

The Financial Accounting Model from a System Dynamics' Perspective

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Summary

This paper explores the foundation of the financial accounting model. We examine the properties of the accounting equation as the principal algorithm for the design and the development of a System Dynamics model. Key to the perspective is the foundational requirement that resolves the temporal conflict that resides in a stock and flow model. Through formal analysis the accounting equation is redefined as a cybernetic model by expressing the temporal and dynamic properties of its terms. Articulated in that form the accounting equation is enabled to be defined as a dynamic stock and flow model expressing the two dimensions of the double-entry accounting system. With that formal foundation it is argued that the accounting model is capable to simulate financial dynamics as well as be integrated with models that express operational and world dynamics. Thus we prove that it is possible to design and build a dynamic business model that can meet requirements of management accounting (*ex ante*, before the fact) as well as financial accounting (*ex post*, after the fact). We conclude that the dynamic accounting model can be made relevant for strategic planning and control purposes and be integrated within a System Dynamics model designed for such purposes.

Topical key words: dynamic accounting model, dynamic accounting equation; financial accounting; management accounting; double-entry accounting system

1 Objective of accounting

Most textbooks draw a very distinct line between financial accounting and management accounting. Kieso and Weygandt (1995), for example, define financial accounting as ‘...the process that culminates in the preparation of financial reports on the enterprise as a whole for use by parties both internal and external to the enterprise.’ Management accounting is defined by them as ‘...the process of identifying, measuring, analyzing, communicating financial information needed by management to plan evaluate and control an organization’s operations.’ Horngren (1995, 1996) on the other hand, defines management accounting from a broader perspective:

Management accounting is ...the process of identifying, measuring, accumulating, analyzing, preparing, interpreting, and communicating information that helps managers fulfill organizational objectives.

That definition comes very close to that of management control. Financial accounting in Horngren’s definition is strictly financial in nature and intended only for an external audience:

Financial accounting refers to accounting information developed for the use of external parties such as stockholders, suppliers, banks, and government regulatory agencies.

At this stage an important observation can be made about management accounting regarding the way this activity is delineated in the textbook literature. Management accounting is seen as the process that only uses financial information, or it is seen as the process that provides more than only financial information to management for internal organizational purposes. The important difference, evidently, is the limitation to financial information as a source for internal organizational decision-making and management purposes, or not. The first note that can be made to this is that in the latter situation any kind of information might be suitable as long as it services the need of supporting management ‘fulfilling organizational objectives’. But, to prevent that an increasing volume of data is collected for that purpose, management accounting should indeed provide the framework with which such activities are to be organized. A thesis central to this paper is that this framework is far from completion and can certainly be extended to accommodate an approach that integrates the different views on the company as well as a more proactive and future directed approach to decision making.

Instead of limiting the focus of management accounting to operations management alone more interest is given here to a broader view of how the business functions as a whole; as a system. The assumption is that a more holistic approach to business problems on all organizational levels will offer a greater insight in the causes of business problems and possible solutions to them. Furthermore, the scope should not be limited to problem solving alone. Management accounting supports decision making about business opportunities as well. Balancing such requirements against company objectives raises the issue at hand from ‘operational’ levels to a ‘strategic’ level because of the financial perspective that is involved. Business opportunities require capital. Capital has a cost — interest or dividend. The investment is expected to render a certain result, preferably better than other opportunities available. Thus, lower level financial decisions quickly aggregate to a high level strategic planning effort.

2 Accounting & System Dynamics

The above comments concerning the different focus authors have on the role accounting has are perhaps trivial and irrelevant for the System Dynamics perspective on how a business operates as a 'living entity'. There are conflicting opinions to be noted. We already mentioned that de Geus (1997) points at the need of a sound financial policy. But other authors tend to discard the need for a very detailed financial analysis. Forrester, for example, cautions against including financial information in a System Dynamics model because it 'does not form an integral part of the decision-making functions' but 'is a reporting system to indicate to the investigator how the system has behaved' (Forrester 1961, p. 335). Further to this, he adds that 'the skeleton framework of primary effects within the organization can often be represented without financial and accounting information' (ibid. p. 336). On the other hand, Forrester admits that 'as models become more subtle and begin to deal with the very important aspect of top-management decision making, the accounting system becomes an essential part of internal information loops affecting attitudes and decisions.' In other words, Forrester does recognize the contribution of accounting information to the set of 'loops' that feed into the analysis for business policy. But, having said that, he concludes that '... financial information is but one small part of the total information within the organization. Usually it measures symptoms, not causes. It is dangerous because it is easier to derive than other more important kinds of information' That opinion certainly will be welcomed by Johnson and Kaplan (1991), who in their influential book *Relevance lost - the rise and fall of management accounting*, explain why today's businesses suffer from a lack of good support for managerial decision-making. They argue that at the beginning of the twentieth century, the direct information link was broken between the production environment and managers. The first cause was that, at the time, it was becoming too expensive to collect and consolidate all relevant business information directly and continuously from the business processes. (This situation has obviously changed radically.) The second cause, they argue, was that businesses at the time were confronted with the need to finance their enterprises with public capital. Accurate financial statements on the health of the business were required by outside investors. The consequence of this requirement was that managers also started to use financial statements for internal organizational and management purposes. There is in itself nothing wrong with that method. Indeed, financial statements are today still one of the fundamental requisites for internal and external assessment of (past) business performance. But, the world is changing, and, hence the requirements of managers for tools and techniques to manage their business are not limited to financial accounting statements alone, and need to be enhanced or extended accordingly. Having said that, such enhancements or extensions should be founded on the axioms of the accounting system.

Kaplan and Norton (1996) offer an alternative with the definition of three non-financial perspectives, beside financial, that make up their *Balanced Scorecard*. Their framework is designed on the assumption that behind these perspectives lies a 'set of hypotheses about cause and effect', and that the 'measurement system should make the relationships (hypothesis) among objectives (and measures) in the various perspectives explicit so that they can be managed and validated.' They do not argue that the presentation of information in separate classes means that there is no unified model behind it. On the contrary, Kaplan and Norton take it as a fundamental assumption that this is the case. Nonetheless, like is done in Smith (1995), their format of presentation can lead to a disjoint presentation of quantitative and qualitative information, and certainly of financial and non-

financial data. The relationship between cause and effect should be made tangible through rigid structural analysis adhering to the systems methodology (Legasto 1980, Randers 1980, Richardson 1991, Richardson 1996, Roberts 1978, Sterman 2000). This effort does not rest only with proper modeling methodology and tools but also with the integration of accounting information that originates from information systems with a method of narration, which principally is simulation and scenario analysis (Lyneis 1982). Such effort will benefit greatly should it be proven that it is possible to build a dynamic model that integrates financial *and* non-financial models.

3 Accounting and the business cycle

Let us assume that all firms operate in the same manner (Geerts, McCarthy, 1999). Someone has an idea about how to provide a product or service. Initial financing is acquired (debt or equity for the firm) and then management and personnel engage in a series of economic exchanges (transactions) with other parties (suppliers, customers, the government etc.). Each time the firm sacrifices an economic resource (financial or non-financial) in return for another resource, and hopefully that has a greater value. It is expected that when all transactions with suppliers and customers took place and are paid and done for,

that then a profit can be declared. Be the firm a single person, or a small shop setup, or be it a corporation working globally, the operating principle is largely the same. Continuously, transactions trigger and maintain the *acquisition cycle* for labor and materials, the *conversion cycle* where the procured materials are transformed into finished goods or services to be delivered, and, finally, the *revenue cycle* where the customers are paying for products or services consumed (Figure 3). The model depicts these cycles as a go around but it is also a value chain of parties & business processes:

- ❖ Procurement of materials, labor and services by the *procurer*
- ❖ Allocation of resources to the company's product or service lines
- ❖ Production of products or services by the *operator*.
- ❖ Delivery of products or services by the *seller*.
- ❖ Receipt of payments made by the *customers*.
- ❖ Payment of the *suppliers* (including company personnel) by the *cashier*.

Any model is a simplification and Figure 3 is no exception; it is perhaps the most simplified (ac-

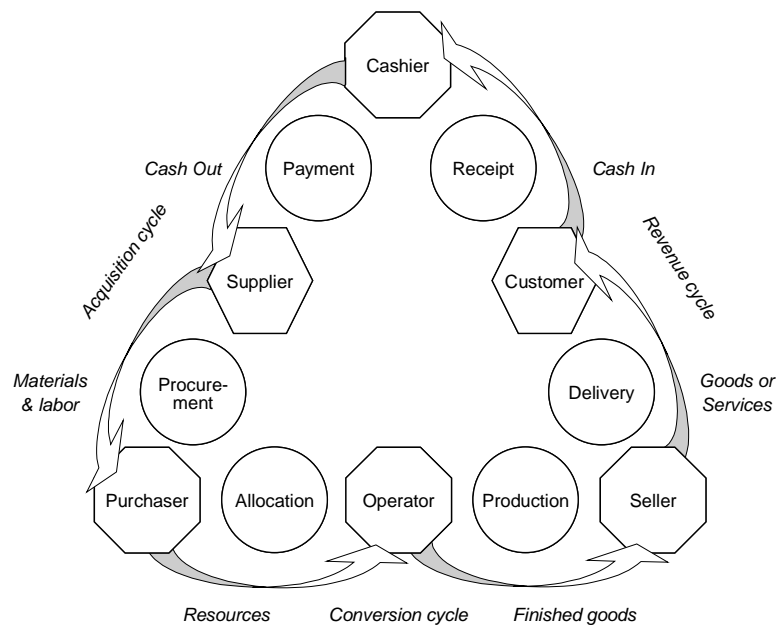


Figure 1 Model of the business cycles that drive the company.

counting) model we can develop of how a business operates. The model of the business cycles is a depiction of the material and immaterial flows of goods, information and money that occur as a result of the company's operation. This model can include any business function and possible depict any particular transformation that occurs in its processes. Such model we can call an operations model and is as such *not* an accounting model. What we have to recognize is that the financial accounting system as such *represents* the business through *economic* dimensions. Maybe it is a confusing statement: the accounting model is an economic model of the business model? But, indeed that is what it is.

