THE COMPARATIVE STUDY OF SYSTEM DYNAMICS AND SYNERGETICS

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ABSTRACT

Being two different branches of the system science, System Dynamics and Synergetics share many common concepts and mathematical manipulation techniques. The comparative study of the two subjects will help accelerate the developments of both subjects as well as the whole system science. In this paper, the authors first introduce the basic ideas and mathematical handling techniques of synergetics, then that of system dynamics. Finally, the two subjects are compared from the angles of both the concept and the mathematical manipulation technique.

The following are the major points of the paper:
1) Synergetics deals with systems that are self-organized. It studies how such systems evolve in a self-organized fashion, how new patterns are brought about, or in a philosophical sense, how the animate and the inanimate world evolve. In contrast, the systems, with which system dynamics deals, are partially self-organized, partially man-made. It studies the relationship between system structure and system function.
2) The general equations of both synergetics and system dynamics are nonlinear, stochastical partial differential equations of high order.
3) Synergetics focuses its attentions on nonequilibrium phase transition, i.e. pattern formulation, while system dynamics endeavors, with most its efforts, to study the dynamic behavior of a system with some kinds of structure.
4) There are many concepts as well as mathematical handling techniques that are counterpart and can be used as a reference to each other, such as order parameter vs. sensitive parameter or minor structure; order parameter equations vs. dominant loop equations; slaving principle vs. the principle that dominant loops mainly determine the dynamic behavior of a system; adiabetic elimination vs. the insensitivity of system behavior to most parameters; linear stability analysis vs. Lyapunov method I & II, gain and phase shift analysis, etc.

INTRODUCTION

System dynamics, founded by Prof. Jay W. Forrester, deals with systems that are complex, of high-order and multi-loop information feedbacks. It studies the structure-function relations of such systems.
Synergetics, founded by Prof. H. Haken in 1970s, deals with systems composed of many subsystems which may be of quite different natures. It studies how the cooperation of subsystems brings about spatial, temporal or functional structures on macroscopic scales. Such kind of ordered structures, with which synergetics deals, arises in a self-organized fashion.

Being two different branches of the system science, system dynamics and synergetics share many common things with respect to the concepts they use, the principles they expound and the mathematical approaches they adopt. The comparative study of the two subjects will help accelerate the developments of both subjects as well as the whole system science. In this paper, the authors first introduce the basic ideas and the mathematical approach of synergetics, then that of system dynamics. Finally, the two subjects are compared from the angles of both the concept and the principle as well as the mathematical approach.

SYNERGETICS

1. Synergetics: Problems, ideas and methodology

1.1 The problems to be solved by synergetics

Synergetics deals with systems composed of many subsystems, which may be of quite different natures, such as electrons, atoms, molecules, cells, neurons, mechanical elements, photons, organs, animals or even humans. It studies how the cooperation of these subsystems brings about spatial, temporal, or functional structure on the macroscopic scales. In particular, synergetics focuses its attention on those situations in which these structures arise in a self-organized fashion.

The systems in the world may be quite different in nature. But still there are common things for all systems: they are all composed of many subsystems, and these subsystems may interact with each other, for example they may exchange material or energy or information. But by means of this interaction, they produce a total action or new type of structure. At the level of system, the total action acquires a new quality which may not be present at the level of individual parts. In many cases the total action is meaningful and serves a purpose.

But systems may have also some basic differences. One of these differences can be characterized by organization and self-organization. For instance, on one hand, a car as a system can serve the purposes like transportation, but such system is designed and constructed by people, thus can be called man-made system. On the other hand, an animal like a bird may also have purpose of its own: the wings of a bird serve also the purpose of transportation. But in a way birds or any animals are self-constructed or self-organized. We would like to call such systems self-organized systems, or synergetic systems according to the terminologies of synergetics.
One of the main problems to be studied by synergetics is: "are there general principles which govern the self-organization of systems irrespective of the nature of these parts?"

