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STRUCTURE ANALYSIS OF COMPLEX SYSTEMS  
AND ITS APPLICATION  
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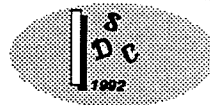
ABSTRACT

With the continuing rise of the complexity of objects, it becomes more and more important and urgent to study the complexity of systems. However we still feel difficult in treating large scale and complex systems in technique because of the high order, multiloop and nonlinearity. In the light of synergetics, a new method of structure analysis is developed. It may have not only the theoretical, but also the practical meanings in the parameter estimations, system optimization, model simplification, dominant loop determination, policy tests, etc.

THE PROBLEM

On principle system dynamics can be used to deal with a lot of relatively complex problems. It has achieved great successes in the fields of enterprise management, city planning, global development, etc. However, the traditional method of system dynamics has some limitations both in theory and practice. It excessively rely upon the personal knowledge and experience in the process of modelling and upon the trail and error of computer simulations in analysis techniques. Thus the whole analysis becomes indisorder and time-consuming and the key problems can not be effectively grasped. It may especially exhibit powerless when the system under analysis possesses complex characteristics such as bifurcation, catastroph, chaos, etc. Therefor it is extremely important to explor a new approach more effectively to analysis complex systems.

System dynamics and synergetics have their own strong points respectively in dealing with the problems of complex system. System dynamics is a kind of "structure form" method. It lays stress on the relationships between loops and on the effects of structures on behaviours. And synergetics is a



kind of "function form" method. It places emphasis on the relationships between variables and on the effects of parameters on behaviours. Now we combine the strong points of both system dynamics and synergetics and produce a new method of structure analysis to study the characteristics of complex system.

We all know that a complex system is usually combined of several subsystems which lay at different levels. The behaviour of a complex system is determined by both the features of the subsystems and the interactions between them. According to the view of the system grades, we first break the system down into some individual feedback loops, next determine the effective intensity for each loop by calculating their loop polarities, loop gains, loop eigenvalues and their distributions of the linear parts. Without simulation and approximation, the new method can supply the exhaustive information of complex systems including basic behaviour modes, dominant loops, key points of behaviour changes, leverage points of policies, etc.

#### STRUCTURE ANALYSIS

The approach of structure analysis mainly contains four relative parts: mathematical description, boundary property, loop polarity and loop gain, and loop eigenvalues and their distributions.

##### 1. Mathematical Description

According to the characteristics of system dynamics model, the following equations can be drawn out:

$$dL/dt = P * R, \quad R = F(A), \quad A = G(A, L) \quad (1)$$

where  $dL/dt$  stands for the vector of net rates,  $R$  for the vector of rates,  $A$  for the vector of auxiliaries,  $L$  for the vector of states,  $P$  for the transmission matrix (normally constant),  $F$  and  $G$  respectively for the vector functions of relative variables. Acturally the vector functions should involve exogenous variables and control parameters. They are not included in equations (1), because environment factors almost have nothing to do with the inherent features of loops. In addition, for the convenience of expression the time variable is also not put in. From equation (1), after some proper arrangements, the partial derivative of vector  $dL/dt$  with respect to the transformed state vector  $L$  is calculated as follows:

$$\partial(dL/dt) / \partial L^T = P * \partial F / \partial A^T * (I - \partial G / \partial A^T)^{-1} * \partial G / \partial L^T = W(L) \quad (2)$$

where  $I$  stands for unit matrix and the inverse exists, otherwise the system would degenerate into an open form. The system matrix of  $W(L)$  reflects the elasticity of states.



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$W_{i,j}$  stands for the elasticity of state  $i$  with respect to state  $j$ . Clearly, the factors of the sensitivity are the functions of states and time, usually not constants except for linear systems. Further more what in the brackets of (2) is a diagonal matrix if the elements of auxiliary vector  $A$  are ranked by the rule:

$$A_i = G_i(A_{i+1} \dots A_m, L_1 \dots L_n), \quad i=1 \dots m \quad (3)$$

where  $m$  and  $n$  are the numbers of auxiliaries and states respectively.

## 2. Boundary Property

A complex system always contains some positive loops and some negative loops. These two kinds of loops connected and interacted each other. When positive loops are dominant, the system becomes boundless. Otherwise, when negative loops are dominant, the system becomes bounded. Lyapunov direct method is usually used to determine the boundary property of nonlinear systems. That is, a constant positive function of  $V(L,t)$  is first defined. And then the rate of  $V(L,t)$  is calculated. If  $(dV/dt) \geq 0$ , the system becomes boundless. And if  $(dV/dt) < 0$ , the system becomes bounded.