What we observe of accounting as an organizational function is that the accounting system administers documents on which basis financial transactions are recognized about which data is registered and entered into a recording system (usually software). With that system financial statements are produced that report what has happened during a particular period in time (income statement) as well as what the impact is on the firms' state at a particular moment in time (balance sheet). It is this, more administrative-procedural, function that is responsible for the impression people have of accounting: a system to count 'beans', i.e. to 'crunch' numbers. However, this is a gross simplification of what accounting is and what accountants do. Behind accounting as a 'technical' or 'organizational' system lays a conceptual model. It expresses a social reality, namely the relation between the organizational entity, i.e. the company, and other entities in its environment. It is from this fundamental interpretation of social and economic reality from where the accounting model has to be understood. It has also to be starting point from which a System Dynamic model can be built as to explain the structural relationship between business dynamics and financial dynamics.

4 The accounting equation

The financial accounting system is foremost designed to measure the value of a firm at a particular moment in time. The design is based on the notion that external transactions and internal events pertaining to a firm — a real world entity — are made accountable in financial terms, i.e. in an economic or monetary value. This is a common sense thing to do because it offers the advantage to 'translate', 'register' or 'account' a multitude of real world phenomena. The beauty of the financial accounting system lies in its capability to transform multidimensional real world phenomena into a one-dimensional measurement: their financial value. From a modeling perspective this means that it is possible to quantify the financial value of material and immaterial assets and equity in terms of a single measurement.

There is a mathematical advantage to have a one-dimensional measurement in the accounting system. Once a formula, model or data set has a homogenous dimension, mathematical operations are allowed to test their validity.¹ In the case of a formula, value changes can be explained formally when the measuring unit itself is transformed. Since the financial accounting system uses a homogenous or uniform measurement system, one can argue that this parameter — the monetary value — is not anymore a foundational dimension. The system is indifferent in respect to the notation of data entered and processed.² This does offer the advantage of computational simplicity in spreadsheets or in System Dynamics models. But, it does not alleviate us from the need to recognize and articulate the foundational dimensions of the financial accounting system. Contrary to what many believe, these foundational dimensions are not the 'debit' and 'credit' entries of an accounting transaction into the bookkeeping system but instead are:

1. *The source of capital*, or the mound of equity: *claims*.
Capital raised, in the past, is administered so that the past flows are known of stockholders' equity, reserves, liabilities (debt), as well as those that report for the company's operational result of a given accounting period.
2. *The use of capital*, or the composition [EM2]of equity: *assets*.
Capital in use is administered with stock accounts so that the current composition of capital invested can be determined. It is where capital is activated.

The two foundational dimensions are material as well as conceptual. Mattessich (1995) argues that behind every business event we encounter a material phenomenon in the empirical sense (e.g. a product exchanged for cash), and an immaterial phenomenon in the conceptual sense (e.g. depreciation reducing the value of an asset that also reduces its ownership claim and thus the mound of equity). Many authors discuss the principles of the accounting equation (Kieso *et.al.*, 1995, Porter *et.al.*, 1998). Blommaert (1994) formalized the logic of the two foundational dimensions of the double-entry financial accounting system. He asserts that *assets* are equal to *claims* in |E 1| and differentiates the claims in two basic elements *stockholders equity* and *liabilities* in |E 2|.

E 1	assets = claims	<i>accounting equation</i>
E 2	assets = owners' equity + liabilities	<i>claim elements</i>
E 3	assets – liabilities = owners' equity	<i>wealth</i>
E 4	net assets = owners' equity	<i>wealth</i>

By subtracting the liabilities (debts of a company) from the assets, the accounting equation is formulated in |E 3| as to determine the *wealth* of a company. Next, |E 3| is rewritten as |E 4| to express how much of the invested capital is *owners' equity* and how that is distributed, or spread, over *net assets*. Ijiri (1989) calls both terms of the accounting equation in this form: *wealth*. He explains that conceptually wealth can be determined from either assets or claims, the equity owners are thought to have, in the sense that they are equivalent measurement-wise. This precludes, and is the first reason why we take in the next paragraph the step to introduce in |E 5| the equivalence sign between the terms (\equiv) of the accounting equation instead of the equal sign ($=$). Ijiri (1989) states that 'wealth denotes *real economic goods*' at the left term of the equation whereas 'capital refers to an abstract, nominal concept of residual claim'. Asset accounts are, therefore, called *real accounts* and capital accounts *nominal accounts*. Every liability that is present in |E 1| or |E 2| is in this interpretation understood to be *negative wealth* in the sense that proprietary ownership lies outside the firm. Therefore, equations |E 3| or |E 4| are to be preferred to determine the true value, or wealth of a company, because they express the absolute value of the company and it can be determined at any moment in time. It should also be noted that this is a strategic perspective because the accounting equation expresses the *value* of an entity instead of 'past success' (Forrester 1961).

From a mathematical perspective the accounting equation has on the highest level of abstraction a very simple structure. |E 1| has the straightforward algebraic form $\alpha = \beta$ (α for assets, β for claims). But behind this equation lies a sophisticated world of layered connotations. Quantitatively the algebraic form states that the figure found at the left term is equal to the right term. And it has to maintain equality because in principal any operation on the accounting equation affects both terms. Hence, the famous implication that 'the accounts must balance', which can be found in every accounting textbook and perhaps is the best known accounting axiom known to the public at large.

It is also here where in the logic of the accounting system the concept of *duality* is introduced. Because each transaction that changes either the (past) mound of claims will also, automatically, change the (current) composition of assets. As a result of this logic—when all financial transactions of a given period are accounted for—the aggregated value of each term of the equation is equal to the other. But, the algebraic notation does hide a serious implication. It does not explain the temporal position of the terms nor the dynamic structure that follows from it. This leads to a computational referential integrity error that must be prevented. Only after we formulate the accounting equation as to express these properties, more can be said about the possibility to use it as the foundation for a System Dynamics model for strategic accounting and management control.

5 Temporal perspective

Net assets of a company can be determined at any point in time because they refer to a particular *state*. However, for the right term of the accounting equation the accounts cannot be specifically identified. For example, in a going concern situation, under ‘normal’ circumstances, reserves or profit are not distributed continuously. Therefore, the state of such accounts can only be determined by describing their development *through time* up to the present moment at which point certain ‘one-off’ decisions have to be made. Likewise, retained earnings will discretely be determined depending on the fact if it is distributed to the owners or added to the reserves. This means that the right term of the accounting equation should be understood to describe the *past* whereas the left term describes the *present*. During a brief bridging moment in between points in time, internal accounting transactions occur but the balance is always maintained. Thus, |E 4| can be rewritten to introduce the temporal perspective of the accounting equation as:

$$|E 5| \quad \text{present} \equiv \text{past} \quad \textit{temporal perspective}$$

This perhaps appears to be a redundant step, both methodologically and mathematically, but most certainly it is not. With |E 5| the temporal characteristic is articulated of the *dimension* of each term of the accounting equation. The model *structure* of the accounting system is thus defined as will be explained in the text that follows. When the static accounting equation is made dynamic it has to abide to this temporal structure. Therefore, the methodological and mathematical foundation of a dynamic accounting tool must adhere to, and include:

1. The computational logic as expressed in |E 3|
2. The temporal logic as expressed in |E 5|

A question that rises from the above constraints is if the structure of the accounting equation is to be put before its dimensions or vice versa. It is somewhat difficult to provide a straightforward answer because we are still at the point of describing the foundation of a dynamic financial accounting system and are not yet working towards its full augmentation for business simulation. But, this question can briefly be addressed. When the past and the present are the foundational dimensions of the double-entry accounting system, as formulated with |E 5|, how does this translate to the structure of the accounting model? Ijiri (1989) touches upon this issue but he understands it to be the *structure of accounting measurements* with which he means ‘a set of rules that unifies numerous accounting measurements’. In his definition the term structure receives its meaning from its capacity to ‘understand the existing system but also an attempt to explore the possibility of building onto

the existing system by adding higher dimensions to it.’ Ijiri denotes ‘two fundamental axes that exist in the structure of accounting measurements, namely the *time axis* and the *component axis*.’ It is on the component axis where he identifies the left term and the right term of the accounting equation. Thus, for Ijiri, the dimensions of the accounting equation are determined *within* the structure of accounting measurements. Nonetheless, another view is taken here on how we should understand the relation between the dimensions and the structure of the accounting system.

