# Bistable reactivity: A coupled oscillator circuit model of an agent organization


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**Abstract**

A coupled oscillator approach to game theory has been designed to resolve two of its major problems: The arbitrariness of valuing cooperation greater than competition in determining social welfare; and the lack of interdependent uncertainty. The bistable approach means that agents in relationships are more likely to be found in reactive states that correspond to their observation-action or energy-time levels. In our view, games are initialized, evolved to a state that solves a target problem, and then measured, consequently creating a measurement problem. In past research, we have addressed the measurement problem, leading to the development of metrics that have been applied to organizations in the field (we briefly illustrate an application to military Medical Department Research Centers). In this paper, we focus on modeling control in bistable close and market relationships to produce evolvable systems. One of our goals is to produce a model that can be used to study the information flows associated with congestion. We speculate that as congestion increases, it can be characterized by standard deviations of decreasing frequency and increasing time.


**Introduction**.

Two significant problems exist with traditional game theory, also known as methodological individualism (Nowak & Sigmund, 2004): The arbitrary assignment of relatively greater preference values for cooperation to assure that cooperation has superior value in comparison to competition in promoting social welfare; and the lack of interdependent uncertainties in relationships (for a more complete list of problems with game theory, see Lawless & Grayson, 2004). Quantum Game Theory (QGT; e.g., Arfi, 2005) overcomes the latter but not the former problem. In this paper, by introducing bistable relationships placed within a reactive social circuit, building on past research (Lawless et al., 2006; Lawless & Whitton, 2007), we begin to develop a means to resolve both problems based on a simple model of coupled exchanges between partners engaged in an evolving interaction.

**Social circuit between partners *A-B*.**

In this model of a game, we assume that relationships have complementary aims (Winch, 1958); e.g., partner *A* wants sex more than *B* but *B* wants the relationship more than *A*; or in the marketplace, partner *A* wants the market power afforded by controlling *B*'s market leading products, but *B* wants the technological skills provided by *A* (e.g., SBC's successful alliance in 2001 with Yahoo to sign up SBC's—renamed AT&T—customers for new broadband services, now changing with Yahoo's loss of market dominance to Google). The result is a set of bistable agents operating within a bistable relationship as they cycle between higher and lower energy, *E*, states. As one of the partners gains control, its energy consumption is reduced, while the one losing control increases its energy consumption, producing a bistability in each agent and also a bistable relationship. If in addition, the agents are reactive (e.g., alternatively acting like an information capacitor and inductor inside of a series or parallel circuit), a delicate dance is generated between the two partners that can be modeled with a reactive circuit dynamically moving between resonance and dampening. In a series circuit, resonance represents the maximum positive action by one partner in response to the other's minimum driving stimulus, and critical dampening (or anti-resonance in a parallel circuit) inverts the action to become the maximum resistance in response to the other's minimum input stimulus. Each agent reacting alone to a stimulus from the other agent is unstable, but collectively stable and can be modeled as coupled harmonic oscillators. The combination leads to a relationship with evolving dual controls: As partner *A* moves away from the relationship, partner *B* offers more opportunities to draw *A* back into the circuit, but as *A* exploits the situation, *B* begins to shut *A*'s opportunities off.

A circuit consists of at least two coupled harmonic oscillators; an organization links circuits into a lattice structure to accomplish a specific function or set of functions. We set aside the circuit, lattice, and construction of bistable agents for future research. Instead, in this paper, with reactance borrowed from Brehm's (1966) theory of reactance, we model the concept of controlling opportunities between agents *A* and *B* in a relationship using May's (1973) system approach and oscillatory responses. Reactance is the overvalued preference for a target action, possibly the best-fitted action for survival, in reaction to a *perceived* threat precluding a desired action, motivating an agent to regain control by countering the threat. We propose that reactance can by used to model two reactive agents or organizations seeking control of a mutual relationship.