This question may seem a strange one because the parts of different systems may be quite different in nature, but it turns out that this general principle can be found when we narrow down this situation and focus our attention on the situation when qualitative changes on macroscopic scales happen.

As a by-product, one special aspect of this problem can be considered, namely "what we can learn from nature or simple model systems on the process of self-organization?"

1.2 Why synergetics is needed

a) Thermodynamics (irreversible thermodynamics included) can not explain the formation of ordered structure such as life. There are two aspects here. On one hand, for closed systems the second law of thermodynamics predicts that entropy will increase, in other words disorder will increase and structure will disappear. Boltzmann had argued that the life was brought about by giant fluctuation, but by statistical mechanics we know that this is extremely impossible. Furthermore, Boltzmann could not explain why the world as a transient state can persist and it can evolve and show increasing complexity. On the other hand, for a system far from equilibrium, the thermodynamics definition of entropy remains an open question. And in principle, the thermodynamics offers no principle or mechanism how order arises. Therefore, it is necessary to find some new principle which is not in contradiction with thermodynamics but in addition to it. This is just one of reasons synergetics is needed.

b) As mentioned above, for different systems there exists one important difference which can be characterized by organization and self-organization. By means of either organization or self-organization, a system can acquire a new macroscopic structure which is of new quality, meaningful and can serve a purpose. Cybernetics studies how such macroscopic structure is brought about by means of organization, while the formation of macroscopic structure in a self-organized fashion is to be studied by synergetics. The basic difference between cybernetics and synergetics lies in the ways the external controls act on the system: the kind of control studied by cybernetics is sometimes called hard control or direct control or specific control, while the kind of control studied by synergetics is, in contrast, called soft control or indirect control or unspecific control. In this sense, synergetics is in parallel with cybernetics.

1.3 The basic ideas of synergetics

a) nonequilibrium phase transition and self-organization
In Haken's first introductory book on synergetics, there is a subtitle: nonequilibrium phase transition and self-organization in physics, chemistry and biology. In a way, this subtitle defines the topics to be studied by synergetics and provides some hints to the understanding of synergetics.

The study of laser has given ideas to the generation and development of the ideas of synergetics. Laser is a device or a system which is fed continuously by the external with energy and is far from equilibrium. When the external pump power is small, it produces light which is like noise and acts like an usual lamp. But when the pump power increases to a threshold a kind of regular light suddenly emitted out, which is like a pure tune and is a kind of ordered structure. Therefore there is a transition here which is quite similiar to the equilibrium phase transition of thermodynamics. Furthermore, the formation of such ordered structure or specific collective mode results from the competition and selection of different collective modes, and this phenomenon is quite similiar to what Darwin's evolution theory predicts which states that selection and mutation determine the origin of new species.

The laser paradigm has provided some hints to the development of synergetics. First, phase transitions are wide-spread phenomena for systems in thermal equilibrium, why shouldn't such phase transition be also a wide-spread phenomena for systems far from equilibrium? Second, Darwinism is quite a general feature in the animal world, why shouldn't it be also a basic principle in the inanimate world such as laser. The study of these two questions has led synergetics to develop as a subject which studies the animate and inanimate world and provides general principle for the formation of new structure. "In a way, synergetics is some kind of generalized Darwinism which acts even in the inanimate world, namely, the generation of collective modes by fluctuation, there competition and finally the selection of the 'fittest'collective mode or a combination thereof, leading to macroscopic structures."

b) Slaving principle

The above ideas can be obtained through rigorous mathematical derivation, which can be presented as the slaving principle of synergetics: "the behavior of a system at the neighborhood of critical points is dominated by a few degrees of freedom which slave the subsystems, and the detailed nature of subsystems become unimportant." In fact, these collective degrees of freedom correspond to the slowly varying modes or configurarations or motions, and are named order parameters in the context of synergetics. Thus we can put slaving principle in more common language: "fast decaying (relaxing) configurations are forced to follow slowing growing configurations (modes)." For instance, language as a medium for communication, changes relatively slowly compared to the change of language in the life time of each individual, and can thus be viewed as slowly varying modes, while
the latter is a fast decaying mode and is slaved or influenced by the social language.

