# Predictability and Forecasting

Foster Morrison
Turtle Hollow Associates

ABSTRACT

Forecasting for complex nonlinear systems has proven to be elusive. Investigators have assumed the causes to be too little data and overly-simplified models. Recent studies in climatology reveal that nonlinear systems behave in ways quite different from the linear or static systems of traditional science and engineering. The behavior of nonlinear systems can be cyclical or essentially stochastic and usually is a mixture of both. New techniques, such as "attractors," are being devised to facilitate analysis. Methodologies must be applied with due consideration to the structure of the system under investigation.

## INTRODUCTION

Currently, System Dynamics is a methodology of broad application, in that it competes with economics, some social sciences, and some parts of management science. Practitioners of these disciplines, whether in the academic world, industry, or government, usually have chosen to retain their traditional methods of analysis (Leontief 1982). In fact, a whole new generation of economists has been trained in techniques that now should be considered archaic. This is due only in part to the usual conservatism of all scholars. The burden of proof still rests with the System Dynamics community, which has the responsibility to deliver quantitatively better models and forecasts to the customers.

My own observation is that social and economic systems exhibit pronounced dynamical behavior, as well as the stochastic component acknowledged by all. This dynamical behavior, however, is precisely what has so far prevented System Dynamics practitioners from producing models significantly better than the competition. The model components, nonlinear differential equations, are more appropriate than the usual static equilibrium techniques. What has been lacking is an effective strategy for constructing the models.

The direct application of integrating initial value problems or even the tuning by multiple regression (least-squares adjustment of parameters and initial conditions) will not suffice for any but the simplest nonlinear systems. These techniques serve adequately for small linear systems or near linear ones such as orbits and ballistic trajectories. Much more analysis must be done for the subject areas of System Dynamics. For one thing, the data are much more noisy and less precise than in physical science and engineering. Even more critical is that the solutions of these equations are not linearizable to a degree sufficient for direct and easy model building.

Mathematicians have made steady, but slow and very painful progress on nonlinear dynamics since the days of Poincare (Hirsch 1984). Their results, unfortunately, are not usually intelligible to practitioners such as scientists, engineers, and economists or even mathematicians with other specialties and are less than adequate for model building and verification. The insights of the mathematicians can be combined with the experiences of scientists and computer experimenters to provide a basic strategy for building better System Dynamics models. For want of a name, I have called this ad hoc synthesis "The Theory of Predictability."

## PHYSICAL SYSTEMS: THE MEDIUM FOR INTUITION

Certain dynamic systems are difficult or almost impossible to model even with today's large-scale computers. A rigorous mathematical explanation of this is not feasible and perhaps not possible. From an intuitive or physical viewpoint though, this is easy to understand. To maintain a complicated structure something has to have considerable rigidity. Sculptors like to use materials such as wood, stone, metal, glass, and ceramics. Fluids, be they liquids or gases, do not hold much shape. The interactions in most dynamical systems are fairly weak. For example, the orbits of the planets are almost independent of each other, especially if one limits one's observations to a short time period. The mechanics of fluids is impossibly complicated because they so easily break down into eddies. At small scales structure is replaced by behavior that is akin to randomness.

As interactions grow stronger they eventually become constraints. Constraints reduce the dynamical complexity of a problem, replacing it with the sort of fixed structure one can have in a rigid body. Rigid bodies and very simple systems can be modeled very nicely by traditional methods: geometry and differential equations (Luenberger 1979). What is sufficiently unstructured and disordered can be modeled by statistical methods. In some cases the tools of classical analysis such as integral transforms, special functions and linear systems allow the work to be done without benefit of numerical analysis or computers.

Figure 1: The Hierarchy of Dynamical Systems

| Type | Constraints | Type of System |
|---|---|---|
| Zero | Absolute | Rigid body |
| I | Analytic integrals | Integrable dynamic |
| II | Approximate analytic integrals | Amenable to perturbation theory |
| III | Embedded structure | Complex dynamic |
| IV | No structure | Turbulent/stochastic |

The hierarchy of dynamical systems (See Figure 1) is well established for Types Zero and I, which are largely mathematical ideals, but useful approximations for many purposes in science and engineering. Types II and III can be approached with numerous analytic or numerical techniques. Complex dynamical systems represent a transition between Types Zero, I, and II behavior and random behavior. The same system can demonstrate these extremes. It may have a nice, stable periodic solution and chaotic transitions to other stable periodic solutions. It may also have non-stable periodic solutions. All these occur in various special cases of the restricted three-body problem, a favorite topic of dynamical astronomers (Szebehely 1967).

