Firm Behavior Optimal Control on Mobile Service Market

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Abstract

Paper present a system dynamics approach for modeling mobile service competitive market and forecasting market development. The model includes dynamic competition between operators. Pricing policy, service quality, subscriber base, potential subscribers, marketing, etc influence their number of subscribers.

The task of defining the market share carrying capacity to forecast sales is described. The method for the saturation level identification is considered and the optimal control problem is formulated and solved. Actual data from several regions of Russia are used in this paper.

Keywords: system dynamics, model, forecast, optimal control, mobile service, telecommunication, firm behavior

Introduction

The application of system dynamics to study economic processes became very popular [1, 6, 14]. The aim of this paper is to use system dynamics approach to analyze mobile service market model and to consider the problem of optimal price strategy synthesis. The paper continues previous work [2–4, 7, 8, 10, 11] and develops approaches [1, 5, 9, 12, 13].

We use a system dynamics model of a mobile service market and formulate a model of a competitive mobile market through considering different dynamic growth models. Our model includes competition between some mobile companies (operators). We derive numerical results of the modeling market penetration process and formulate the problem of the operator optimal price strategy synthesis.

Mobile Service Market Model

The model includes dynamic competition between some operators (Figure 1). The pricing policy, service quality, subscriber base, potential subscribers, marketing, etc influence their number of subscribers. The system dynamic model can be described by the set of equations [9, 13]:

for the developing market (adoption level under 100%):

$$\frac{1}{x_i} \cdot \frac{dx_i}{dt} = \left(\sum_{j=1}^{6} p_{ij} s_{ij} + p_{i7} \frac{x_i}{z} + p_{i8} + w_i \right) \cdot \left(1 - \frac{z}{Z}\right), \quad i = \overline{1,n} \ , \tag{1}$$

for the saturated market (adoption level ~ 100% or more):

$$\frac{1}{x_i} \cdot \frac{dx_i}{dt} = \left(\sum_{j=1}^{6} p_{ij} s_{ij} + p_{i7} \frac{x_i}{z} + p_{i8} + w_i \right) \cdot \left(1 - \frac{z}{Z}\right) + \sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_{ij} x_j, \quad i = \overline{1,n} \ , \tag{1`}$$

where n – number of operators; x_i – subscribers number of the i-operator; Z – potential market capacity; $w_i \in W_i$ – uncertainty in current market situation; $z = \sum_{i=1}^{n} x_i$; p_{ij}, $j = \overline{1,8}$ – parameters; s_{ij}, $j = \overline{1,6}$ – control units.

As control actions the following parameters are used: s_{i1} – users payment; s_{i2}, s_{i3}, s_{i4} – cost of radio time minute for different bells directions; s_{i5} – sales efficiency; s_{i6} – radio cover zone. The square brackets in (1) can be interpreted as influence of the i-operator in the market.