2. The kind of equations synergetics studies

To study the evolution of a system in time, we have to take into account the parameter "time" which plays a crucial role for an evolving system. Therefore, the kind of equations studied by synergetics is sometimes called evolution equation.

The structure of the general equation synergetics studies goes like following:

\[ \dot{q} = N(\alpha, q, \nabla, x, t) \]

which is a set of nonlinear stochastic partial differential equations. Whereby q is a vector of state variables which are usually variables at the mesoscopic level, \( \alpha \) is control parameter, x is a vector of space coordinate, \( \nabla \) is differential operator (which describes spacial gradient) and t is time. The equation is of the following characteristics:

First, this is a set of first-order partial differential equations. The state variables evolve in the course of time, therefore this is an evolution equation.

Second, this is a nonlinear equation. In fact, all equations of synergetics are nonlinear, and nonlinearity is absolutely necessary for a system to acquire self-organization.

Third, there is a control parameter \( \alpha \) here. All synergetic systems are open systems, with control parameter \( \alpha \) indicating the action of the externals on the systems. By changing control parameter, we can study the evolution behavior of a system. But remember that the meaning of control here is somewhat different from that in cybernetics. The control here doesn't set any prescribed constraints to the individual parts of a system. Instead, it only provides conditions for a system to evolve. In contrast, the kind of control used in cybernetics is to optimize a system or to regulate the behavior of a system so that the system can reach a desired goal. As we will see later, the concept of control parameter in synergetics is rather similar to that in system dynamics.

Fourth, stochasticity. Fluctuation plays a crucial role for a system near a critical state. Since stochasticity is a ubiquitous phenomenon in the world, we have to take into account this feature. For a physical system, there exist at least three kinds of stochastic factors:

1) Statistical fluctuation. Fluctuations occur whenever we pass from a microscopic description to one which uses more or less macroscopic variable, for instance when we describe a liquid not by the position of its individual molecules but rather by the
local density and velocity of its molecules.

2) Quantum fluctuations. With the advent of quantum theory, it becomes clear that it is impossible even in principle to make prediction with absolute certainty. This is made manifest by Heisenberg's uncertainty principle which states that it is impossible to measure at the same time the velocity and the position of a particle with absolute precision. This impossibility is cast into its more precise form by Born's probability interpretation of the wave function in the quantum theory.

3) Chaos. Even if there is no quantum fluctuation and the equation describing temporal evolution are entirely deterministic, future development can also proceed along quite different routes. This rests on the fact that some systems are extremely sensitive in their future developments to initial conditions. In synergetics, chaos is defined as irregular motion stemming from deterministic equations.

The state variables $q$ in the evolution equation are usually variables at the mesoscopic level. Synergetics starts from the mesoscopic level and devises methods to predict the evolving macroscopic patterns.

3. Outlines of the mathematical approaches devised by synergetics

Since synergetics focuses its attention on the situation when macroscopic qualitative change is brought about or an old state is replaced by a new one under the change of control parameter by the means of self-organization, the mathematical approach devised is accordingly centered on the resolution of this problem.

In the following, we would like to outline briefly the steps to the mathematical approach of synergetics. Remember that the original evolution equation goes like following:

$$\frac{dq}{dt} = \dot{q} = N(q, \alpha, x, \nu, t) + F(t)$$

where $F(t)$ is fluctuation force and we write out here in an explicit way.

We assume that for specific control parameter we have found the solution which can be denoted as $q_0(t)$. Its time dependence may be either constant, or periodic, or quasiperiodic which means that it contains several basic frequencies. On the other hand its space dependence may be quite arbitrary. A simple example for such a space dependence is provided by a liquid at rest. Now we study what happens when we change the control parameters, for instance when we heat up the liquid.

