

A System Dynamic Approach to A Chaotic Market Economy

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1 An Exchange Economy

A couple of years ago, I happened to encounter a chaotic price movement in a traditional price adjustment process when I was doing a simulation of a market economy [1, 2]. To be more specific, market prices turned out to be chaotic for some values of an adjustment coefficient and consumers' preferences. This simulation was done by using S language in a UNIX environment. Due to this environment, the analysis turned out to be hard for introductory students of microeconomics to play with.

The simulation model of the economy, however, is very simple and could be easily understood by most introductory students. It consists of two consumers who bring their products (called initial endowment in economic jargon) to a market for exchange. At the market, an auctioneer quotes product prices so that consumers can figure out the market values of their initial endowment. With these market values as their tentative incomes, consumers wish to purchase the products in the market so as to maximize their utility according to their tastes. If the total demand of the consumers were not equal to the total supply of their initial endowment, the auctioneer quotes another price. The rule of a new price quotation is to raise a market price if demand is greater than supply, and vice versa. This price adjustment process continues until demand and supply become equal

and a market equilibrium is attained. It has been generally believed in microeconomic theory that, under some reasonable conditions, the auctioneer can eventually attain a market equilibrium by changing prices little by little. In this sense, market prices become stable. The above simulation turned out to be contrary to this well-believed stability of price adjustment. Specifically, price movement turned out to be chaotic.

2 A System Dynamic Approach

About a year ago, I had a chance to use the Stella II software¹ (which is now upgraded to version 4.0 and called Stella Research), through which I was also guided to a system dynamic methodology of analysis. I was very amazed to find how easy it is to solve systems of difference and differential equations numerically with this software. Not only that, I was also impressed by a wholistic approach of the system dynamics. I started modeling the above market economy in order to see whether chaotic price movements could be more easily revealed so that introductory students can understand the mechanism of price adjustment in a market economy. One of the nuisances of the software was its incapability to handle vectors and arrays. Accordingly, a mapping diagram of the economy becomes very complicated as the number of consumers and commodities increases. Fortunately, its latest version 4.0 solved this problem, and a system dynamic diagram of the market economy is now compactly drawn in Figure 1.

3 Chaotic Price Movements

With this new Stella version I'm now in a position to show a price adjustment mechanism of a market economy in a very neat fashion. Due to a space limitation of the paper, only a portion of price adjustment mechanism in the Stella model is copied as follows:

$$\text{Prices}[2](t) = \text{Prices}[2](t - dt) + (\text{Price_Change}[2]) * dt$$

¹Stella is a trademark of High Performance Systems, Inc.

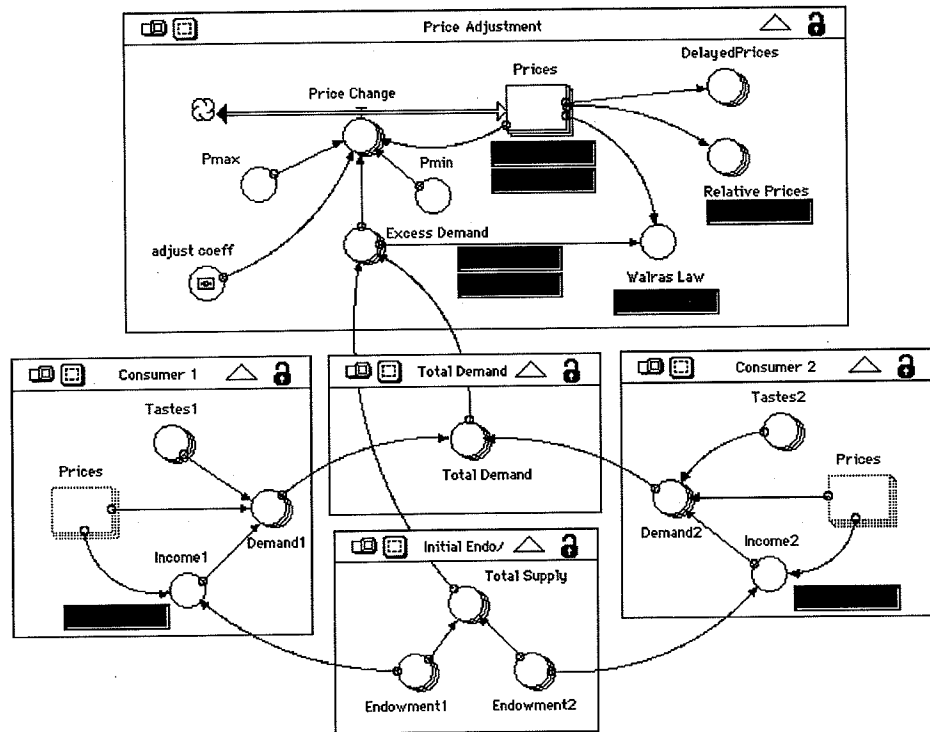


Figure 1: A System Dynamic Diagram of the Market Economy

$$\text{Price_Change}[2] = \text{MIN}(\text{Pmax}, \text{MAX}(\text{Prices}[2] + \text{adjust_coeff} * \text{Excess_Demand}[2], \text{Pmin})) - \text{Prices}[2]$$