**Control opportunities between *A* and *B*: A system approach.**

Let $N$ be the total control opportunities between *A* and *B*, where $N^*$ is equilibrium and $x$ is a deviation from equilibrium due to a perturbation to $N$. Then,

$$dN/dt = dN^*/dt + dx/dt = dx/dt = F(N(t)). \quad (1)$$

Assume that the rate of change in a perturbation is related to $x(t)$ as

$$dx/dt = a\,x(t) = F(N(t)), \quad (2)$$

then from Equation (2), a solution for $x$ is:

$$x(t) = x_0 \exp(a\,t), \quad (3)$$

with the rate of change, $a$, found near equilibrium from

$$a = \partial F/\partial N. \quad (4)$$

Putting Equation (3) back into (2) gives

$$dx/dt = a\,x_0 \exp(a\,t) = a\,x(t). \quad (5)$$

In matrix form for multi-organizations or multiple groups, $m$, Equation (5) becomes

$$x_i(t) = \sum_{j=1}^{m} C_{ij} \exp(\lambda_j\,t) \quad (6)$$

Taking the derivative of Equation (6) and rearranging to $(A - \lambda\ I)\,X(t) = 0$ *gives*

$$det\,|\,A - \lambda I\,| = 0. \quad (7)$$

<u>Example</u>:

Let $m = 2$ (agents $A$ and $B$), $a$ the change rate of $N_1$ and $b$ that of $N_2$ (e.g., a negative sign in front of $b$ means that $N_2$ is decaying), and let $\alpha$ and $\beta$ represent the competition between $N_1$ and $N_2$ for control (e.g., when $\alpha$ is preceded by a negative sign, then $N_2$ is taking control from $N_1$) then

$$F_1(N_1,N_2) = N_1\,(a - \alpha\,N_2),$$
$$F_2(N_1,N_2) = N_2\,(-b + \beta\,N_1). \quad (8)$$

Setting $F_1 = F_2 = 0$ gives

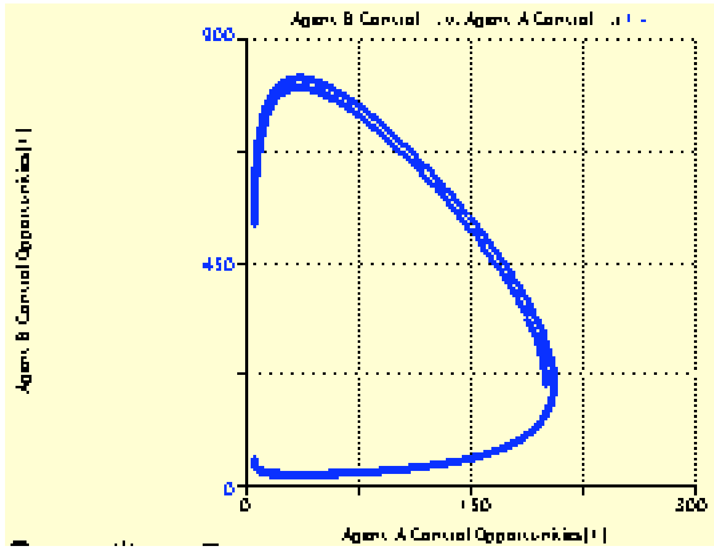
$$N_1^* = b/\beta,\ N_2^* = a/\alpha. \quad (9)$$

$$\partial F_1/\partial N_1 = a - \alpha\,N_2 = a - \alpha\,a/\alpha = 0 = a_{11},$$
$$\partial F_1/\partial N_2 = -\,\alpha\,N_1 = -\,\alpha\,b/\beta = a_{12},$$
$$\partial F_2/\partial N_1 = \beta N_2 = \beta\,a/\alpha = a_{21},$$
$$\partial F_2/\partial N_2 = -b + \beta\,N_1 = -b + \beta\,b/\beta = 0 = a_{22}. \quad (10)$$

$$\det \begin{vmatrix} -\lambda & -\,\alpha\,b/\beta \\ \beta\,a/\alpha & -\lambda \end{vmatrix} = 0 \quad (11)$$

The eigenvector equation, $\lambda^2 + ab = 0$, gives the purely imaginary eigenvalues $\underline{+}\,i\omega = (-1)^{1/2}(ab)^{1/2} = i(k/m_r)^{1/2}$, representing a classical neutral system oscillating with a period of $2\pi/\omega$ (and virtual spring constant, $k$, with reduced inertia $m_r$). See Figure 1. Superpositions result from the linear combination of solutions, and a Lyaponuv function can be built so this neighborhood analysis describes its global character (i.e., $\lambda$ in Equation (6)). In this system, if $\lambda = \varphi + i\omega$ and with $\varphi$ real and negative, perturbations die out; for $\varphi$ real and positive, perturbations grow.