1) Linear stability analysis
First we have to study whether the old state is stable or now become unstable. To this end we perform a conventional linear stability analysis. We put the new solution we seek equal to the old state plus a small deviation:

\[ q = q_0 + w \]

Then we insert this hypothesis into our original equation and linearize so that we obtain the equation for \( w \):

\[ \dot{w} = Lw \]

And we can find two types of solutions to \( w \): solutions which will grow exponentially when \( \lambda \) is positive (\( \lambda_u \)) and other solutions which are dumped when \( \lambda \) is negative (\( \lambda_s \)):

\[
\begin{align*}
\dot{w} &= \begin{cases}
    e^{\lambda_u t} v_u(x,t) & \lambda_u > 0 \\
    e^{\lambda_s t} v_s(x,t) & \lambda_s < 0
\end{cases}
\end{align*}
\]

At the same time this equation determines the form of \( v_u \) and \( v_s \).

2) Transform \( q \) to new basis \( v \) with amplitude \( \xi_u(t) \), \( \xi_s(t) \)

Now \( q \) can be represented as a superposition of all these possible modes \( v \) while the coefficients \( \xi_u(t) \) which refer to the modes \( v_u \) and \( \xi_s(t) \) which refer to the modes \( v_s \) are still unknown.

\[
\begin{align*}
\dot{\xi}_u &= \lambda_u \xi_u + N_u(\xi_u, \xi_s) + F_u(t) \\
\dot{\xi}_s &= \lambda_s \xi_s + N_s(\xi_u, \xi_s) + F_s(t)
\end{align*}
\]

This basis tranformation will simplify the manipulation of original equation.

3) Slaving principle

Remember that \( \lambda_u \) is a positive quantity and \( \lambda_s \) is a negative quantity. Now by rigorous derivation we can obtain the mathematical form of the slaving principle, which manifests that the slaved modes \( \xi_s \) can be explicitly expressed by means of the so-called order parameters:

\[ \xi_s = f_s(\xi_u, t) \]

\( f_s \) depends also on \( t \) because the fluctuation force depends on time.

4) Order parameter equations

In practical cases there are only very few order parameters while the number of slaved modes may be enormous. This means that we can reduce the original highly complex system as described by the original equation to relative simple equations which refer to only few order parameters:
\[ \hat{x}_n = M (x_n , F) \]

By solving the order parameter equations, we can predict the new structure to be formed.

SYSTEM DYNAMICS

System dynamics is a subject which studies the problems of a information feedback system. It is founded in 1950s by Prof. Jay W. Forrester at MIT. The origin of this subject is to solve the problems existed in many systems like social-economic systems which are characterized by high-ordered, nonlinear and of multiple information feedbacks. For this kind of problems, no single subject such as cybernetics, information theory, computer simulation, decision theory, economics theory etc. can by itself provide solutions to them. Only when these subjects are synthesized and a new interdisciplinary subject is developed can such large-scale systems charaterized by high-order and nonlinearity be solved. This is the background for the emergence of system dynamics.

The systems dealt with by system dynamics are nevertheless limited to social economic systems. But in order to easily present our ideas, we would rather project our description of system dynamics on the basis of its application to social-economic systems.

1. The basic principles of system dynamics

System dynamics defines system as the grouping of parts that work together for a common purpose. The reason that the parts of a system can coordinate rests on the fact that there exist many information feedbacks within a system. Therefore, the information feedback structure is the fundamental structure of a system dynamics system. System dynamics studies the structure-function relationship of such systems.

Based on the assumption that the structure of a system determines the behavior of the system, the following basic principles of system dynamics can be obtained:

1) The nature of the dynamic behaviors of a system depends on its internal structure.

2) The internal structure of a system include two aspects: one is the subsystems as components of a system and their interactions, another is the structures of the internal feedback loops existed in a system.

3) Among the many feedback loops of a system there exist some dominant loops. It is these dominant loops and their interactions that determine mainly the dynamic behaviors of a system.
4) Usually, the parameters and structures of a system vary with time.