We argue on the basis of [E 5] that the structure of the accounting system follows from its dimensions because the terms of the accounting equation are equivalent. Both terms add up to the same sum value but have a different connotation in time. Computationally the two dimensions are the same but temporally they are different. That is the second reason why [E 4] is formulated as [E 5] with equivalent terms ($\alpha \equiv \beta$) instead of equal terms ($\alpha = \beta$). That aspect is completely lost in the current practice of the accounting system, and its models, because of the fact that the negative sign of the right term of the accounting equation is written and read as a positive (instead of the negative sign it has). As a result, the structure of the accounting equation remains hidden in equations [E 1] up to [E 4] in the sense that the temporal characteristic of the terms is not made explicit. Therefore, in these formulae the equation is still ‘static’ or ‘timeless’. The inherent dynamic of the accounting equation, and the model that originates from it, remain hidden. Nevertheless, the accounting equation is dynamic because the dimensions have a temporal property as defined in [E 5]. Therefore, its structure necessarily must be dynamic too. We thus conclude that the dimensions of the accounting equation determine the structure of the accounting system. The dimensions of the accounting equation constrain the structure of the accounting system and its model. Ijiri (1989) actually draws a similar conclusion: ‘the fact that accounting measurements are functions of time allows us to develop new measurements from existing ones by taking their time derivatives and time integrals’. Having recognized the temporal property of the terms of the accounting equation we now can proceed to explain the dynamic property that unmistakably follows from them in terms more familiar to System Dynamics modeling, namely as a stock and flow model.

6 Stock & flow in the accounting model

The temporal—dynamic aspect that follows from the equivalence of the two terms of the accounting equation is better explained when it is formulated as a stock and flow model. For this, the accounting equation is redefined from an economic perspective in [E 6]. Mattessich (1991, 1995) argues that the accounting equation defines between entities, beside an economic relationship, also a social relationship. Indeed, these two connotation layers, properties if you like, are intertwined and as a methodological premise not only have to be addressed simultaneously in any effort to define the accounting system, but they also explicate the computation of its model.

Double-entry financial accounting administers claims that owners have on equity *indirectly* through the constituent elements with the *real accounts* (the first dimension) and *directly* through company result with the *nominal accounts* (the second dimension). This is due to the fact that the left term of the equation refers to stock or point values while the right term refers to flow or period values. Real accounts *accrue* or *accumulate value* continuously while nominal accounts determine the *value change* periodically. When the value of the term at the right term of the accounting equation is computed so that all the flows that occurred from the start of a business are accrued, then the accounting equation will balance when the value at the right term is compared with the left term.

Therefore, the accounting equation can be formulated as a stock and flow model with |E 8|.

E 6	use \equiv source	<i>economic perspective</i>
E 7	real \equiv nominal	<i>accounts</i>
E 8	stocks \equiv Σ flows	<i>accounting equation</i>

The dimensions of the accounting equation are recognized with their temporal characteristic because we can read the left term as a point value at τ , the *present*, and the right term as a period value, the *past*, determined between $\tau - \pi$ and τ , the sum of flows. Next, we explore further the structure of the dynamic accounting system.

The right term of the accounting equation |E 8| is differentiated in |E 9| so that the flow at a particular point in time is computed by subtracting from the current stock value the value it had at the previous moment of measurement (i.e. at the start of the period).

E 9	stock τ - stock $\tau - \pi$ = flow $\tau\pi$	<i>period measurement</i>
E 10	\square stock = flow	<i>mutation</i>

This equation is rewritten in |E 10| using the symbol \square that stands for ‘change of’ or *mutation*. Ijiri (1989) states that since we have to understand that the right term of the equation |E 8| is the cumulative value of each flow that occurred in the past, both terms of the equation are ‘put in the same time frame and the symmetry between the two is thus made clear’. Ijiri has his reasons to do this, to ‘somehow convert flows into stocks’. He wants to set the stage for moving away from periodic income determination and toward continuous income determination. This is a fundamental step to take because it would allow for the elevation of the accounting system as merely a tool for record keeping (of ‘past success’) to a general-purpose system for strategic planning, management control and accounting. This objective is not unfamiliar to System Dynamics (Lyneis 1982, Bianchi 1996).

Although |E 8| up to |E 10| are conceptually and mathematically correct, formal and computational logic is violated because their terms are disparate in the perspective of temporal logic. This is explained through the following analysis of the structure of the accounting equation expressed in a stock and flow model. First, we elaborate on the definition of the mutation in terms of inflow and outflow, or in accounting terms, the debit and credit of accounts:

E 11	\square stock = inflow - outflow	<i>input less output</i>
E 12	\square stock = debit - credit	<i>debit less credit</i>
E 13	mutation = debit - credit	<i>mutation</i>

In |E 11| the right term of |E 10| is differentiated into its constituent parts: *inflow* and *outflow*. This is symmetrical with the left term of the equation in the sense that the mutation (change) of a stock can be an inflow, an outflow or both. It is here where in the structure of the accounting measurements the terms *debit* and *credit* are introduced. Debit refers to the flow into an account and credit refers to the flow out of an account. Although not used as such in the literature, in |E 13| we can safely coin the term *mutation* to be the sum value of the debit and credit entries of an account at a particular moment of measurement. Equations |E 11| up to |E 13| are correct when assessed from the viewpoint of computational and temporal logic because they equate the measurement of income and the change of wealth of *one particular point in time*. This means that the determination of the value of either term of the accounting equation can be done from two different sources made at a

moment of measurement. The determination itself is mathematically timeless. But, as soon as we accrue such values, as is done under normal going concern circumstances with the accounting equation as defined with [E 8], a very serious problem arises. Then a referential integrity error will occur in the model that must methodologically be prevented and technically not be possible.

It is not possible to compute in a dynamic model at the same moment of measurement the sum flow value that mutate a stock and the value of the stock itself. When the flow value has mutated a stock, that new stock value will be different from the sum value of all the past flow values. How this operates can be only understood by taking into consideration the temporal logic of the accounting equation. With [E 9] the value of a flow was determined by subtraction of the value of a stock from the previous point in time of the current value. Thus, the computational logic runs from the left term of [E 9] to its right term: $\alpha \Rightarrow \beta$. This is mathematically sound, but not computationally in a dynamic accounting model. The value of a mutation, the sum of flow values, is determined in [E 9] from the difference between stock values of two moments in time. In other words, the flow value is differentiated from two measurements of value accumulated in that stock. Forrester (1961) disagrees with the formulation of dynamics through differentiation because ‘nature only integrates’. Certainly for the accounting model he has a point. Under going concern conditions computational logic runs from the right term of the accounting equation to its left term: $\alpha \Leftarrow \beta$. In a dynamic model the flow values are determined first and next they are integrated, accrued, with the stock value of the previous moment of measurement. Thus the accounting measurement is formulated structurally from [E 9] into [E 14].

$$\text{[E 14]} \quad \text{stock}_{\tau} = \text{stock}_{\tau-\pi} + \text{flow}_{\tau} \quad \textit{accounting measurement}$$

This explains that at the right term of the equation the temporal position of a flow value is always one step before that of the stock on which it operates. The flow represented here is a generalized value: it is the mutation of the account, the debit, credit or both.

Now we can also better understand how to read equation [E 8] because its form is identical to [E 14]. Also in [E 8] the computational logic runs from the right term to the left term: $\alpha \Leftarrow \beta$ when we want to understand this equation to adhere to the temporal logic of equation [E 5]. An example might explain this further. Table 1 lists row-wise five points in time when measurement is made of inflow to be added, outflow to be subtracted (together they are the flow or mutation) and the stock that accrues them (accumulation, integration). There are two columns that list a value at each moment in time. The column with the title ‘stock _{$\tau-1$} ’ has the value of the stock *before* the current flow (mutation) is accumulated. The column with the title ‘stock τ ’ has the value of the stock *after* the flow is accumulated. That column lists the value that the account has when all transactions of that moment in time are accounted for. The present value of the account is equal to the sum of the present flow value (mutation) added to the previous value of the account. Note that the row-wise direction of computation in Table 1 runs from the left to the right. Although it is a matter of choice how to present the example values, in this case it is done on purpose in this manner as to explain that the present value of the account is necessarily the end *and at the end* of the computation of each time step. It is not possible to use the result value in any flow computation of a time step at that same time step. Otherwise a referential integrity error will occur — a mathematical snake that bites its tail. Therefore, the last row in Table 1, which has no time step identification, with the sum values of the inflow, outflow and mutation has to be understood to be ‘stocks’ as well. In that row we can see that the sum of all flows is 140, which is, of course, the same value as that of the stock

at time step five. This is in perfect agreement with [E 8]. But, note that to be able to demonstrate that [E 8] is correct, a tabular listing of time steps is required on which a sum calculation must be performed. Clearly Table 1 proves that this is possible and in any spreadsheet this can be done. But, not in a dynamic simulator, unless we resort to using stocks to accumulate the flows of accounts separately.³ The additional sixth row of Table 1 acts as this ‘stock of flows’ that is ‘outside’ the time step. Hence, [E 14] not only explains us how to understand [E 8] and thus the foundational accounting equation [E 1], it also shows us how to meet the requirements necessary for its dynamic simulation. Furthermore, [E 14] explains the temporal position of the two dimensions of the accounting system in a structural manner. It means that the present value of any stock or account that is part of the left or the right term of the equation is explained by computing the sum of the stock

time	inflow	outflow	mutation	stock _{τ-1}	stock _τ
1	100	0	100	0	100
2	50	10	40	100	140
3	20	10	10	140	150
4	0	0	0	150	150
5	10	20	-10	150	140
Σ	180	40	140		

Table 1 Example computation of inflow and outflow of a stock adhering to the accounting equation.