Many practical problems fall into this category of complex systems. Weather prediction and climatology, econometric modeling, and mathematical ecology all share in common some general qualitative behavior. Until now these problems have defied scientific analysis and led some people to the conclusion that this always would be the case. High-speed digital computers, which have become incredibly powerful, seemed to offer a direct way to cope with complexity. A dissenting voice, the mathematician Garrett Birkoff (1975), pointed out that any problem can grow in complexity beyond the limits of physically achievable computing systems. This Malthus of mathematical modeling called his principle "the combinatorial explosion." In practice the properties of dynamical systems usually prevent this from ever being a problem. The disintegration of structure in complex systems, often called "turbulence" from the fluid mechanics example, puts a much more severe limit on the capabilities of modeling and forecasting than does Birkoff's principle, which is basically geometric and static in nature (Sugarman and Wallich 1983). Turbulence may be said to be the dynamical limit to modeling. At the other extreme image processing and language translation are areas that are characterized by sheer quantities of data and are quite amenable to sheer computing power. Basically, they are Type Zero systems with much detail.

Some scientists and economists have come to the conclusion that complex systems cannot be modeled, let alone predicted. This would leave us, with all our highly developed data processing technology, little better off than the ancients, who at least had their astrology (maybe it didn't work, but they believed it did). The real problem can be summarized in the aphorism, "He who computes much, thinks little." In the past this has been interpreted to mean that the problem at hand could be solved through the rigors of classical analysis. What this implies for complex systems is that the structure must be identified in the system before one attempts to do any serious modeling. One cannot throw together an enormous model, even if one knows all the basic interactions from some physical principles and makes no errors (both are rare occurrences), and expect it to work.

Let's consider, for a moment, how turbulent behavior prevents the application of the stock tools of the computer modeler. On a global scale, what is known as "turbulence" is produced at the local scale by the divergence of nearby trajectories. What this implies is that small changes in the initial conditions can produce very large changes in the state. The usually useful state-transition matrix does not produce a satisfactory description of the behavior of the system no matter how small the variations in the initial conditions may be chosen. Another way to put this is that the system cannot be linearized even very locally in either time or state space. There is no nice trick, such as one sees in the literature for solving very specific problems. What is needed is a strategy instead of a recipe, and that is what we shall describe.

PREDICTABILITY

In addition to turbulence, there also is "random" behavior. Traditionally, random behavior has been thought of as "noise," or rather unmodeled effects. Actually, the quality of being random is something that cannot be determined mathematically or empirically (Kac 1983, 1984). Nothing is completely deterministic either. The case is that every dynamical system has some inherent degree of predictability of state between absolute certainty and white noise. From this observation follows the "obvious" conclusion that the first part of a scientific investigation should be to determine the predictability of the variables. Model makers should first attempt to define the most stable or predictable variables, not necessarily ones identical to the measurables, but functions of them. An optimal model would concentrate the largest amount of predictability into the smallest number of variables. In the traditional physical sciences this now seems God-given. It was not always so. Ancient civilizations never developed modeling beyond geometry and counting (Type Zero dynamical systems). Western Civilization added dynamical systems of Types I and II, with the help of calculus and differential equations, and later Type IV with probability and statistics. Quantum mechanics is a Type IV system, not a mystical one.

System Dynamics (and its cousin, fluid mechanics) dominate the still thinly explored region of Type III. "The Theory of Predictability" presented here is largely a collection of generalizations and ad hoc procedures. This is not so bad in that the methods can be used by engineers, social scientists and M.B.A.s without enduring years of (additional) graduate study.

Rigorous mathematical analysis has done little for Type III systems and not because any effort has been spared (Hirsch 1984). But mathematics is primarily the use of exact symmetries as a labor-saving device. Statistical analysis of Type IV systems makes use of the symmetries arising from the applications of limit processes (infinite numbers of state variables or parameters). Complex dynamic systems are a transition type least amenable to mathematical analysis because there are no exact symmetries to exploit and usually too few approximate symmetries. Mathematical concepts as novel as probability in its day may be devised in the future. Meanwhile, scientists, engineers and modelers must struggle with ad hoc methods and trial and error to solve practical problems and to provide enough special cases to suggest new concepts to mathematicians.

A System Dynamics model, among other things, may be summarized in vector notation by

$$d\underline{x}/dt = \underline{f}\left[\underline{x}(t);t\right], \qquad (1)$$

where $\underline{x}$ is a vector of state variables and t the time. In most serious models the components of $\underline{f}$ are nonlinear functions of $\underline{x}$; t may be absent or occur as a linear or nonlinear variable, where its presence is used to indicate the influence of exogenous effects such as astronomical or climatic systems.