Although some of approaches, such as Lyapunov index, fractal dimension and so on, can be used for studying complex features of systems, a very simple and direct method can be introduced to do the same task for a bounded system. Actually a bounded nonlinear system, an unautonomous system or a more than three-ordered autonomous system, may give rise to a limit circle behaviour when all the equilibrium points of the bounded system become unstable, because the trajectory of the system can neither stop at some equilibrium points nor go so far as to exceed the system boundary. Further more, under some conditions the limit circle may lose its stability and the behaviours of bifurcation and chaos may appear.

## 3. Loop Polarity and Loop Gain

The definitions about loop gains and loop polarities are given below:

Def.1: A loop involving one state variable is called a one-stated loop;  $G_i = W_{i,i}$  is its loop gain, and  $SIGN(G_i)$  is its loop polarity.

Def.2: A loop involving two state variable is called a two-stated loop;  $G_{i,j} = W_{i,j} * W_{j,i}$  is its loop gain, and  $SIGN(G_{i,j})$  is its loop polarity.

By the same way  $n$ -stated loop gain and loop polarity can be



defined. All the definitions are identical with the common used concepts. Loop polarity reflects the developing direction of loop behaviours. positive polarity means growth and negative polarity means being attracted. Loop gain reflects the effective intensity of loops. The bigger the loop gain, the stronger the effect of the loop to the system.

#### 4. Loop Eigenvalues and their Distributions

Loop eigenvalues concern the linearization in a small area of the equilibrium points of the system. In equations (1) we take  $dL/dt=0$ , then

$$P * F(Ae) = 0, \quad A = G(Ae, Le), \quad W(Le) = \text{constant} \quad (4)$$

where  $Le$  and  $Ae$  stand for the state vector and auxiliary vector at the equilibrium points respectively.

Clearly, for an one-stated loop the loop eigenvalue is equivalent to its loop gain. For an n-state the loop the loop eigenvalue equation can be written as:

$$(LE)^n - G(n) = 0 \quad (5)$$

where  $G(n)$  stands for the loop gain of the n-stated loop. Positive real part of loop eigenvalues contributes growing modes; negative real part contributes attractive modes; and imaginary part is responsible for oscillatory modes. For a well-structured system different loop eigenvalues may appear in a same loop. Then various complex behaviour modes take place including the shift of loop polarities, catastrophic changes, chaos and self-organisations, etc.

In addition, loop gains determine not only loop eigenvalues but also system eigenvalues and their distributions. Obviously for an one-ordered system the system eigenvalue is identical with the loop eigenvalue. For a two-ordered system the system eigenvalue equation is:

$$(E - G_1) * (E - G_2) - G_{1,2} = 0 \quad (6)$$

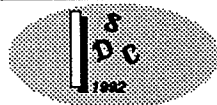
where  $E$  stands for the system eigenvalue. For a three-ordered system we get:

$$\prod_{i=1}^3 (E - G_i) - \sum_{i,j,k=1}^3 G_{i,j,k} (E - G_k) = 0, \quad i < j, i \neq k \neq j \quad (7)$$

By the same way we can get the system eigenvalue equations for higher order systems. In practice, it is convenient that a complex system is first decomposed into some lower order subsystems on the principle of system grades, and then the eigenvalue descriptions of the system can be analysed.

#### APPLICATION

Now we consider a simplified model of the economic long wave (see Appendix). It was developed by J. D. Sterman and analysed by E. Mosekilde, etc. Here we study it by a new way of structure analysis. Figure 1. shows its system