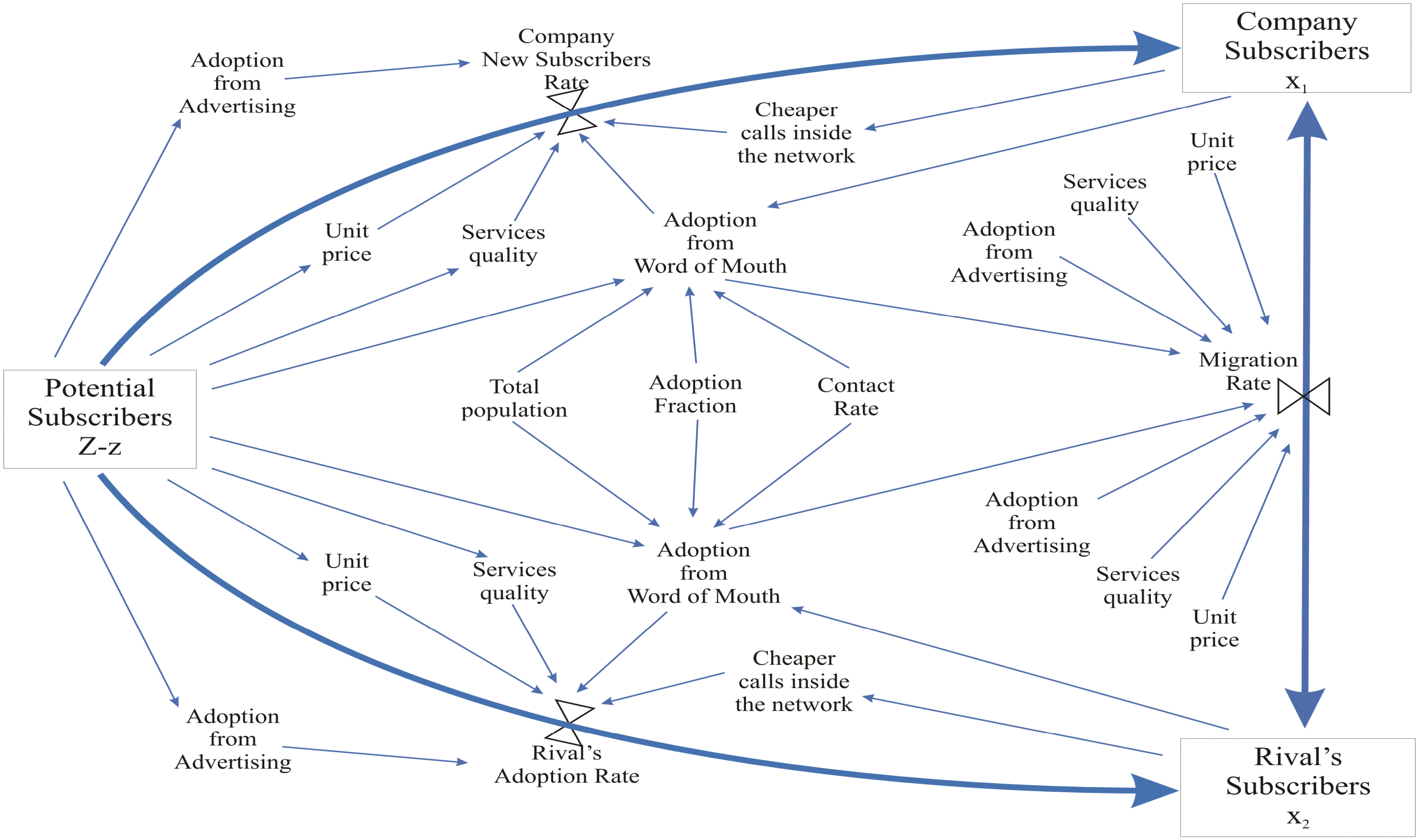

Figure 1. Mobile service competitive market SD model

Thus, parameters $p_{ij}, j = \overline{1,6}$ determine the degree of influence of the appropriate control actions in the market; p_{i7} takes into account the influence of the current market share of a company; p_{i8} considers the impact of other operator activities (advertising, quality management etc.). For income, operational costs and profit of the i-operator we have accordingly: $v_i = p_{i9} \cdot x_i$, $c_i = p_{i10} \cdot x_i$, $s_i = v_i - c_i$. The parameters p_{i9} and p_{i10} also correspond to the income (ARPU – average revenue per user) and costs of an operator in account per user.

The parameter identification of the model is based on the example of the cellular communication GSM-standard market of one Russian region (Figure 2) using mathematical software package MatLab.

