0+STEP(change_in__shipments,2))*availability
capacity_adjustment_time = 2
change_in__shipments = 40*1+0*5*sin(time)
D_Control = trend_in_discrepancy*D_Wt
D_Wt = 1
goal_change = 0
inventory_discrepancy = inventory_goal-inventory
inventory_goal = 200+STEP(goal_change,2)
I_Control = capacity*I_Wt
I_Wt = 1
production_orders = inventory_discrepancy/time_to_change_production
P_Control = production_orders*P_Wt
P_Wt = 2
smoothed_discrepancy = SMTH1(inventory_discrepancy,2,0)
time_to_change_production = 2
trend_in_discrepancy = (inventory_discrepancy-smoothed_discrepancy)/2
availability = GRAPH(inventory/inventory_goal)
(0.00, 0.00), (0.2, 0.558), (0.4, 0.756), (0.6, 0.87), (0.8, 0.942), (1.00, 1.00), (1.20, 1.04), (1.40, 1.06), (1.60, 1.07), (1.80, 1.07), (2.00, 1.08)