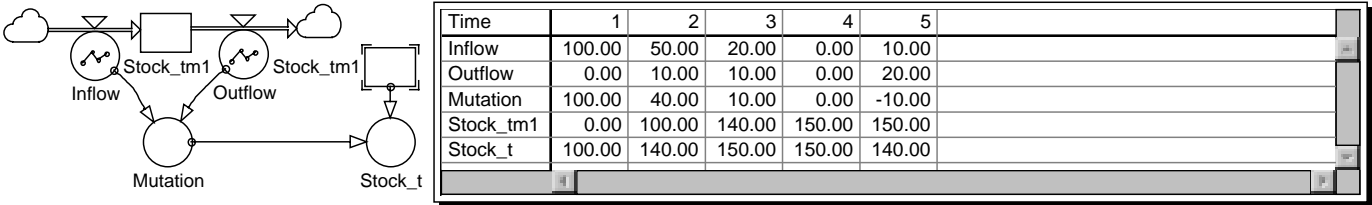


Figure 2 Stock and flow model of Table 1 modeled with Powersim™ Constructor version 2.51. (See DAM_T2.SIM.)

value of the previous time step with the flow value of the current time step. This is an important property of the accounting equation because it — in Ijiri’s words — allows us to ‘convert flows into stocks’. Ijiri (1982, 1986, 1989) strives to extend of the double-entry accounting system so that managerial contributions that improve a company’s capacity to earn income are disclosed instead of past performance. This means that the system that today is used principally for financial accounting may be extended and used for management accounting. The already difficult to draw borderline between the two functional areas of use of accounting information would then disappear. Both uses, accounting for regulated disclosure to an external audience (shareholders, government, banks, etc.) and accounting for regulated disclosure to an external audience.

7 The duality principles of accounting

Perhaps double-entry accounting is ‘...simply a recording system which records the dual effect of each business transaction’ (Chadwick 1996, 35). But others disagree. Blommaert (1994) points at the widely held misunderstanding that the *double-entry*, the left and right hand entry into financial accounts, the debit and the credit, refer to the *two dimensions* of the accounting system. The opera-

tion of double-entry bookkeeping indeed involves the recording of each business event with two accounts. But those refer to the two dimensions of the accounting system, assets and claims. This is usually done by means of two columns, for debit and credit entries. But this can be done just as well using one column only with *positive*, or debit, and *negative*, or credit, entries. Also Mattessich (1995, 96 note 11) disputes the accounting literature on this subject: ‘For decades I have tried to show that the crucial event in accounting is not double-entry — which is a mere technique — but the *logical structure* behind it. ... A set-theoretical analysis of this flow or input – output structure in terms of ownership and debt relations ... need not manifest itself in a twofold entry but may be represented in form of a single-matrix entry or a network relation between two points, or a vector, or an algebraic equation, or the like.’

What then are the foundational dimensions of double-entry accounting? What is the logical structure of accounting? For Mattessich it is *duality* that governs the structure and logic of the accounting system. For him, the double-entry of each business transaction, that is required in the accounting ledgers, is not a foundational property of the financial accounting system per se but a necessary instrumental outcome of its principles. This will be discussed with some depth in the following paragraphs because the foundational properties of the financial accounting system also, by logical necessity, govern any effort to design a dynamic accounting model, be it for financial or management accounting.

Mattessich (1995, 61-68) convincingly argues that a property claim between two parties is always to be found to exist behind an accounting event, internal or external. This is the duality principle he recognizes as the essential concept of double-entry accounting. Its manifestation in the empirical world Mattessich sees just as real as any transaction of conceptual nature because all are a *social reality* behind which lies an ‘ingrained behavior pattern’. Which makes financial accounting, according to Mattessich, an applied science rather than a pure or cognitive science. Which does not mean that financial accounting should not develop an instrumental theory by finding the means – end relations of its methodology. He identifies three duality principles of the financial accounting system: the input–output principle, the symmetry principle and the change principle. Mattessich underlines the importance of the first two principles and discusses the change principle more as an extension of the symmetry principle. However, it seems appropriate to distinguish the change principle from the symmetry principle more explicitly because it relates the claim dimension *dynamically* to the wealth dimension of the financial accounting system. It will be demonstrated that the change principle applies temporal logic of *dynamic behavior* to accounting. The symmetry principle, on the other hand, applies a mathematical logic of *model structure* to accounting. In the paragraphs below it will be discussed that the symmetry principle not only is expressed through the structural relationship between the two dimensions of the accounting equation, but also between accounts of one dimension. It will be argued that the input–output principle is responsible for the symmetric structure of the accounting equation and the dynamics of change that it drives.

8 The input — output principle

Mattessich formulates the input–output principle as pertaining to ‘the transfer of a *concrete* economic good (i.e., non-monetary asset) from one “location” (e.g., accountability center) to another’.⁴ A relevant attribute, like substance, quantity, volume, value is preserved ‘in regard of which the

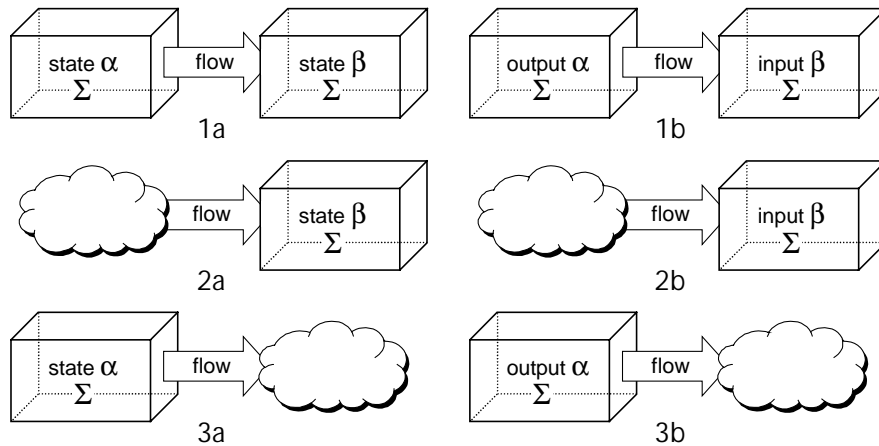


Figure 3 binary transfers define a data flow between two expressions.

- 1a Flow from state variable α to β .
- 1b Flow from state variable α to β is output of α and input of β .
- 2a Flow from environment to state variable β .
- 2b Flow from environment to state variable β is input of β .
- 3a Flow from state variable α to environment.
- 3b Flow from state variable α to environment is output of α .

output from one location corresponds to the input in the other.’ Through this definition, actually, attessich articulates the scope of the input – output principle broader than that of the financial accounting model alone. His definition applies to all system models that are based on the principle of stock and flow analysis. It is assumed in this study that a dynamic model has relations between expressions that involve the flow of data between them. In cybernetic terms this attribute is *information*; data viewed from the perspective of a system in the context of its environment. To model such an informational attribute means that it must represent a *material* or *immaterial* flow of ‘things’ between expressions, or variables, that represent real world entities. In either case ‘numbers’ are used. For example, when cash is exchanged for goods between two entities, a company and its customer, both cash and goods are material. The ‘number’ then refers to the face value amount of cash exchanged for the amount, or number, of items, of goods.⁵ But this material flow can also be modeled as an immaterial flow, namely as the flow of value that *represents* the material flow of cash and the goods. The latter is what the financial accounting system does. It shows the impact of both material (and immaterial) flows (assets) on the ownership rights (claims), of one accounting dimension on the other using only one attribute: *value*. The ‘numbers moved’ are a reference to money (cash value or book value). But, for modeling purposes it is not that relevant because the numbers are technically dimensionless in the accounting model.

For any dynamic business model the input – output principle is fundamental. It means that an amount — in the case of the financial accounting model *value* — is ‘transferred’ from one ‘position’ to another. In modeling terminology that position or location is called a *state variable*. The state variable is able to ‘keep’ its value during the run of the simulation from one simulated time step to the next step. This *state variable* is called *stock* in the System Dynamics methodology because it ‘stores’ amounts over time like a stock of goods. But it can just as well be called an *account* because that is the variable by which the financial accounting system accrues or stores amounts over

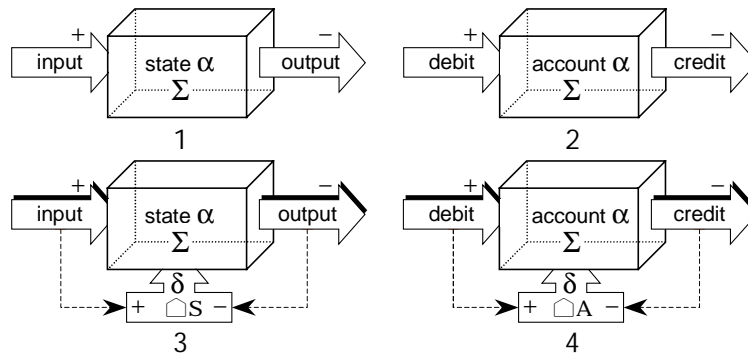


Figure 4 State mutation with both input and output. Ideal (1), and applied in the accounting system (2). Input and output added together as the mutation of a state, stock or account (3, 4).

time. Actually, *account* is the preferred term because it also best expresses the working of the input – output principle. Figure 3 illustrates this with state variables and a flow. The state variable is pictured as a 3-dimensional box and the flow as a broad arrow. A flow occurs from state α to state β and the amount that flows in between is the *output* of state α as well as the *input* of state β (1a, 1b). The input – output principle thus means that an event — that *causes* the flow between the expressions α and β — has a *dual effect*:

1. *output*, or outflow, of the first state variable, stock or account, and
2. *input*, or inflow, of the second state variable, stock or account.

