Disaggregating a Simple Model of the Economic Long Wave

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ABSTRACT

A multi-sector, input-output version of Sterman's simple Long Wave Model is developed to investigate the validity of the capital self-ordering theory for a more realistic system with diverse capital types. Simulation experiments with varying capital lifetimes and input-output coefficients tend to reproduce the characteristic fluctuations in capital production, caused by self-ordering, with a period in the 30 to 70 year range. However, complex patterns of oscillation with wide variance in period can emerge, explained by varying dominance of self-ordering loops. The analysis thus confirms the destabilizing effect of self-ordering and its significance for long term fluctuations while raising issues and generating new insights about the long wave.

INTRODUCTION

The System Dynamics National Model project at M.I.T. represents a unique approach to macro-economic theory, in that it aims to explain the aggregate behavior of the economy from the bounded rationality of internal decision rules of firms and households (Forrester, 1979). But while such decision rules are represented with great richness and detail in this approach, whole industries and sectors in the economy are aggregated into single "firms", to allow for internal detail while keeping the model at a manageable size.

This aggregation, which occurs in both the National Model and the simple long wave model as well as in a host of other system dynamics economic models, is based on the assumption that interactions occurring within each individual firm in a particular sector are more or less "in phase" with that of other firms in the sector, and that these interactions are more important than the interactions among firms for the sector's overall behavior. While this assumption may be adequate, there have been few attempts to test it explicitly.

One of the major outcomes of the National Model is the comprehensive theory of the economic long wave, the large cycles in economic activity with a period of about half a century (see e.g. Sterman, 1984a). The mechanism of "capital self-ordering" seems to play an important role in the long wave. A simple model of a one-sector capital production system built by John Sterman shows how the self-ordering of capital is sufficient to cause long waves.

Sterman's model provides a convenient focus for a first attempt to tackle the aggregation issue; apart from being simple, it also raises some obvious questions about capital aggregation: In the simple model, the physical and technical features of the system, namely the capital/output ratio and the average lifetime of capital, are very important parameters: they significantly affect the period and amplitude, and even the existence of fluctuation. But at the same time, these aggregate parameters relate to an aggregate capital stock that is supposed to represent the real economy, where production takes place in a complex input-output structure of different industries, producing and using many different kinds of capital with widely different physical characteristics. The average lifetime of capital, for instance, may vary from a few years, in the case of automobiles and tools, to fifty years or more, in the case of buildings, roads, and other infrastructure.

To address the importance of the physical diversity of capital in the real economy, it is necessary to replace the one-sector model with a multi-sector system, where each sector may use several different kinds of capital in production and may ship its own output to several other sectors, in addition to shipments to the "household", or final demand sector. The economy becomes an input-output system of capital producing firms, where the technical coefficients, average lifetimes of the capital product, and substitution possibilities may be different in each sector.

It is assumed in the following that the reader is already somewhat familiar with Sterman's model (for a more extensive description, see Sterman, 1984b). The next section only briefly recapitulates the model. The following section discusses some issues in the approach of this study and sketches the disaggregate model. Some simulation results are then presented, followed by conclusions and suggestions for future work.

THE SELF ORDERING THEORY

The notion of self-ordering, i.e. the idea that it takes capital to produce capital, is well known in economics, mostly under the name of the "accelerator" mechanism (see e.g. Samuelson, 1939). Mathematically, it can be expressed as follows: if production is proportional to capital stock by a factor of 1/COR (capital-output ratio), and capital depreciates at a rate of 1/ALC (average lifetime of capital) then, to produce a <u>net</u> flow of capital goods HO, the capital sector must produce a <u>gross</u> flow PR of:

$$PR = HO/(1-(COR/ALC)).$$

However, this observation per se does not explain the existence of cycles. The long wave is inherently a disequilibrium phenomenon arising in the transient adjustment, where the effect of self-ordering is greater than in equilibrium, due to several factors, the most important being the need to restore backlogs after an unanticipated change in demand.