Solvable examples of (1) are the exception rather than the rule. Perturbation techniques sometimes can extend these ideal cases for application to actual data reduction and analysis problems. Celestial mechanics is the classic example.

## AN ARCHETYPAL SYSTEM

The literature of dynamic systems and differential equations is replete with specific examples of the application of "powerful techniques" for solution. Simple problems can be reduced to evaluating an integral (analytic quadrature) and then finding the inverse of that function. Power series methods are a variation of this. The concept is simple, but the implementation is usually complicated and laborious. Few ever master even a portion of the literature of special functions such as Bessel functions and elliptic functions.

A few coupled nonlinear systems can be solved by separation of variables techniques. Problems in mechanics often can be expressed by a Hamilton-Jacobi partial differential equation, of which about a dozen examples are known of separable problems. An explicit solution is obtained by solving several "simple" problems of analytic integration and function inversion.

Perturbation methods sometimes can be applied to problems dominated by solvable systems. Considerable progress has been made in extending the applications beyond simple methods of averaging, but the subject, like special functions, is intricate enough to constitute a specialty in itself. Most practitioners are satisfied to resort to the computer and run off a few calculations with a good single-step numerical integrator in the span of a few minutes. By contrast, Delaunay took 20 years to derive his theory of the moon's motion and, though often applauded, it has never been used.

The problem with all the analytic and perturbation methods is that they depend on symmetries that do not exist in complex systems. The trajectories (i.e., solutions) are knotted and twisted and tangled so badly that they cannot be sorted out by any simple process. This description may recall the legend of the Gordian knot. Alexander the Great, finding the knot impossible to undo, simply cut it. To preview our conclusions, that essentially is what is recommended here. What will follow will be suggestions for surgery as opposed to blind hacking.

Perhaps the best archetypal system for system dynamics is the Lorenz equations. This is a system of three simultaneous equations (Hirsch 1984; Sugarman and Wallich 1983)

$$\begin{aligned} dx/dt &= -10x + 10y \\ dy/dt &= 28x - y - xz \\ dz/dt &= -8z/3 + xy \end{aligned} \qquad (2)$$

where we let $\underline{x} = (x,y,z)^T$. The first equation is linear and the rest have one simple nonlinear term which is quadratic. These equations were discovered, so to speak, during some pioneering experiments using computers to integrate numerically a drastically simplified problem in fluid mechanics.

Analysis is often complicated, but not very "powerful." The first thing one can attempt is to find equilibrium points, which are those values $\underline{r}$ where

$$\underline{f}(\underline{r}) = 0 \tag{3}$$

so that

$$d\underline{x}/dt = 0 \tag{4}$$

for $\underline{x} = \underline{r}$. Close to such points some information can be obtained about the behavior of the solutions $\underline{x}(t)$ by linearizing (1). The process, of course, is Taylor's Theorem for several variables

$$d\underline{x}/dt = J(\underline{r})(\underline{x}-\underline{r}) + O(\underline{x}-\underline{r})^2, \tag{5}$$

where J is the Jacobian matrix of partial derivatives of $\underline{f}$ and O a common (often British) notation for the higher-order error terms. By ignoring the error terms in (5), which usually is a valid procedure for some neighborhood of $\underline{r}$, the behavior of the system there may be deduced by computing the eigenvalues of the Jacobian matrix J (Luenberger 1979). The system (5) is approximated thereby by a linear system with constant coefficients.

The equilibrium points for the Lorenz system are easy to find. From the first equation in (2) one concludes x = y and the evaluation follows easily that

$$\begin{aligned} \underline{r}_1 &= (0,0,0)^T \\ \underline{r}_2 &= (6\sqrt{2},\ 6\sqrt{2},\ 27)^T \\ \underline{r}_3 &= (-6\sqrt{2},\ -6\sqrt{2},\ 27)^T. \end{aligned} \tag{6}$$

Computing the Jacobian matrix is straightforward:

$$J(\underline{x}) = \begin{bmatrix} -10 & 10 & 0 \\ 28-z & -1 & -x \\ y & x & -8/3 \end{bmatrix}. \tag{7}$$

The eigenvalues of $J\left[(0,0,0)^T\right]$ are easy to find, since the characteristic polynomial can be partly factored; they are all real: -8/3, 11.8275..., and -22.8275.... The large positive value indicates that the origin is not a stable point and that the trajectory will soon depart the region.