Another way of describing this principle is that the first state variable is the *source* of the amount that is transferred (flow), while the second state variable is the *sink*. In mathematical terms, the amount that is output, or outflow, is *subtracted* while the input, or inflow, is *added* to the count of the state variable, stock or account. Thus we conclude that:

The input–output principle implies that all events expressed by the accounting equation involve two, or more, state variables.

The operation of the input – output principle through two state variables and a flow explains how the duality principle of the financial accounting system manifests itself: how does the dynamic model compute flows between a model expression and the environment? In exactly the same way as it would do with two or more internal expressions, by assuming an expression that represents that environment but without counting with a state variable. In Figure 3 the environment is pictured by means of a cloud. When a flow occurs from the environment into an expression it is input (2a, 2b). In that case the state variable only adds an amount to its last count. Likewise output from a state variable to the environment results in the subtraction of an amount to its last count (3a, 3b). A state variable, or account, can receive input and provide output at the same point in time. This means that the amount that is input *will be added* and the amount that is output *will be subtracted* (Figure 4*1). The sum of both is added to the count of the state variable of the previous moment in time. In terms of financial accounting, the input, or inflow, is *debited* while the output, or outflow, is *credited* (Figure 4*2). Their sum is accrued to the balance of the account of the previous point in time as to count the balance of the current moment in time. The *sum of input & output, inflow & outflow or credit & debit* of an account is called *mutation* and the pictogram \square is used to depict it.

input	output	mutation	input	output	mutation
10	20	-10	-10	-20	10
20	10	10	-20	-10	-10
10	10	0	-10	-10	0
-10	20	-30	10	-20	30
20	-10	30	-20	10	-30

Table 2 Mutation from positive & negative amounts of input and output.

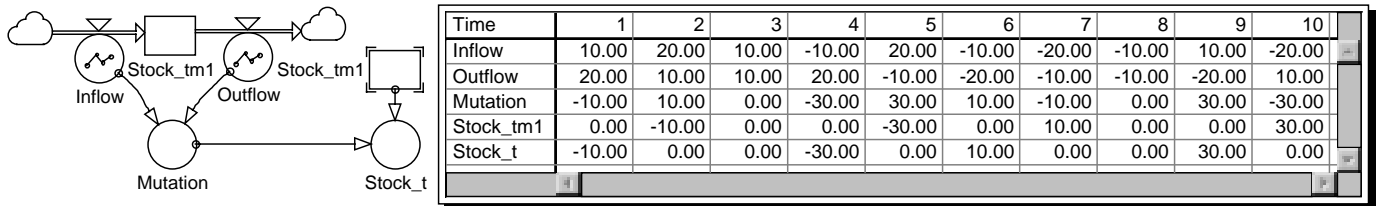


Figure 5 Stock and flow model of Table 2 modeled with Powersim™ Constructor version 2.51. (See DAM_T3.SIM.)

This was already explained and used during the analysis of the accounting equation. The mutation is added to the previous value of the account. When the amounts involved are positive and the output is larger than the input, the mutation will be *negative*. When the amounts are positive and the input is larger than the output, the mutation will be *positive*. Naturally, when the input is equal with the output the mutation is zero. Table 2 lists the examples that are possible including when negative amounts of input or output occur. Negative amounts result in positive account value when they are outflow (credited) and negative account value when they are inflow (debited). But, what is very important to understand and remember for the discussion in this thesis is that it is not possible to accrue a mutation to a state variable and, at the same time, use that result value to compute any flow value that is input or output of that stock. The important temporal difference between the flow value and the stock value, at any particular moment of the dynamic simulation of an input–output system, is that when the flow values are computed, the stock values have not the same temporal position. At any moment, any time step, of a dynamic simulation, the value of the stocks is of the previous time-step whereas the value of the flows is of the current time step. The mutation value of the stock thus is accrued ‘in between’ time steps to their stock. This is the reason why the two terms of the accounting equation formally have a different temporal position, as was discussed above. This explains why a temporal conflict necessarily resides in the accounting equation and its dynamic structure. But, more importantly, this is not just a technical constraint that has to be solved to be able to design and build a dynamic business simulator for management control and accounting. We should also appreciate the input–output principle as foundational to the operation of the accounting system as a whole. That provides us with a solid basis to investigate its dimensions and its structure. It will prove a fine instrument to test our assumptions as well as provides the means to extend usability of the system for management control purposes and strategic planning.

9 The symmetry principle

The duality expressed by the symmetry principle, according to Mattessich ‘arises from the fact that an asset “belongs” to a person [or entity] and thus corresponds with some owner’s equity (or part) of it’ (Mattessich 1995). This means that any change in the composition (allocation) of assets that results in the change of its value is expressed immediately in one of the claim accounts. Every

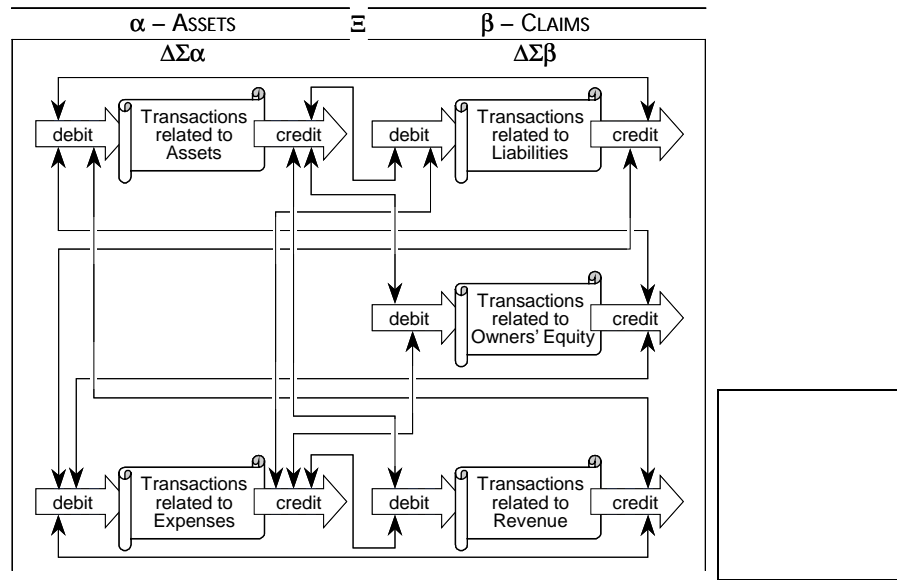


Figure 6 The symmetry principle made manifest through transactions that causes a vector (debit, credit) between asset and claim accounts. A *change of capital composition* will occur in the left model examples because the vector then unifies the two accounting dimensions (inter-dimensional transaction). This does not occur in the right model examples because each transaction is a *change of capital distribution* (intra-dimensional transaction). Compare with Figure 7.

Use of capital		assets	≡	Source of capital		claims
Cash		50		Liabilities		60-
Goods		50		Owners' Equity		40-
	Total	100	τ_1	Total		100-
<hr/>						
Cash		50		Liabilities		50-
Goods		50		Owners' Equity		50-
	Total	100	τ_2	Total		100-

Table 3 Transfer of liability claim of 10 to owners' equity in the model of capital accounting (Figure 6, right) hat is an one-dimensional transaction and thus does not result in a change of total capital (τ_2).

transaction that results in a change of assets is also accounted with a change of a claim account or vice versa. Hence, the dual-entry—the debit and credit—must be understood as a vector, a flow, of a particular amount between such accounts that is debited or credited at one of the accounting dimensions. Thus the symmetry principle, which is about a social relation (assets versus claims) that is expressed through a transaction is made manifest by a vector between accounts that sets the input and output of the related amount as depicted in Figure 6 and Figure 7. However, we can argue that this is only half the story because transactions are possible that change the distribution of assets or claims without changing their total value. For example, when a loan is converted into stockholders equity a debt claim is transformed into an ownership claim (Table 3). The social relation between the outside entity and the company is changed from a lender to an owner. In Figure 6 this is depicted by the vector between the debit flow of liabilities and the credit flow of owners' equity. It is interesting to observe that this particular example does not change the position of capital of the firm because it is an *intra-dimensional transaction*. The vector only causes a transfer between two claims accounts (Table 3). But, when we put this transaction in the perspective of wealth accounting, it does change the wealth position of the firm because it is now recorded as an *inter-dimensio-*