5) In the course of the evolution process of a system, the dominant loops which play more crucial roles than the other loops are not invariable. The old dominant loops may be replaced by the new ones because of the internal interactions among various generalized forces.

2. The basic equation of system dynamics

To study the dynamic behaviors of a system we have to take into account the parameter 'time' because it plays a crucial role for a evolving system. In principle, the basic equation of system dynamics is also a evolution equation, which goes

\[ q = f (q,u,t) \]

This is a high-ordered nonlinear stochastic differential equation, where \( q \) is various level variables or state variables, \( f \) is the net rates of level variables, \( u \) is control control variables and \( t \) is time. This equation is of great similarity to the equation of synergetics.

1) The problems dealt with by system dynamics are also the evolution problem of a system over time. Space coordinate \( x \) does not appear here, which is of course a simplification of the reality. In fact, if the systems system dynamics deals with are large ones and if the distribution of variables over space is important, it is also necessary for system dynamics to take into account the space coordinate and its gradient, for instance the world is divided into several regions in the world model of system dynamics. Examples of such spacial inhomogeneity or spacial structure can easily be found, for instance the imbalance of the economies of different regions, the migration of population etc.

Therefore the generalized system dynamics equations are the evolution equations which take into account and bring about the temporal and spacial structure.

2) Closed loops and openness. System dynamics emphasizes a great deal that the models built should have their cause-effect loops closed. This is because one of the basic principles of system dynamics is that the behavior of a system depends on its internal structures. In fact, this manifests itself the nature of self-organization. Meanwhile, system dynamics believes that in order for a system to acquire a alive structure the system should be open.

3) Nonlinearity. Strictly speaking, all complex systems are of the nature of nonlinearity, for instance the nonlinear coupling among their components and variables.
4) Control variables. The control variables in system dynamics are quite similar to that in synergetics. They function as the connection between environment and system and mean only the action of environment on system. They do not mean to control or regulate each individual or optimize the whole system.

5) Massive components and variables at the mesoscopic level. The systems studied by system dynamics are usually composed of enormous components which may be the individuals or firms in a society. But on the other hand, the variables used in system dynamics are actually variables at the mesoscopic level, which may be the description of the ensembles or sets of those individuals belong to the same category or of the same nature, for instance the set of people or population is chosen to be a variable in a system dynamics model, and the set of firms can be used as the description of a production system. The idea to start from mesoscopic level is similar to that of synergetics.

6) Stochasticity. In fact, social systems show greater stochasticity than nature systems. The stochasticity of a social system stems from the following sources: First, statistical deviation. As mentioned above most descriptions of a social system are based on the variables at the mesoscopic level (except subjects like psychology which study the individual behavior), thus when we pass from the variables at the individual level to variables at the mesoscopic level there definitely exist a statistical deviation. Second, the uncertainty principle of individual. Generally speaking, the individuals in a social system show much greater uncertainty. The reason for this is that the components of a social system are themselves composed of people, while people may have their own will, their different style of decision making, their different experiences and learnings, their different expectations about future etc.. All these will lead to their different responses even when they are exposed to a same environment. Third, chaos. In fact, for some social systems, their future behaviors may show great uncertainty even when their evolution equations are deterministic. The reason lies in the fact that the behaviors of such systems depend not only on the internal feedback mechanisms but also on their initial conditions and fluctuations to which some systems may be quite sensitive. For instance, the outcome of a competition between two opposite political campaigns, of close appeals or a war between two opposite militaries of equal strengths may depend also upon some stochastic factors as weather, geographical condition etc..

3. The mathematical approach of system dynamics

The basic equation dealt with by system dynamics is a high-ordered nonlinear stochastic differential equation. Although many efforts were made to find the analytical solutions to such problems, there remains a long way from finally reaching this goal. Therefore, system dynamics adopts computer simulation i.e.
DYNAMO to study this equations.