$$\text{Excess_Demand}[2] = \text{Total_Demand}[2] - \text{Total_Supply}[2]$$

$$\text{Pmax} = 100$$

$$\text{Pmin} = .01$$

When the value of an adjustment coefficient is very close to 0, the adjustment process tends to be continuous and an equilibrium is attained at $\text{Prices}[2] = 0.7619$; that is, the process becomes globally stable. As the value of coefficient increases, however, the

behavior of prices turns out to be unstable. At the value of 0.146, the process produces a clear bifurcation, or an oscillation of prices in period 2.

This confirms the existence of a critical value between local stability and instability calculated above under a linear approximation, that is, $\lambda = 0.1451247$. Beyond this critical point, a bifurcation or an oscillation of period 2 clearly emerges, and an equilibrium price becomes globally unstable. The adjustment process in this way continues to create new bifurcation or price oscillations of period $2n$, $n = 1, 2, \dots$ until it becomes totally chaotic around $\lambda = 0.2$. (λ here denotes a value of the adjustment coefficient.) [1, p.130]

By assigning the adjustment coefficient to the Slider Input Device of the Stella software, we can easily select different values of coefficient by a mouse, and observe these simulation processes visually. Figures 2 and 3 show two typical cases of these price adjustment processes; that is, stable and chaotic processes.

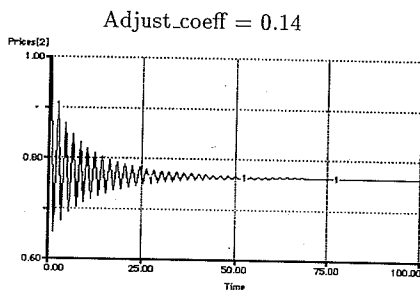


Figure 2: Price Stability

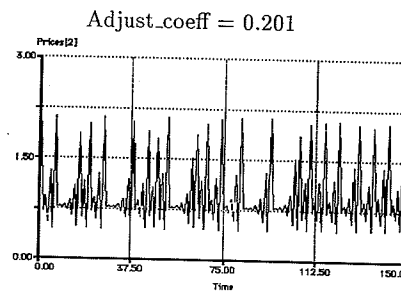


Figure 3: Chaotic Price

4 An Extension to n Commodities

New feature of Arrays in the latest version of the Stella software allows us to increase the number of commodities in the economy, as well as the number of consumers, without changing a basic structure of the economy's diagram. When we handle a general case

of n commodities, $n = 1, 2, \dots$, the mathematical formula becomes very complicated so that an essential structure of the economy is easily lost in the midst of mathematical complication, though it remains the same. In this sense, a system dynamic approach with Arrays is very heuristic for learning fundamental workings of the market economy. Figures 4 and 5 illustrate a chaotic price movement of Prices[2] and Prices[3] at the adjustment coefficient value of 0.15, one with connected prices and the other without connected lines, for the market economy extended to 3 commodities.

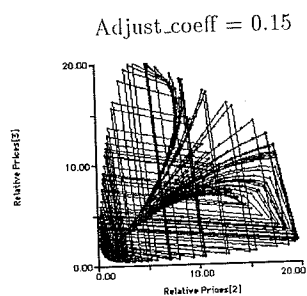


Figure 4: Chaotic Prices (Connected)

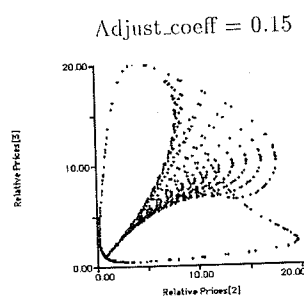


Figure 5: Chaotic Prices

References

- [1] Kaoru Yamaguchi. *Simulations of Equilibria and Chaotic Price Movement in an Exchange Economy - Using S/UNIX Environment*. Bulletin of Nagoya University of Commerce, Vol. 38, No. 1, pp.101-138, September 1993.
- [2] Kaoru Yamaguchi. *Simulations of Equilibria and Chaotic Price Movement: A Sited*. Journal of Economics and Management, Nagoya University of Commerce, Vol. 39, No. 1, pp. 57-69, July 1994.