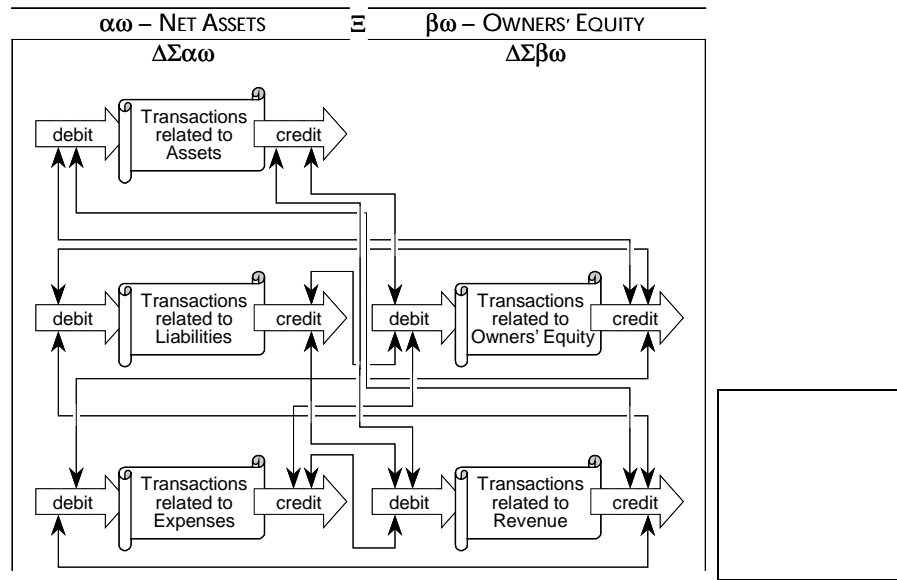


Figure 7 The symmetry principle made manifest through transactions that cause a vector (debit, credit) between *net asset accounts* and *owners' equity accounts*. A *change of wealth composition* will occur in the left model examples because each vector then always unifies the two accounting dimensions, whereas this will not occur in right model examples that are *changes of wealth distribution*.

Use of wealth		assets	≡	Source of wealth	equity
Cash		50			
Goods		50			
Liabilities		60-		Owners' Equity	40-
	Total	40	τ_1	Total	40-

Cash		50			
Goods		50			
Liabilities		50-		Owners' Equity	50-
	Total	50	τ_2	Total	50-

Table 4 Transfer of liability claim of 10 to owners' equity in the model of wealth accounting (Figure 7) that is a two-dimensional transaction and thus does result in a change of total wealth (τ_2).

nal transaction (Figure 7, Table 4). It is the same debit of liabilities and the same credit of owners' equity but because liabilities are now subtracted from assets to determine wealth, any change of it will cause a change of wealth accordingly.

Likewise, transactions that are related to nominal accounts will cause inter and intra-dimensional vectors between a debit flow and a credit flow. For example, compare Figure 8 with Table 5. Here, revenues (15) and expenses (13) of a certain period are accounted for, respectively in nominal accounts, as well as in the cash account (assuming both are cash transactions). Next, the nominal account result is used to determine the period final result (2), which annuls in the process the nominal revenue and expense accounts through the transaction vector. Finally, the result account itself is annulled through transfer of the booked result to the real account owners' equity, which has increased to 2. This simple and small example demonstrates that the symmetry principle is upheld for transactions between nominal and real accounts. The previous paragraph already demonstrated that the symmetry principle not only applies to inter-dimensional transactions (composition) but also to intra-dimensional transactions (distribution). Furthermore, it was demonstrated

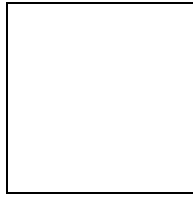


Figure 8 The symmetry principle made manifest through a series of example transactions that cause a vector (debit, credit) between nominal accounts and real accounts. See Table 5 for the related amounts.

Use of capital		assets		Source of capital		claims
(RA) Cash		0		(RA) Owners' Equity		0
	Total	0	τ_0		Total	0
<hr/>						
(RA) Cash		2		(RA) Owners' Equity		0
(NA) Expenses		13		(NA) Revenue		15
	Total	15	τ_1		Total	15
<hr/>						
(RA) Cash		2		(RA) Owners' Equity		0
(NA) Expenses		0		(NA) Revenue		0
				(NA) Result		2
	Total	2	τ_2		Total	2
<hr/>						
(RA) Cash		2		(RA) Owners' Equity		2
(NA) Expenses		0		(NA) Revenue		0
				(NA) Result		0
	Total	2	τ_3		Total	2

Table 5 Nominal accounts (NA) and real accounts (RA) in the model of capital accounting (Figure 8).

τ_0 Opening balance of the real accounts.

τ_1 Intra-dimensional transaction between Cash and Expenses [13].
Inter-dimensional transaction between Cash and Revenue [15].

τ_2 Inter-dimensional transaction between Expenses and Result [13].
Intra-dimensional transaction between Revenue and Result [15].

τ_3 Intra-dimensional transaction between Result and Owners' Equity [2].

At τ_3 all nominal accounts are closed, i.e. period values are transferred to real accounts.

that the symmetry principle is maintained when the accounting equation is formulated in terms of wealth instead of capital. But, in the case of wealth accounting that leads to a structural change of the expression of the symmetry principle from intra-dimensional to inter-dimensional for transactions between liabilities and owners' equity. All these observations sustain the more fundamental statement that the symmetry principle seems to confirm that the input – output principle is the dominant principle of the accounting equation. Or, rather, analysis of the symmetry principle leads to the acknowledgement that the input – output principle is determining the *structure* of the accounting system whereas the symmetry principle explains the dynamic logic of its dimensions. In contrast to Mattessich, it is observed here that when we subscribe, and we do, to his notion that the accounting system is inherently symmetrical, which is about the 'conservation of capital' in its *manifestation*, as assets, and in terms of its *force*, as claims, that this symmetry is not limited to the expression of the social relationship between assets and claims (Mattessich 1995, 66). The symmetry principle is clear and present for transactions that are related to only one accounting dimension. Intra-dimensional transactions are as symmetrical as inter-dimensional ones. This is neither a superficial conclusion nor just an academic appreciation of the foundational properties of the ac-

counting equation itself. The symmetry principle is an important guide, if not a golden rule, for the design of a dynamic accounting model. It explains that any transaction must be accounted for symmetrically. The debit and credit entries are only the ends of the vector that is caused by a business transaction. Thus, when made dynamic in a System Dynamics model, the accounting system will have to adhere to the structural design that necessarily follows from the symmetry principle.

10 The change principle

The principle of change, or gain or loss, is, according to Mattessich, closely related to the symmetry principle. In his words: ‘If there is an ownership claim on an asset, and if its relevant attribute does change, then this change is also reflected in the corresponding ownership claim’. This definition ties in closely with the more general principle of conservation that Mattessich sees as fundamental to accounting: ‘Accounting is not merely concerned with physical transfers but also with the change of wealth over time. Just as the conservation laws of physical sciences are giving account of what happened to the input of energy and matter, momentum, spin, and so on, in terms of the corresponding output; so accounting tries to give account in terms of commodity utilization and financing. For now, in this paper, only the fact is pointed at that the change principle, like the symmetry principle, is not limited to inter-dimensional phenomena. A *change* of value of an asset account, or its symmetrical partner found in a claim account, is to be recorded by the two accounting dimensions at either end of the vector that it causes. Such inter-dimensional transaction meets the definition of Mattessich perfectly. It was extensively discussed in the previous paragraph, and depicted in Figure 6 and Figure 7, that there is indeed a conceptual difference between a transaction that is inter-dimensional or intra-dimensional. But there exist not a methodological, logical or technical difference between these types of transaction. Like with the symmetry principle this applies also for the change principle. Again, the explanation is found in the working of the input – output principle. It is the vector, triggered by any transaction, which links accounts inter-dimensionally or intra-dimensionally and determines the logic of the debit and the credit. Next, that same vector transfers an amount between the accounts. This transfer is the change of both accounts. For the account that is debited, the change is positive because it is added, whereas for the account that is credited, the change is negative because it is subtracted. This is so because the debit is an inflow and the credit is an outflow, as was explained above (Table 1, Figure 2). This observation should not surprise the System Dynamic modeler. It is perfectly natural to understand that an outflow depletes a stock and thus a negative value is computed (assuming the stock start value was 0). However, for accountants this might be something of a surprise.

Again, like with the symmetry principle, it is claimed that the input – output principle determines the structure of the accounting model. The accounts are the bricks whereas the vectors are the mortar. But, instead of a firm wall made of bricks with solidified mortar, there is no accounting wall in this model because the vectors are dynamic. They are drawn between accounts, nominal and real ones, inter-dimensional or intra-dimensional, on a transaction basis. That is a dynamic phenomenon observable only in time, subject of the final paragraphs of this paper.