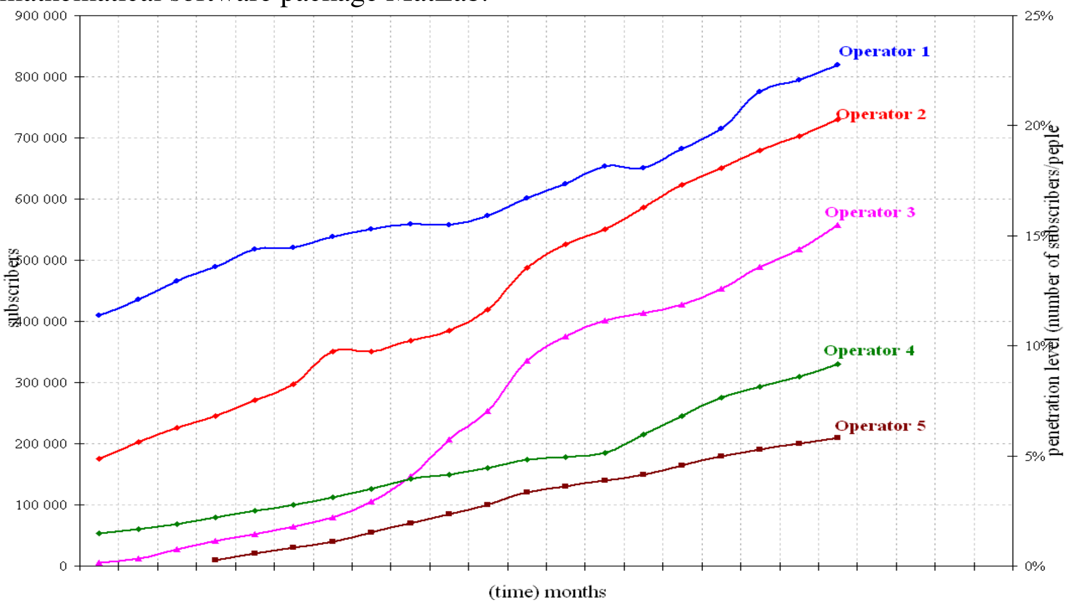

Figure 2. Modeling of the Mobile Service Market Development

Figure 3 illustrates the results of modeling the market share distribution with model (1) where parameters are constant. The latter are described in Table 1. Approximation errors are: Operator 1 – 0.6%; Operator 2 – 1.6%; Operator 3 – 2.2%; Operator 4 – 3.6%; Operator 5 – 18%. The basic data on Operators 4 and 5 have errors.

Table 1

Parameters	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$
Z	3,5 million of subscribers							
p_{1j}	-0,211	-0,333	-0,528	-0,508	0,536	0,571	0,536	1,063
p_{2j}	-0,058	-0,160	-0,609	-0,581	0,477	0,393	0,474	0,975
p_{3j}	-0,500	-0,440	-0,462	-0,423	0,486	0,493	0,504	1,055
p_{4j}	-0,102	-0,159	-0,540	-0,524	0,519	0,536	0,501	0,982
p_{5j}	-0,500	-0,469	-0,475	-0,446	0,468	0,482	0,493	1,054