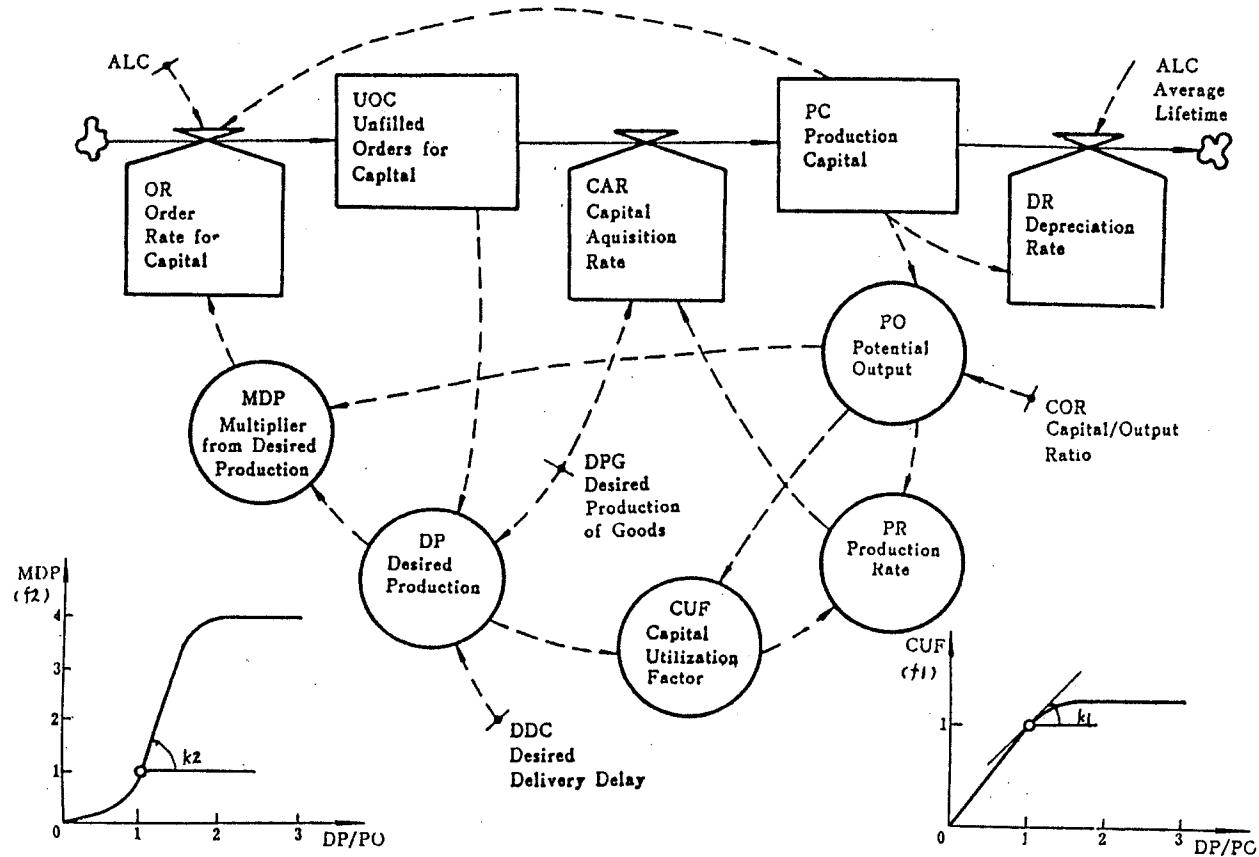
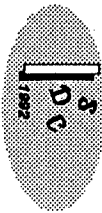


Figure 1. System Dynamics flow-diagram for a simplified oneselector Kondratieff-wave model

dynamics flow diagram. This is a typical nonlinear system with two levels and two table functions. First of all, we write down the mathematical expression of the model:

$$\begin{bmatrix} dL1/dt \\ dL2/dt \end{bmatrix} = \begin{bmatrix} dpc/dt \\ dUOC/dt \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} CAR \\ DR \\ OR \end{bmatrix} \quad (8a)$$

$$\begin{bmatrix} CAR \\ DR \\ OR \end{bmatrix} = \begin{bmatrix} PR-DFG \\ PC/ALC \\ PC/ALC*MDP \end{bmatrix} \quad (8b)$$

$$\begin{bmatrix} PR \\ MDP \end{bmatrix} = \begin{bmatrix} PC/COR*f1(z) \\ f2(z) \end{bmatrix} \quad (8c)$$

$$z = (COR/PC)*(UOC/DDC+DFG) \quad (8d)$$

where  $f_1$  and  $f_2$  are table functions (see Figure 1.). The system is bounded. In fact we note:

$$dPC/dt + dUOC/dt = PC/ALC*(f_2-1), \quad 0 \leq f_2 \leq 4 \quad (9)$$

The following function of  $V$  is taken as a Lyapunov function:

$$V = 0.5*(f_b-1)^2 * PC^2 / ALC^2 \geq 0.5*(dPC/dt + dUOC/dt)^2 \geq 0 \quad (10)$$

where  $f_b$  stands for the boundary of  $f_2$ . Then the rate of  $V$  becomes:

$$dV/dt = (f_b-1)^2 / ALC^2 * PC * [PC*(f_1/COR - 1/ALC) - DFG] \quad (11)$$

It can be seen from Figure 1. that if  $PC \rightarrow \infty$ , Then  $f_1 \rightarrow 0$ . In fact if  $PC$  grows so big as to make  $f_1 < COR/ALC$ , the coefficient sign of  $PC^2$  becomes negative. So the tracks of the system can not exceed a limit range at any time.