11 Time and the accounting system

Is time a dimension of the financial accounting system? Ijiri notes that ‘one common property of

accounting measurements is that they are all functions of time and are recorded and reported as time series data, $x(1), x(2), \dots, x(t)$, rather than as isolated measurements at a single point in time' (Ijiri 1989, 1996). Mattessich introduces as his second basic assumption the '*Time measure*: there is a sequence of relatively small time intervals (e.g., dates) that can be ordered, added (to longer time periods), measured, differentiated, and so on, by means of a number system.'⁶ Both Ijiri and Mattessich, however, tend to see time as a dimension external to the financial accounting system. It is introduced to it by means of the measurements of accounting transactions propagated along a time-axis. However, no dynamic model will compute without a step-based time difference and that difference is there from the moment the first relation is defined in the model. Time is a necessary and intrinsic property of the financial accounting system because it is automatically propagated by its structure. This means that any measurement is related to a single point in time but only in subordination of the relation between accounts as set by the transaction vector (Figure 6, Figure 7). Whether or not we treat such measurements in isolation or as time series data depends on the purpose of the model and the nature of the transaction. Regular bookkeeping will account for each transaction individually (e.g., a sale), but the mutation of most nominal accounts does occur continuously (e.g., depreciation of fixed assets or interest charges of a loan). Therefore, also such transactions have to be accounted, in a dynamic model, individually and continuously on a daily basis.

Beside the notion of the two dimensions of the financial accounting system, for *capital accounting* these are claims and assets whereas for *wealth accounting* these are owners' equity and net assets, we have to include model sub-structures for the real, or permanent, accounts and for the nominal, or temporary, accounts. This is necessary to be able to account for period results (Table 5) that are most peculiar to models based on the premises of System Dynamics. The data of nominal accounts accrues then with that of the real accounts at the start of the accounting period (change). These structures have to be integrated into the super-structure of the dynamic financial accounting model. Both sub- and super-structures are input – output models when made dynamic. As to enable the integration of such accounting models that necessarily compute on a daily basis and could be seen as discrete state space models ($\Delta\tau=1$) with 'classic' System Dynamics models that regularly compute with smaller or larger step changes ($\Delta\tau\neq 1$) consolidation is required. The accounting model expects that its flows are daily based. Therefore, when a System Dynamics model is used for policy analysis that has a different step value, then transaction values have to be accrued or differentiated to a single step change value for proper accounting purposes. This requirement also underlines again the need for a technical solution to the temporal inequality between the stock and flow values of a System Dynamics model.

12 The dynamic accounting model: an example

The solution to the temporal inequality between the stock and flow values of a System Dynamics model was already discussed and demonstrated above (Table 1 & Figure 2 , Figure 5). What is required for dynamic financial or management accounting is the inclusion of a report variable that sums the mutation (inflow – outflow) of an account (stock) with that account. In System Dynamics terminology: an auxiliary that sums the value of a stock and the rates that change that stock at τ . Or, in temporal terms, the stock value of the next time step is computed. It should be noted, most carefully, that this approach is to be applied only within the accounting model structure for reporting

purposes. It enables the availability of the values of both accounting dimensions at each (discrete) time step. It is noted that such auxiliary values can *never* be used to compute rate values of related accounts or any other account. That would instantaneously cause a referential integrity error.

An example financial accounting model was developed with Powersim™ Constructor (Figure 9, Figure 10) and with a prototype dynamic accounting simulator (Figure 11, Figure 12). Kefford (1995) discusses the structure of such a model very similar to the business cycle model presented

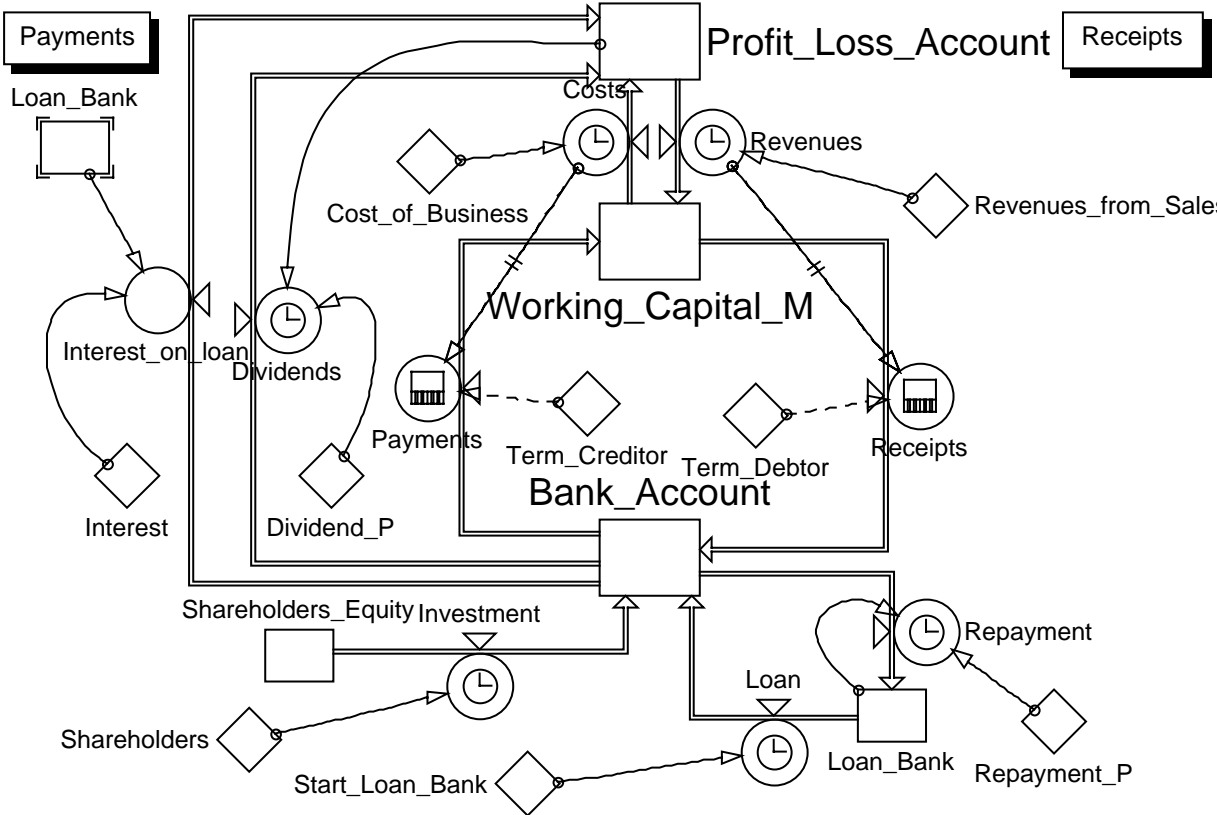


Figure 9 Example financial model modeled with Powersim™ Constructor version 2.51. (See DAM_F11.SIM).

Day	Revenues	Costs	Payments	Receipts	Working_Capital_Now	Working_Capital_Last
0	0.00	0.00	0.00	0.00	0.00	0.00
1	120.00	100.00	0.00	0.00	20.00	0.00
2	120.00	100.00	0.00	0.00	40.00	20.00
3	120.00	100.00	0.00	0.00	60.00	40.00
4	120.00	100.00	0.00	0.00	80.00	60.00
5	120.00	100.00	0.00	0.00	100.00	80.00
6	0.00	0.00	100.00	0.00	200.00	100.00
7	0.00	0.00	100.00	0.00	300.00	200.00
8	120.00	100.00	100.00	120.00	300.00	300.00
9	120.00	100.00	100.00	120.00	300.00	300.00
10	120.00	100.00	100.00	120.00	300.00	300.00
11	120.00	100.00	0.00	120.00	200.00	300.00
12	120.00	100.00	0.00	120.00	100.00	200.00
13	0.00	0.00	100.00	0.00	200.00	100.00
14	0.00	0.00	100.00	0.00	300.00	200.00
15	120.00	100.00	100.00	120.00	300.00	300.00

Figure 10 Result data of Figure 9. Note that time step 0 is required for proper initialization of the model as to enable the computation of financial accounts so that they can report the

above (Figure 3). As can be expected the basic structure of the financial accounting model is that of a closed system (Figure 9). A more sophisticated design will maintain this structure because the financial accounting model has to be understood as a recording system primarily albeit with its own