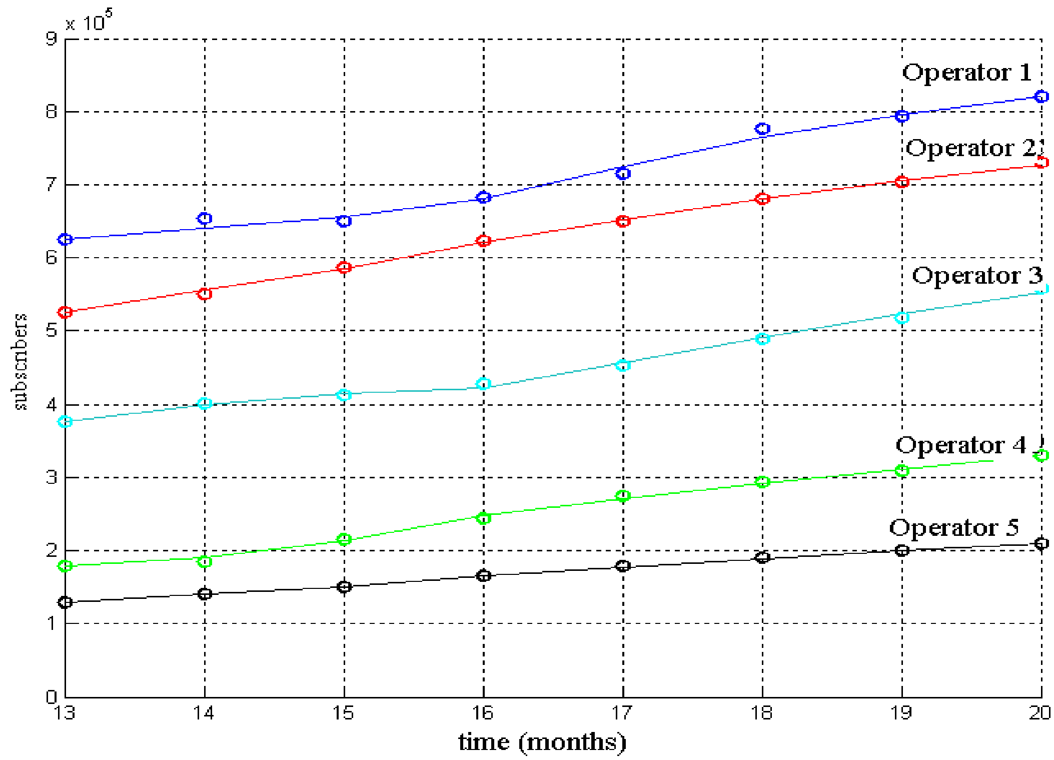

Figure 3. Modeling Market Share Distribution

The influence of control actions of each operator in the market was stabilized since the beginning of 2005 (Figure 4). The absolute values of the model parameters for various operators are close. That is stipulated by an identical response from the consumers on control actions of the market participants.

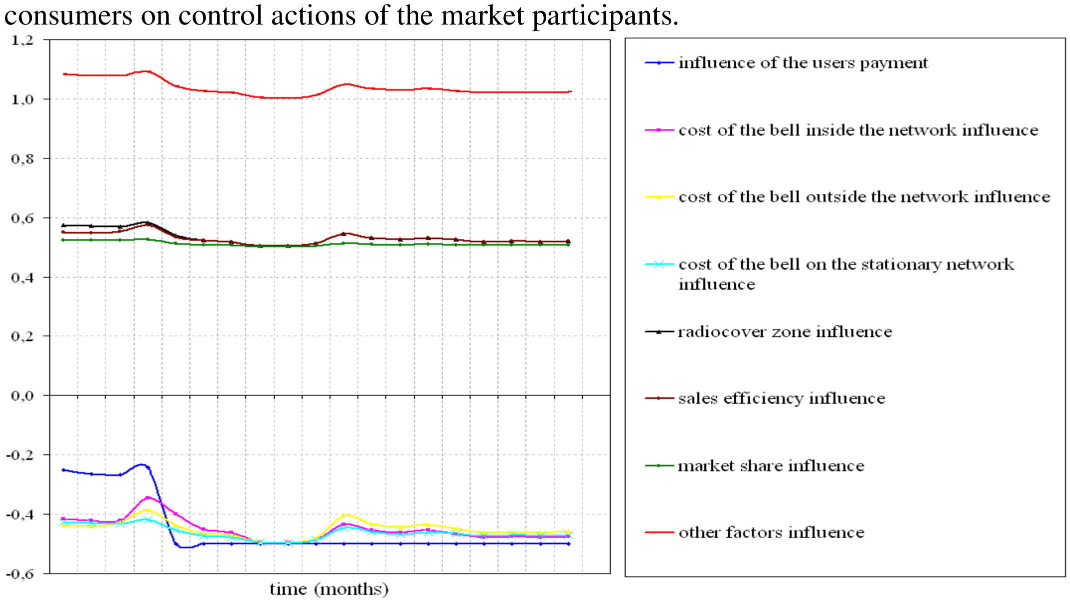

Figure 4. Parameters p_{ij} of the model (1) for one operator

Forecasting of the Mobile Service Market Development

The results of forecasting market share distribution on the June-August, 2005 interval with model (1) where parameters are not constant are illustrated on Figure 4. The errors are: Operator 1–1.7%; Operator 2–1.1%; Operator 3–4.2%; Operator 4–8.2%; Operator 5–5.8%. The basic data on Operators 4 and 5 have errors.