The partial derivative of  $dL/dt$  with respect of  $L^T$ , that is the matrix of  $W(L)$ , can be calculated as follows:

$$W_{1,1} = f_1/COR - 1/ALC - k_1/PC * (UOC/DDC + DFG) \quad (12)$$

$$W_{1,2} = k_1/DDC$$

$$W_{2,1} = f_2/ALC - f_1/COR - COR/PC * (UOC/DDC + DFG) * (k_2/ALC - k_1/COR)$$

$$W_{2,2} = COR/DDC * (k_2/ALC - k_1/COR)$$

where  $k_1$  and  $k_2$  are the slopes of  $f_1$  and  $f_2$  respectively. There are two one-stated loops and one two-stated loop inductively in the system. Their loop gains and loop polarities are:

$$G_1 = W_{1,1}, \text{ SIGN}(G_1) = \text{the polarity of the loop;} \quad (13)$$

$$G_2 = W_{2,2}, \text{ SIGN}(G_2) = \text{the polarity of the loop;} \quad (13)$$

$$G_{1,2} = W_{1,2} * W_{2,1}, \text{ SIGN}(G_{1,2}) = \text{the polarity of the loop.}$$

Obviously the loop gains and loop polarities may vary with the changes of states, table functions and parameters, and the dominant loop may shift from one to another under different conditions. The system possesses an unit unnormal equilibrium point:

$$PC_e = DFG * COR * ALC / (ALC - COR) \quad (14)$$

$$UOC_e = DDC * DFG * COR / (ALC - COR)$$

$$f_{1e} = f_{2e} = z_e = 1$$

At the equilibrium point,  $k_1 = 1/4$ ,  $k_2 = 3$ , and  $W(L_e) = \{W_{ei,j}\}$  is a constant matrix. That is:



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$$We_{1,1}=0.75/COR-1/ALC \quad (15)$$

$$We_{1,2}=0.25/DDC$$

$$We_{2,1}=-2/ALC-0.75/COR$$

$$We_{2,2}=COR/DDC*(3/ALC-0.25/COR)$$

We can get the system eigenvalue equation:

$$F(E)=(E-G_1)*(E-G_2)-G_{1,2}=E +b*E+c=0 \quad (16)$$

where

$$b=1/ALC-0.75/COR-COR/DDC*(3/ALC-0.25/COR)$$

$$c=0.25/DDC*(0.75/COR+2/ALC)$$

Now we first take

$$ALC=20, DDC=3, COR=6 \quad (17)$$

The loop gains, loop polarities and loop eigenvalues at the equilibrium point can be calculated as follows:

$$G_1=LE_1=-0.008<0 \quad (18)$$

$$G_2=LE_2=0.286>0$$

$$G_{1,2}=-0.012<0$$

$$LE_{1,2}=\pm 0.11i$$

And the two system eigenvalues are:

$$E=0.042\pm 0.187i \quad (19)$$

The equilibrium point is unsatable. The tracks of states show limit circle behaviour because the system is bounded. Frome this expriment we can conclude the second one-stated loop is dominant. Its loop gain is much bigger than the others.

Next we take COR=4 and the other parameters remain their original values as obviusse. We can get:

$$G_1=LE_1=0.0125>0 \quad (20)$$

$$G_2=LE_2=0.179>0$$

$$G_{1,2}=-0.0135<0$$

$$LE_{1,2}=\pm 0.116i$$

And the two system eigenvalues are:

$$E=-0.019\pm 0.2i$$

The equilibrium point is a stable focus. The tracks of states may convergent to it in a spiral way. From this test we can know the two-stated loop is dominant, though the polarity of the other two-stated loops is positive. The conter-intuitive properties of complex systems appear here. Practically there existssa critical value of COR between 4 and 6 at which the behaviour of the system may bifurcate from a damped oscilation to a limit circle. That means the real partof the system eigenvalues of Re(E) changes from negative into positive.