SYNERGETICS AND SYSTEM DYNAMICS: THEIR COMPARISON AND WHAT THEY CAN LEARN FROM EACH OTHER

The above description of both subjects from the angles of purposes, problems to be solved, fundamental ideas, basic equations and their mathematical approaches shows that there exist many common things between the two subjects, in the concepts they use, the principles they hold, and the mathematical approaches they devise. On the other hand, there are also some differences between the two subjects, and these differences provide areas for them to cooperate and learn from each other.

1) Organization and Self-organization

The problems dealt with by synergetics are the phenomena of self-organization which exist abundantly in the world. One characteristic of such systems is that by means of the interaction among subsystems and the natural constraints formed a system can acquire a temporal, spacial or functional macroscopic ordered structure by the way of self-organization.

The systems dealt with by system dynamics show also the nature of self-organization. In fact, the capability of human beings to efficiently control the world is rather limited. Though to some degree, human being can shape the world, control or navigate the world, or change the structure of the world, it is still inevitable that the human beings are subject to many constraints from the objective world.

The distinction between organization and self-organization or the distinction between man-made systems and synergetic systems are not strict and absolute, it may conclude differently when we change the angle from which we view the problems. Generally speaking, when we consider a system which by itself is man-made in the context of a larger one, this system can now be viewed as a component of a larger self-organized system.

In principle, a perfect market economy, which Adam Smith used the catchwords invisible hands to describe, shows mainly the nature of self-organization, while a perfect planned economy is more or less a man-made one.

If we distinguish systems according to the degree to which the human beings exert interferences on them, we may obtain a distribution of systems over this "coordinate" with synergetic systems at one polar, system dynamics systems at the middle and cybernetics systems at the other polar (purely man-made systems).

2) The greatest common point of synergetics and system dynamics stems from the similarity of their equations which are all high-ordered nonlinear stochastic differential equations.
3) Synergetics focuses most of its attentions on the evolution of a new state from an old one, namely nonequilibrium phase-transition, while system dynamics pays much of its attentions to the dynamic behavior of a system within the framework of a prescribed structure, with less attentions to the evolution between two structures.

To study the behavior of a system at the critical point or phase transition point, synergetics employs the linear stability analysis method to analyze the stability of the old state and use slaving principle to find order parameters and determine their equations, while system dynamics devises its ways to find sensitive parameters and/or sensitive structures which make the behavior modes of a system change (The change of the behavior modes of a system corresponds to the phase transition in the synergetics notions).

4) The similarity in the mathematical manipulation techniques between the two subjects can be summarized neatly by the following table:

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<thead>
<tr>
<th>Synergetics</th>
<th>System Dynamics</th>
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<tbody>
<tr>
<td>Order parameters</td>
<td>Sensitive parameters or</td>
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<td></td>
<td>Sensitive structures</td>
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<tr>
<td>Order parameter equations</td>
<td>Dominant loop equations</td>
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<tr>
<td>Slaving principle</td>
<td>The dominating loops dominate the behavior of a system</td>
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<tr>
<td>Adiabatic elimination procedures</td>
<td>The behavior of a system is insensitive to the change</td>
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<td>and minor loops</td>
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<td>Stability analysis:</td>
<td>Stability analysis:</td>
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<td>Linearity stability analysis</td>
<td>Lyapunov methods I &amp; II</td>
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<tr>
<td>Time-domain method</td>
<td>Frequency-domain method</td>
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<td>(gain &amp; phase shift study)</td>
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We feel that it is necessary to explore further the similarity of the detailed mathematical manipulation techniques between the two subjects.

CONCLUSIONS

The general system theory should include both the theory of organized systems and the theory of self-organized systems.
Synergetics and system dynamics are two different subjects at the different levels of the general system theory. Though each has its own features, the two subjects have many things in common, not only in the concepts and principles they put forward but also in the mathematical manipulation techniques they adopt. Thus there exists a great area for the two subjects to cooperate and a common territory for them to explore. It is necessary to make more efforts on this cross-subject study.

REFERENCES

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