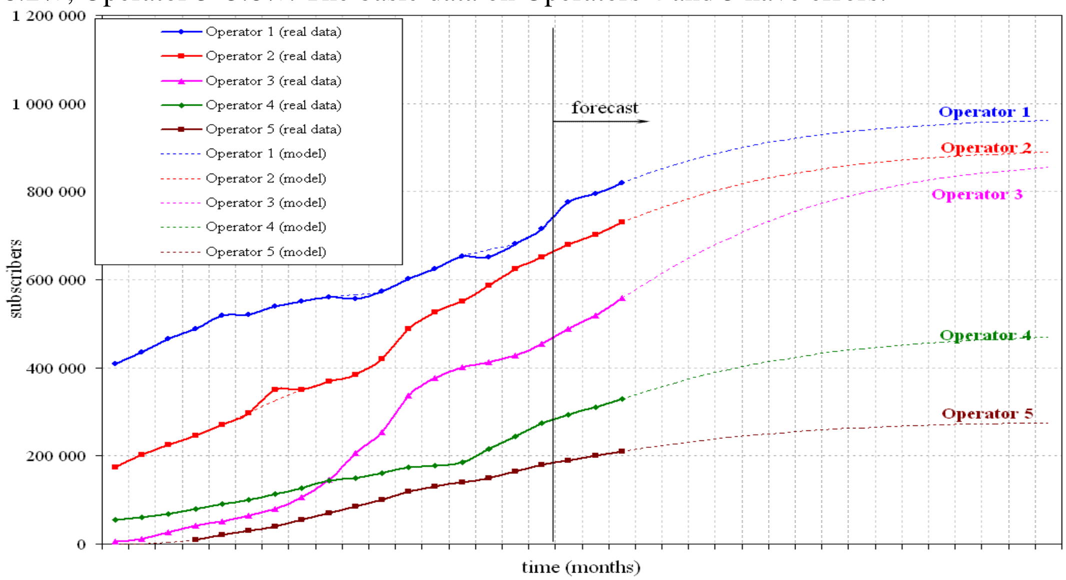

Figure 5. Forecasting of the Mobile Service Market Development

The next part of the paper is based on the system dynamics growth models [1, 12]. The obtained demand estimation should be used for analysis of the market situation and determination of the price strategy, which is optimal in the sense of firm development criterion [8, 11].

Dynamic Growth Models

The growth model, built on the basis of the first-order differential equation is proposed in [1, 12]. This method is not applicable to the S-shaped curve [6, 9, 10]. In this case we used a growth model [1, 12], built on the basis of the second-order differential equation:

$$T^2 \frac{d^2 x(t)}{dt^2} + 2\xi T \frac{dx(t)}{dt} + x(t) = x_{max}, \quad \xi > 1, \tag{2}$$

where $x(t)$ – cumulative sales of the mobile services; x_{max} – estimation of the potential market share carrying capacity; T, ξ - constants, relating to the curve of the cumulative increase in total volume of services' sales. Unlike the first–order differential equation, equation (2) offers more possibilities to look at available experimental data and thus increase the accuracy of the approximations. This results in a more exact determination of the course and capacity of the market for goods [1, 12].

Another dynamic growth model:

$$\Delta x(t) = \frac{x_{max}}{A \cdot t^2} \cdot exp\left(\frac{-1}{A \cdot t}\right), \tag{3}$$

where A is a parameter, which depends on the fractional growth rate; and t – time (days, months, years, etc.).

The next growth model:

$$\frac{dx(t)}{dt} = [\alpha_1 + \alpha_2 x(t)] \cdot (x_{max} - x(t)), \qquad (4)$$

where α_1, α_2 – parameters, depending on company marketing activity and the "word of mouth" effect.

Figure 6 illustrates the real company statistical data (real subscribers) and the result of modeling the regional mobile service market using growth models (2)–(4). Experimental data can be approximated with a high degree of accuracy through the parameter identification.

The growth model includes the process of goods distribution to the customers and allows the estimation of the market share carrying capacity – x_{max}. This method also enables us to forecast the demand for services. The receipt of new sales volumes allows updating of the model parameters continuously.