Further more we explor the chaotic property of the system behaviour. Suppose DPG (desired production of goods) has a sinusoidal fluction instead of a constant:

$$DPG=1+AMP*\cos(6.283*TIME.K/PER) \quad (21)$$

here PER is the period of the exgenous excitation and AMP is the amplitude of the excitation. At the equilibrium point the

system matrix of  $WS(L_e)$  can be rewritten as follows: (22)

$$WS(L_e) = \begin{bmatrix} We_{1,1} & We_{1,2} & -1 & 0 \\ We_{2,1} & We_{2,2} & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & SS & 0 \end{bmatrix}$$

where  $L_e = [PC \quad UOC \quad DPG \quad RDPG]$ ,  $SS = -(6.283/PER)^2$ , and RDPG stands for the rate of DPG. Then we can get the system eigenvalue equation:

$$FS(E) = [E^2 + (6.283/PER)^2] * F(E) = 0 \quad (23)$$

That means the eigenvalues of the sinusoidal excited system remain the original eigenvalues and at the same time increase a pair of new net imaginary eigenvalues of  $\pm 6.283/PERi$ . If the equilibrium point loses its stability under some conditions, the behaviour of bifurcation and chaos may occur in this bounded system (see E. Mosekilde 1986).

### CONCLUSION

Traditional system dynamics has some limitations in dealing with complex problems of systems. It is ineffective and time-consuming during the analysis because of its excessive dependence upon personal knowledge and the trail and error in computer simulations. A new approach of structure analysis is developed in this paper. It draws the cream from system dynamics and synergetics. By the view of system grades a complex system can be broken down into a series of one-stated loops, two-stated loops, etc. Then the contributions of each loop to the system can be comprehensively analysed by calculating the loop gains, loop polarities, and loop eigenvalues and their distribution. The new approach can supply an effective way to study the structural origins of behaviour in complex systems, to determine the leverage points in policy tests, and also do good to model simplification. It need not simulations and is low cost. The practical tests show that it is an useful tool to explore the complexities of systems.

### REFERENCES

- Brons, M. and J. Sturis. 1991. Local and Global Bifurcations in a Model of the Economic Long Wave. *System Dynamics Review*, Vol.7, No.1, Winter:41-60.
- Forrester, N. B. 1987. *Theory and application of System Dynamics*. Beijing: New Times. 112-122.
- Hahen, H. 1989. *Advanced Synergetics*. Beijing: Science (in Chinese).





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Mosekilde, E. and S. Rasmussen. 1987. Bifurcations and Chaitic Behaviour in a Simple Model of the Economic Long Wave. Theory and Application of System Dynamics. Beijing: New Times. 367-383.

Steman, J. D. 1989. Deterministic Chaos in an Experimental Economic System. Journal of Economic Behaviour and Organization. 12:1-28.

Thomsen J. S., E. Mosekilde and J. D. Sterman. 1990. Mode-Locking and Chaos in a Periodically Driven Model of the Economic Long wave. Proceedings of System Dynamics Conference. 3:1137-1151.

Toro, M. and J. Aracil. 1988. Qualitative Analysis of System Dynamics Ecological Models. System Dynamics Review 4:56-80.

#### APPENDIX

The complete DYNAMO program for the simplified model of the economic long wave:

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* ONE SECTOR KONDRATIEFF MODEL
L PC.K=PC.J+(DT)(CAR.JK-DR.JK)
N FC=FCI
N FCI=(DPG*COR*ALC)/(ALC-COR)
R DR.KL=PC.K/ALC
C ALC=20 years
R CAR.KL=PR.K-DPG
C DPG=1.0 unit/year
L UOC.K=UOC.J+(DT)(OR.JK-CAR.JK)
N UOC=(PCI*DDC)/ALC
R OR.KL=(PC.K/ALC)*MDP.K
A PO.K=PC.K/COR
C COR=6 years
A PR.K=PO.K*CUF.K
A CUF.K=TABHL(CUFT,DP.K/PO.K,0,2,0.2)
T CUFT=0/0.2/0.4/0.6/0.8/1/1.1/1.15/1.18/1.19/1.2
A DP.K=(UOC.K/DDC)+DPG
C DDC=3 years
A MDP.K=TABHL(MDPT,DP.K/PO.K,0,2,0.2)
T MDPT=0/0.1/0.2/0.3/0.5/1/2/3/3.5/3.9/4
SPEC DT=0.2
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