Unexpected symmetry is revealed by analysis of the two other equilibrium points. Even though the Jacobian matrices are different, they have identical characteristic polynomials

$$P(s) = s^3 + 41s^2/3 + 304s/3 + 1440. \tag{8}$$

The polynomial P(s) has one real and two conjugate complex roots, which are approximately

$$\begin{aligned} e_0 &= -13.8545... \\ e_1 &= \ 0.09395... + 10.1945...j \end{aligned} \tag{9}$$

$$e_2 = 0.09395\ldots - 10.1945\ldots j, \qquad (9)$$

where j is used for the imaginary unit ($j^2 = -1$). The small, positive real parts of (9) imply that the trajectory will gradually spiral away from these equilibrium points. This analysis does not tell much and it cannot, since it is "local" in nature. Useful global analysis is more the exception than the rule in nonlinear systems.

Numerical studies have revealed that the trajectory can jump back and forth between conditions of oscillation about the equilibrium points in a rather unpredictable fashion (Sugarman and Wallich 1983). Such behavior has been observed in System Dynamics models and been viewed by some (Kalman 1980) as indicating that the approach somehow is not valid. Proponents of System Dynamics have argued that the models show qualitative behavior similar to what actually happens and therefore are superior to traditional econometric models (Forrester 1978).

In some ways the debate is another of "oranges vs. apples." Econometric models are locally valid (in time and state space) and system dynamics models are emulators, i.e., they perform like the actual system but cannot be used for data analysis and prediction with the usual techniques of multiple regression. Why this is so is clear from the example of the unpredictable Lorenz system, which is why it is the best archetype for the current System Dynamics paradigm.

Non-mathematicians (or scientists and mathematicians in different specialties) can understand the phenomenon as being essentially the same as an (old-fashioned) pin ball machine. We know where the ball starts, where it will end its journey eventually and the limits on that journey, but no one does know or can know what it will do along the way.

## THE THEORY OF PREDICTABILITY

From some points of view, the turbulence is the phenomenon of interest (Gollub et al. 1978, 1980; Swiney and Gollub 1981). But among most people there is only an interest in forecasting the measurables. Outside the academic world there is little or no appreciation of the differences between the usual models of economists and social scientists and the System Dynamics versions. We would do well to heed the observation of Paul Fussell (1980) that "The Top Out-of Sight class ... is ... entirely devoid of intellectual or even emotional curiosity." Decision makers and managers at lower levels also have little time and patience for elaborate explanations and subtle repartees. They want user-ready products to help them make the decision that will be most likely to promote their goals.

The example of the Lorenz system illustrates why the most common econometric variables cannot be forecast very well. An example from elementary economics can illustrate the same phenomenon without recourse to esoteric mathematics: the demand for two commodities that are readily substituted for one another. A case might be oil from Britain and oil from Nigeria. Since the products are similar in composition, a refinery can use either one without major changes in its operations or equipment. Therefore the crude oil from the two sources will compete head-on with every little factor in cost of delivery and production being important in the purchase decision. Such a market can be very volatile, especially when production exceeds reasonable demand.

At the other extreme, major business cycles persist despite (or perhaps because of) government efforts to eliminate them. A cycle is an indicator of stable, predictable behavior. The most predictable variables should be sought, initially by trying weighted sums (linear combinations) of the measurables. For example, the total of British and Nigerian oil sold might be very stable, but the difference will vary wildly.

Rather than debate the reality of the Kondratieff wave, a better approach would be to determine what quantity is behaving most like a nice, self-regulated system such as the Volterra predator-prey system (Davis 1962, pp. 99-115). A sinusoid would be a poor choice because the harmonic oscillator is not a good model of growth, overexpansion and decline whereas the Volterra system is the "classic." The variable can be christened the "Kondratieff-Forrester (K-F) Index" and be published like the Dow-Jones averages, which are complicated weighted averages whose definitions are unknown to most investors. Would knowledge cause the amplitude of the long-period oscillations of this index to decay?

## A MODEL THAT WORKED

Finding a K-F Index that is reasonable, let alone optimal, is a data processing exercise well beyond the scope of this study. For an example of predictability, let us consider M. King Hubbert's notoriously successful forecast of the decline of U.S. domestic oil production (Pazik 1976).

The mathematical model used is a remarkably simple dynamic one known as the logistic equation. There is only one state variable and three parameters; the general form is (Luenberger 1979, pp. 316-319; Davis 1962, pp. 96-98)

$$x(t) = \frac{c}{1 + b\,\exp(-at)}. \qquad (10)$$

The curve starts at 0 and rises gradually to c and is an "S-shaped" form not unlike the arctangent.