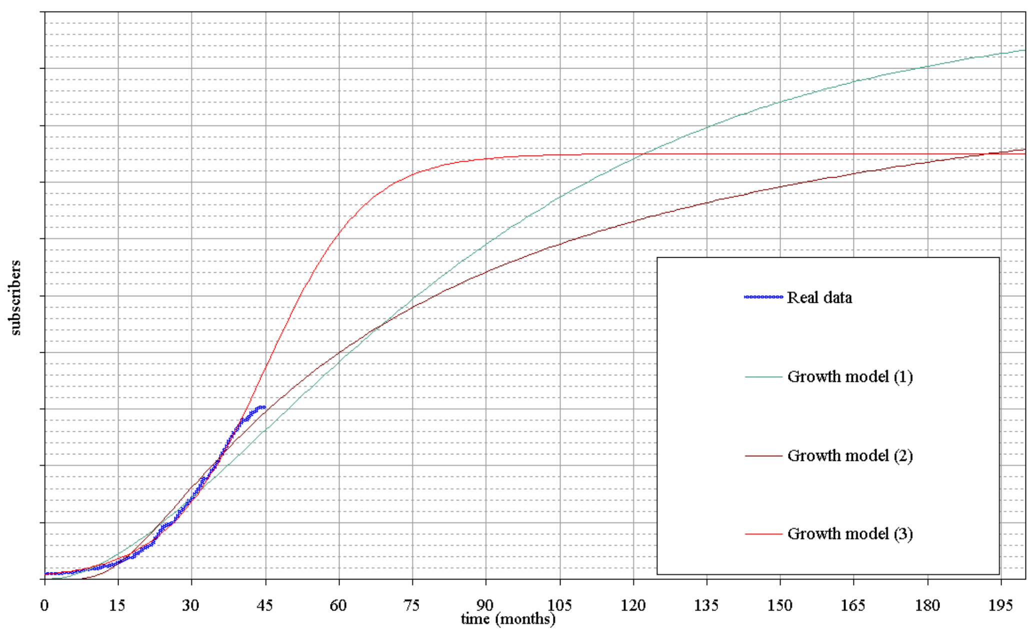

Figure 6. Growth Models and Experimental Data

The growth models (2)–(4) display a process of adoption of the mobile services in the competitive market. They include information about the popularity of the mobile services; this, in turn, provides the growth of the subscriber base at the initial stage when the number of subscribers is significantly smaller than the potential. Marketing, price strategy, dumping, etc, generates such popularity.

The total adoption rate is the sum of adoptions resulting from the "word of mouth" and adoptions resulting from the company's marketing and communication services, advertising and other external influences. Adoptions from the "word of mouth" are formulated exactly as in the logistic innovation diffusion model or the Bass diffusion model. The probability of adoption by potential users as a result of an exposure to a given amount of advertising, the volume of advertising and other external influences at each period remains constant.

The initial growth is driven by feedbacks outside the boundary of the simple logistics models. Existing subscribers of the company, having found out about satisfactory services available, spread the information among the unaware potential customers. As a result of such contact, there is a probability of new subscriptions.

Optimal control problem

We consider the problem of optimal control for a mobile operator in a competitive market. The nonlinear model (1) includes control parameters s_{ij}, $i = \overline{1, n}$, $j = \overline{1, 6}$. Control purposes for each mobile operator at different stages vary by: the subscriber base, revenue, market share, minimized churn rate, increased revenue per user, net income, improvement in financial efficiency, etc.

The model is described by the following linear equations:

$$x_{k+1} = A_k x_k + B_k u_k + \Gamma_k w_k + c_k \ , k = 0, 1, \ldots, N, \tag{5}$$

where $u_{i\,k} = \left[\sum_{j=1}^{6} p_{ijk} s_{ijk} + p_{i7k} \frac{x_{ik}}{z_k} + p_{i8k} \right]$ is a linear function.

Optimal control problem statement – minimization of the criteria

$$I = \| x_N - M_N \|_S^2 + \sum_{k=0}^{N-1} \left(\| x_k - M_k \|_Q^2 + \| u_k \|_R^2 \right), \tag{6}$$

where $M_k, k = 0, 1, \ldots, N$ – company plan trajectory, by the set of control parameters synthesis u_k, $k = \overline{0, N-1}$, $k = \overline{0, N-1}$.

The optimal control problem is solved by the dynamic programming method. The results of the optimal control synthesis algorithm are illustrated in Figures 7 and 8.