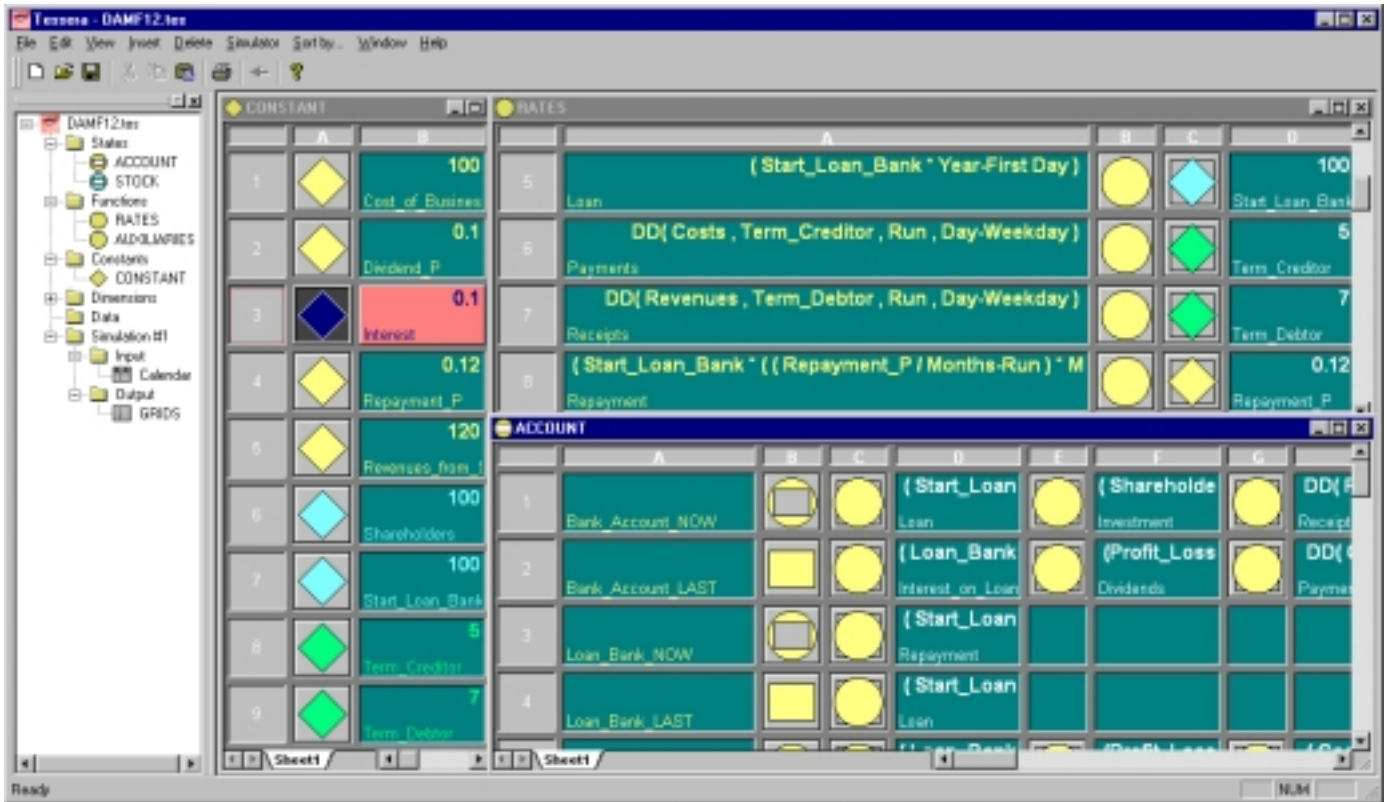


Figure 11 Example financial model modeled with a prototype accounting simulator identical to Figure 9 but with calendar based business dynamics and reporting adhering to accounting requirements.

Day	Revenues	Costs	Payments	Receipts	Working_Capitalflow
Tuesday	120	100	0	0	20
Wednesday	120	100	0	0	40
Thursday	120	100	0	0	60
Friday	120	100	0	0	80
Saturday	0	0	0	0	80
Sunday	0	0	0	0	80
Monday	120	100	200	0	300
Tuesday	120	100	100	120	300
Wednesday	120	100	100	120	300
Thursday	120	100	0	120	200
Friday	120	100	0	120	100
Saturday	0	0	0	0	100
Sunday	0	0	0	0	100
Monday	120	100	300	120	300
Tuesday	120	100	100	120	300

Date	Revenues	Costs	Payments	Receipts	Working_Capitalflow
2002/01/01	120	100	0	0	20
2002/01/02	120	100	0	0	40
2002/01/03	120	100	0	0	60
2002/01/04	120	100	0	0	80
2002/01/05	0	0	0	0	80
2002/01/06	0	0	0	0	80
2002/01/07	120	100	200	0	300
2002/01/08	120	100	100	120	300
2002/01/09	120	100	100	120	300
2002/01/10	120	100	0	120	200
2002/01/11	120	100	0	120	100
2002/01/12	0	0	0	0	100
2002/01/13	0	0	0	0	100
2002/01/14	120	100	300	120	300
2002/01/15	120	100	100	120	300

Figure 12 Result data of Figure 11. Note that initialization of the model and reporting follows the computation algorithm of calendar based business dynamics adhering to accounting requirements. Additional functionality is enabled so that the model is sensitive to the kind of day. E.g. Payments and receipts will not occur during the weekend although they are due. Hence, at the next Monday lapsed amounts fall free.

dynamic properties. There are many sources of dynamic behavior within the accounting system, e.g. interest and depreciation calculations. But, the most important source is the interface between it and the ‘real world’ through the natural calendar. All accounting is measured by discrete events that occur, like closures and disclosures accounted at the end of the year. But even the simple phenomenon of payments or receipts that are due as set by a given term (Figure 9, Figure 11) in relation to the natural calendar cause impulses within the accounting model (Figure 12). Such dynamics cannot be forecasted with larger models and perhaps not even with simple ones. It is here where the inclusion of a temporally balanced dynamic accounting model into a System Dynamics model for policy analysis can offer a new vista to explore.

13 Concluding remarks

The purpose of this paper was to discuss foundational issues that follow from the proper interpretation of the accounting equation, its dimensions and its translation into a dynamic system of accounts. Central is the understanding that any organization is a *system*, a manifestation in the real world of living entities, people organized in purposeful teams, that maintains itself, furthers relations with other entities, and strives to meet its self-determined goals. Because an organization acts purposeful it should be able to rationalize its objectives and the procedures to meet them. Hence, the organization builds and maintains systems to manage itself. The management control system is such a system. It enables people take corrective action when an internal or external disturbance occurs through an agreed set of norms, rules and measures. System theory can offer valuable insights to develop a corporate language and measurement system. Because the universal imperatives of the management control system are truly systemic, both its methodological and technical underpinning has to be based on systems theory.

The purpose of dynamic business simulation is to enable, if not empower, the controller and his or her colleagues in other financial and managerial disciplines to extend the use of the financial accounting system for management control purposes. Two reasons were given why this should be a

good idea. The first reason is that the financial accounting system is used and accepted globally. That is a remarkable feat and has great value because it is in itself a reference, a benchmark, for any effort to introduce a new methodology, technology or praxiology for management control and accounting. The second reason is that the financial accounting system is a system in its own right. It is not made to measure money or time but to express the relation between the firm and entities present in its environment in terms of business events documented in economic terms. It is a holistic image, albeit filtered, of the operation and state of an organization. Nevertheless, it also has serious limitations. The most important is that it only includes ex post information, which means that any information contained in it is about facts that occurred in the past up to the present. Very little, if not anything, can be said about the future of the company using the accounting system. Another limitation is that all information is financial. Solutions to both limitations are offered in the form of a multitude of tools, some solely financial, others more focused on management accounting to plan for operations. What is missing is a predictable model that is based on the foundational principles of the financial accounting system but ties into the requirements of management control. In other words, an extension of accounting system was sought so that ex ante accounting becomes feasible.

The duality of the double-entry accounting system is found in the notion of *cause* and *effect*, or 'simply that of change in something brought about by the observed event or occurrence'. This can be recognized in every foundational aspect of the accounting system, its objective, structure, logic and operation. Mattessich formulated three foundational principles that are the framework that drives the synthesis, the structure and the logic of financial accounting system. It was argued in this paper that of these three the input – output principle is most fundamental. The symmetry principle and the change principle explain the existence of the two accounting dimensions for capital or wealth accounting for the fact that they express a social reality. It is the relationship between the owner and the user of capital, or wealth, that determines the logic of the accounting system. A vector between two accounts, mathematically expresses a transaction between the two accounting dimensions. Thus, any change, any gain or loss, of an asset will symmetrically be reflected in the corresponding ownership account. But, it was also demonstrated in this paper that the symmetry principle and the change principle apply just as well to intra-dimensional transactions, i.e. those that have an impact only on assets, or only on claims or owners' equity. Furthermore, the same rules apply to the treatment of nominal accounts during period accounting and their closure. Although only a small number of simple examples were provided, it can be safely argued that each and every transaction that enters into the double-entry accounting system is accounted symmetrically and that its related amount does change the state of the accounts that are linked by its vector. Most importantly, it was demonstrated that any such vector has a debit and a credit (with equal sum values) and thus any transaction triggers an inflow and an outflow. More poignantly put, any transaction is both inflow to, and outflow from an account. Therefore, the conclusion is that the input – output principle necessarily determines the structure of the accounting model, whereas the symmetry principle and the change principle drive its logic. These foundational principles inescapably determine the design of a *dynamic* accounting model.

¹ This process is called *transformation* and involves the operation of a function on all data of the set or model. An alternative method is *transgeneration* with a logarithmical or asymptotic logarithmical function to get a homogenous dimension, which also resolves the problem of large variances of magnitude possibly present in data (See Lewi, 1989, 23-39).

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- ² From a management accounting perspective this is also a limitation because it can be beneficial to account for business events with more than one dimension. For example, a parameter to express the risk associated with the event or the probability of it to occur.
- ³ It is a technical matter how a 'stock' value is computed in a dynamic simulator. It could be a vector of memory addresses that each save the flow value of a time step on which a sum operator is computing, identical to what is done in the tabular format of a spreadsheet column, like in Table 1. Alternatively, an algorithm can sum each time step flow value to the accumulated value of previous flows and save that in one memory address. Nevertheless, the sum computation can only be done in both cases after the flow value has been computed.
- ⁴ Page 67, italics are his.
- ⁵ In this example the amount exchanged of coins or notes with a certain face value is actually more appropriate. But perhaps overstates the argument and confuses it.
- ⁶ Mattessich (1995) ref. 143, 84, italics are his. Errors can occur when dates are added to 'longer time periods', e.g., when ratios are calculated from the resulting values. Differences occur because of the inequality of period sums are usually not taken into account, e.g., months have a different total number of days.
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