Hubbert did not possess or at least did not demonstrate any spectacular mathematical ability. Despite this, the logistic curve, or more often its "bell-shaped" derivative function, is frequently called "Hubbert's Curve" (Goodwin 1980). What he did do was exemplary scientific work of selecting a minimal model and using it properly. The rest is history, though there remain some whose minds have been made up and are not about to allow themselves to be disturbed by mere facts. Advanced technology which has increased the arena of exploration greatly has not changed the long-term oil supply significantly. The time bought by this technical achievement has been squandered already by ignorance and apathy on the part of both politicians and businessmen. Today's oil glut will not last forever.

## CONCLUSIONS AND RECOMMENDATIONS

Models should be as simple as possible. The uncertainty principle of quantum mechanics asserts a necessary trade-off of precision in position or velocity.

Geophysical inverse theory has a trade-off of precision versus resolution. Nonlinear dynamical systems have an analogous trade-off of predictability versus dimension. Choose too many variables and the model cannot be used for curve-fitting, let alone forecasting. Use the optimum number and type of variables for maximum predictability. Working back and forth between model and data will allow this and that is the essence of the scientific method.

New methods of analysis are being discovered, though not nearly so many as the volume of papers published would seem to indicate. Of particular interest is a technique for identifying an attractor in a system which is known only through a time series of one of its measurable state variables (Nicolis and Nicolis 1984). The literature of many disciplines needs to be scanned and with alarming consistency one finds the same thing done over and over again in a different discipline and described in a different jargon.

System Dynamics is now over 20 years old. Despite early promises, the "New Economics" and der Tag for System Dynamics are not yet here. Electronics went on for two generations but did not come into its own until the perfection of the transistor. That process continues today, not just in the USA, but in Japan, Taiwan, Korea, Singapore, the USSR and elsewhere.

System Dynamics must achieve the perfection of the attractor, or perhaps the elimination of the attractor is a more accurate description. Super computers will be used for grinding through the phase information or nonstationary statistics common to Type Zero systems, typified by image processing and the now popular laser disc player. System Dynamics will be done on small computers, despite some multiple regression exercises, by people who are thinking more than they are computing.

REFERENCES

Birkhoff, G. "Mathematics and Computer Science," American Scientist, Volume 63, 1975, pp. 83-91.

Davis, H. T. Introduction to Nonlinear Differential and Integral Equations. New York: Dover Publications, 1962.

Forrester, J. W. "A great depression ahead? Changing economic patterns," The Futurist, Volume 12, December 1978, pp. 379-385.

Fussel, P. "A dirge for social climbers," New Republic, July 19, 1980, pp. 18-21.

Gollub, J. P., T. O. Brunner and B. G. Danly. "Periodicity and chaos in coupled nonlinear oscillators," Science, Volume 200, April 7, 1978, pp. 48-50.

Gollub, J. P., E. J. Romer, and J. E. Socolar. "Trajectory divergence for coupled relaxation oscillators: measurements and models," J. Stat. Physics, Volume 23, Number 3, 1980, pp. 321-333.

Goodwin, S. "Hubbert's curve," Country Journal, November 1980, pp. 56-61.

Hirsch, M. W. "The dynamical systems approach to differential equations," Bulletin (New Series) of the American Mathematical Society, Volume 11, Number 1, 1984, pp. 1-64.

Kalman, R. E. "A system-theoretic critique of dynamic economic models," Inter. J. of Policy Analysis and Info. Systems, Volume 4, Number 1, 1980, pp. 3-22.

Kac, M. "What is random?" American Scientist, Volume 71, 1983, pp. 405-406.

Kac, M. "More on randomness," American Scientist, Volume 72, 1984, pp. 282-283

Leontief, W. "Academic economics" (Letter), Science, Volume 217, 1982, pp. 104, 107.

Luenberger, D. Introduction to Dynamic Systems: Theory, Models, and Applications. New York: Wiley, 1979.

Nicolis, C. and G. Nicolis. "Is there a climatic attractor?" Nature, Volume 311, 1984, pp. 529-532.

Pazik, G. "Our petroleum predicament" (Editorial), Fishing Facts, November 1976, pp. 1-20.

Sugarman, R. and P. Wallich. "The limits to simulation," IEEE Spectrum, Volume 20, Number 4, 1983, pp. 36-41.

Swiney, H. L. and J. P. Gollub. "Introduction," Hydrodynamic Instabilities and the Transition to Turbulence, Topics in Applied Physics, Volume 45. Swiney and Gollub, eds. Berlin: Springer-Verlag, 1981.

Szebehely, V. Theory of Orbits: The Restricted Problem of Three Bodies. New York: Academic Press, 1967.