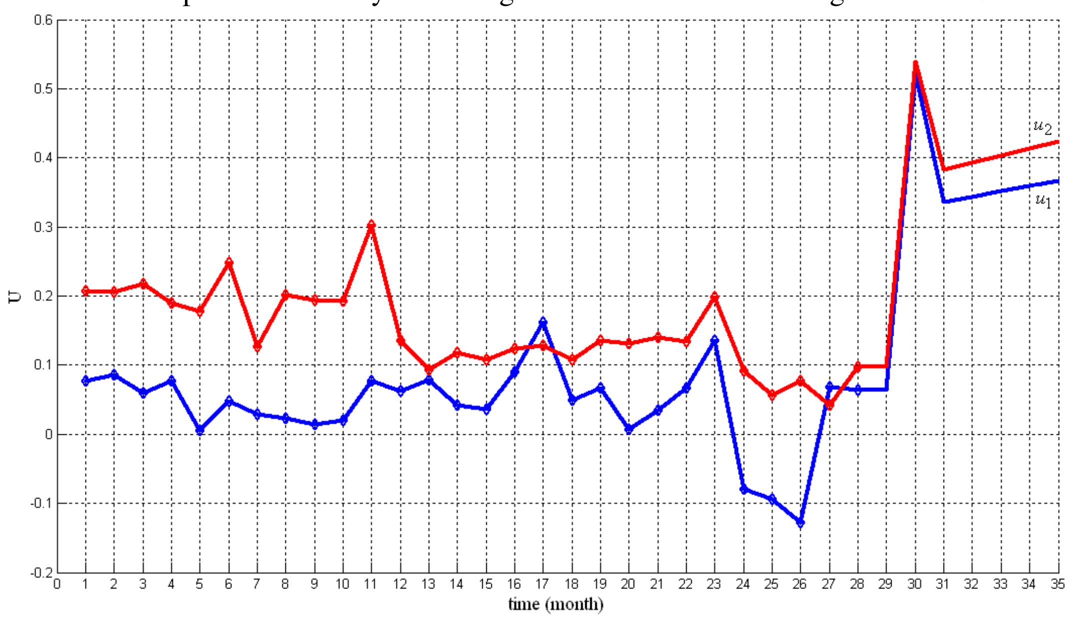

Figure 7. Control u_1, u_2

(actual: 1–29 months, u_1, u_2 –optimal control: 30–35 months)

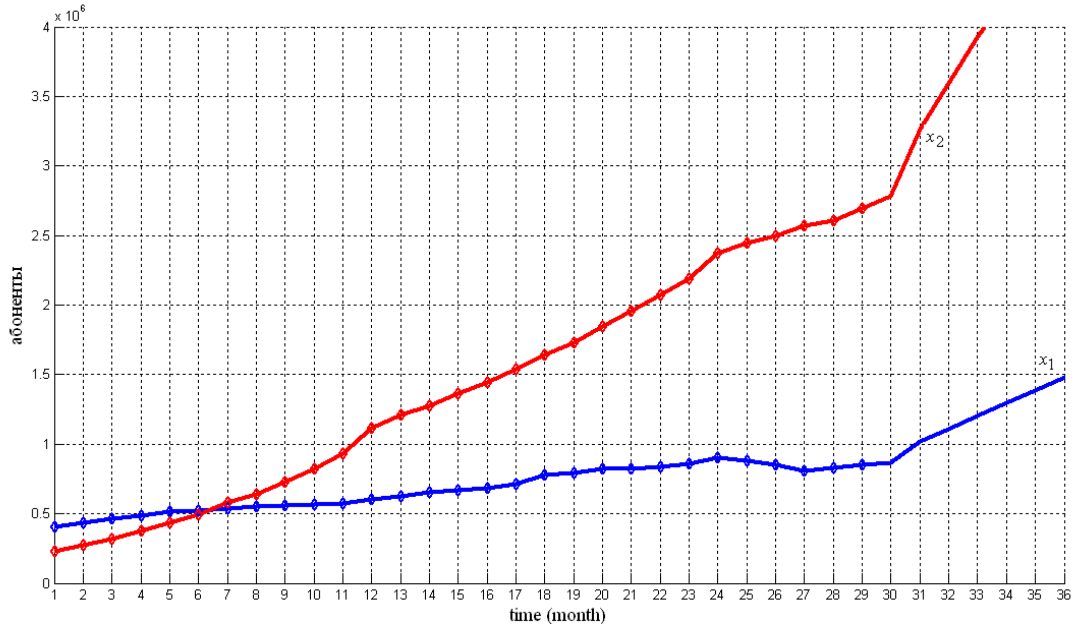

Figure 8. Subscribers x_1, x_2
(actual: 1–29 months, optimal trajectory: 30–35 months)

Conclusion

In this work we presented the system dynamics approach for modeling and forecasting mobile service competitive market of a region. The model includes dynamic competition between some operators. Pricing policy, service quality, subscriber base, potential subscribers, marketing, etc influence their number of subscribers. The task of defining the market share carrying capacity to forecast the sales process is described. Real data from several regions of Russia are used in this paper. The optimal control problem of the mobile operator behavior on a competitive market is considered and solved.

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