Figure 1 (produced with Stella).




Only the real part of $\lambda$ has consequences in the real world; however, the imaginary part affects oscillations, and can represent beliefs, justifications, preferences and other non-real, but reactive factors, which produce the oscillations. The character of oscillations have practical consequences. Compared to competitive decision rules, we have found in field research that neutral oscillations are more likely to arise under cooperative or consensus decision rules, producing by definition the lowest levels of discourse resistance but also less practical results from an increase in inertia to information or worldview change (Lawless & Whitton, 2007). As we have done in previous research (Lawless et al., 2006), comparing two decision systems is straightforward: The greater the inertia to system evolution, resistance to information change, or the more cooperative the organization, then the slower the oscillations that result. In contrast, the faster the oscillations, the greater the conflict and the more resources expended, but, counterintuitively, the fewer the reactive agents:

$$\omega_1/\omega_2 = (m_{r2}/m_{r1})^{1/2} \qquad (12)$$

An important corollary to (12) occurs in the measurement of human or bistable organizations and individuals: Cooperative decision making participants are much more rational in their worldview perspectives, viz., "normatively consistent" (Shafir & LeBoeuf, 2002). Thus, those operating under cooperative decision rules are not obliged to change their beliefs, only to engage them to the extent that protects their turf in the final decision product. The end result is a stable set of beliefs that do not have to compete in the market place of ideas (Holmes, 1919). Conversely, competitive decision makers are not consistent for several key reasons: The conflictual or oppositional nature of the decision drivers; the drivers tend to be more expert in the knowledge of the matter being decided (e.g., prosecutor and defense attorneys); and the matter is often decided by those who are more or less neutral to the subject matter but not to the outcome, a critical requirement that we have argued sets the stage for human subjects to become entangled in the decision-making process (Lawless & Whitton, 2007; Lawless et al., 2006).

When the competition between agents *A* and *B* for control is stable, plotting the phase space of $N_1$ versus $N_2$ produces a limit cycle. Competition occurs when the cross-diagonal terms are less than one (Gauss-Lotka-Volterra criterion). The limit cycle becomes a circle when competition goes to zero (i.e., meaning that the actions of *A* and *B* are mutually independent). As competition increases, their interactions become correlated and the limit cycle becomes elongated. As the cross-diagonal terms approach one, fluctuations caused by their interactions become severe.

**Resource exploitation**.

Equation (1) is generalized below in (13) by replacing $a$ with $r_i$ as the integral over the resource spectrum and its utilization by agents or organizations (14), and by replacing the competition coefficients with $\alpha_{ij}$ as the convolution integral of the utilization functions between the $i$-$j$ competitors (15).

$$dN_i(t)/dt = N_i(t)\ [r_i - \sum^{m}_{j=1} \alpha_{ij}\ N_j(t)] \tag{13}$$

$$r_i = \int R(x)\ f_i(x)\ dx \tag{14}$$

$$\alpha_{ij} = \int f_i(x)\ f_j(x)\ dx \tag{15}$$

$$Q(t) = \int [R(x) - \sum_i f_i(x)\ N_i(t)]^2\ dx \tag{16}$$

In Equation (16), $Q$ measures the squared difference between the available and consumed resources across a resource continuum. It indicates that as $Q$ is minimized, the resource spectrum $R(x)$ is synthesized from the addition of $m$ Fourier components, $f_i(x)$, and Fourier coefficients, $N_i$. The utilization functions include time utilization, resource utilization for the agents-organizations to survive or to overcome barriers, and resource renewal. In addition, the minimum eigenvalue solution to Equation (13) should be larger than the resource variation, $\sigma^2$, existing in the available resource spectrum.

$$\lambda_{min} > \sigma^2/\langle r\rangle \tag{17}$$

At equilibrium, $dQ/dt \leq 0$ is minimum, implying from Equations (13) to (17) that the best fit has occurred to the available resource spectrum. In general, the greater the number of $m$ organizations exploiting a resource continuum, the smaller will be each eigenvalue, the larger the total eigenvalue, and the more stable the system.

**Application: Military Medical Department Research Centers (MDRC's)**

Guided by our theoretical results, we have been studying organizations in the field, including a system of seven military MDRC's. One of two primary goals that we have established with them is to help their Centers to become more productive; e.g., produce more research with greater impact; improve patient care; and reduce the costs of care. However, at the same time, MDRC's want to become transformative; e.g., transform medical treatments in the field; transform physician education in research; and transform publication impacts. These two goals are not just different, but contradictory (Smith & Tushman, 2005). Attempting to satisfy these two goals has led us to propose Figure 1 as an interim solution designed to help the Centers be more productive today, but also to evolve into more transformative organizations in the future.

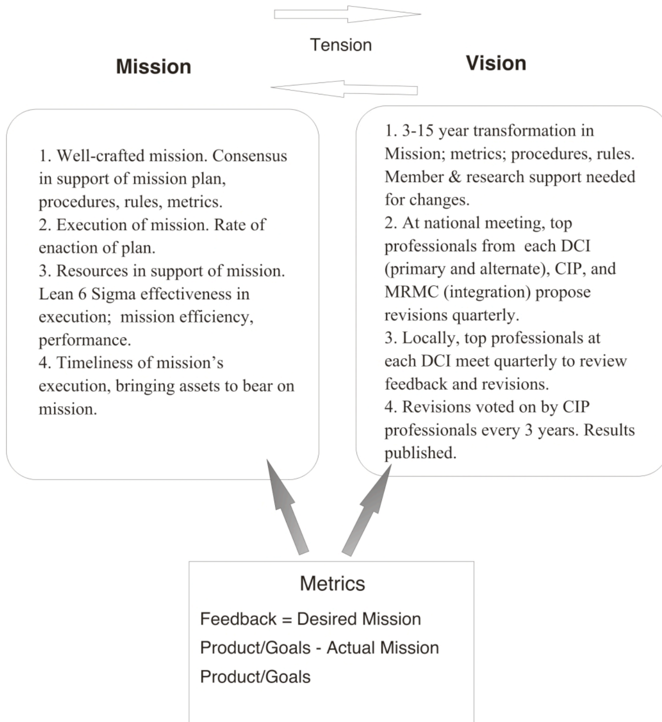



Fig. 1. Preliminary proposal to a system of seven military Medical Department Research Centers. Based on the feedback from a new system of electronic metrics currently being planned, administrators would have responsibility to enact the Mission as effectively and efficiently as possible; e.g., Lean Six Sigma plans. At the same time, a group internal to each MDRC and a national group of elite professionals from all MDRC units would gather regularly to transform the Mission, its goals, and its procedures and rules with the same feedback. As these two systems compete in a bistable relationship to control the Mission, the two systems operate in tension, producing a natural evolution of the system.

**Conclusion**

In this paper, we have introduced coupled harmonic oscillators as the basis of relationships between members of an organization. The advantage of using them is the straightforward mathematics that accompanies this model. We have also illustrated how our model is being applied in the field. While we have restricted our comments to coupled harmonic oscillators, which are likely to not demonstrate chaos but instead linear superposition in line with our quantum approach, non-linear oscillators could be used as an alternative to study chaos (e.g., Van der pol oscillators).

In the new approach we have presented in this paper to modeling social relationships among humans, organizations, and agent systems, social welfare improves when the resources available to a society are utilized effectively to solve the problems that it confronts. Social welfare can also improve when society finds the most efficient means of fully exploiting its resources to fine-tune solutions to problems it has already solved (i.e., the fewest number of steps to solve a particular problem; von Bayer, 2004). In both cases, the combination of competition between groups to find the best solutions to the problems that they face and cooperating to reach compromise between opposition drivers has been shown to have superior value to social welfare.

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