Sterman's model consists of a single capital-producing firm which receives an exogenous stream of orders for capital. In addition to the orders from outside, the aggregate capital sector orders capital from itself. All orders are accumulated in a backlog, which is then depleted as the capital is delivered. The firm's production capacity is proportional to its capital stock, but production is cut back below capacity when there is insufficient demand, i.e. when the order backlog falls below what is compatible with the existing capacity. The firm's orders for capital are based on the need to replace depreciating capital and the desire to increase or decrease capacity. In its planning, the firm takes into account the orders already placed (i.e. the capital under construction) to avoid double ordering. The desired production capacity is based both on (adaptive) expectations of future orders and the need to adjust order backlogs to normal levels.

When, from an initial equilibrium, this system is perturbed by a small change in exogenous demand, it exhibits large oscillations in capital production and capacity with a period of about 50 years--much longer than a typical business cycle fluctuation--that persist without continuous outside triggering.

Self-ordering is directly responsible for the persistence and long period of fluctuation, through several channels. First, if there is a perceived shortage of capacity, more orders are placed, swelling the backlog, reinforcing the perception of shortage of capacity. Second, as delivery delays rise, capacity arrives more slowly than expected, widening the initial discrepancy between desired and actual capacity. Third, to compensate for higher delivery delays, orders are increased, resulting in still higher backlogs and delays.

These self-inforcing mechanisms cause an initial small shortage of capacity to result in a foot-race between desired and actual capacity, leading to a large overshoot of capacity over its equilibrium value. Eventually, capacity catches up with demand, and the process is reversed, causing a rapid fall in demand and production, followed by a prolonged depression with capacity far exceeding demand. The depression lasts for about two decades, as excess capacity is depreciating, until orders once again catch up with capacity to start a new cycle.

DISAGGREGATING THE MODEL

Because of the prevalence of non-linear, highly complex models in system dynamics, it is virtually impossible to derive general mathematical results that could indicate the validity of aggregation. Moreover, system dynamicists stress the fact that validity is a relative concept. Philosophers of science have long emphasized that there is no objective way of judging the performance of a model. Ultimately, then, the validation of a model is a question of using common sense. Moreover, good judgement can only be exercised when one has a clear idea of the purpose of the model (see e.g. Forrester, 1961).

The purpose of Sterman's model was to investigate how self-ordering by itself is sufficient to cause a long wave, and what factors may control the period and amplitude of the cycle. In comparing results from a disaggregate model, three questions should thus be asked:

- Will the instability of the simple model persist, and for the same causes?
- Will the cycles in production be similar, i.e. be of about 35 to 70 years in period with a large amplitude?
- And will fluctuations in different sectors be in phase so that one can meaningfully speak of a single "wave"?

The simplest approach to the aggregation issue is to construct a disaggregate input-output version of the simple model and compare the results. However, in making the transition from one to several sectors, the question arises of how the sectors should interact. In the one-sector model, the "firm" and the "household" have no choice in what capital to order and where to order it from, since there is only one supplier and only one type of capital. This is not the case in a multi-firm setting. How, for instance, will a firm or a consumer react to changing availability (and/or price) of factors, given factor substitution possibilities? Will the decision rules vary depending on what type of capital is in question and what kind of industry the firm is in? For example, the decision to build a new plant may be influenced by other factors than the decision to buy more typewriters for the office.

To keep the focus on the physical aspects of capital, the simple assumption has been adopted that the decision making in all sectors is identical in structure to that of the simple model, i.e. the decision rules, adjustment times, shapes of non-linear functions, and the structure of ordering and production equations are the same, with only minor modifications to allow for a multi-input multi-output situation. Each sector is assumed to order from the other sectors based on a desired production capital mix that remains fixed, regardless of the relative availability of factors and the possiblities for substitution. The only difference between sectors is the production techonology, lifetime of capital product, and the share of this product in "household" or final demand.

While such simplification is done partly to limit the scope of the project, there are also limits to how far one can depart from the decision functions in the simple model, since they are an inseperable part of the theory it represents; at what point does a disaggregate model no longer represent the same theory?

The question of when the two models are "equivalent" relates closely to the old problem of capital aggregation in economics. In order to compare a simple and a disaggregate model in a meaningful way, one must require that the aggregate parameters of the disaggregate model are the same as those of the simple model. But the question is what aggregation rule is appropriate. The economics literature has shown that unfortunately, it is impossible to give exact rules and that the aggregation procedure must be taylored to the particular aspect of capital one wishes to investigate (see e.g. Fisher, 1969).

It is common usage to define the aggregate capital/output ratio as the aggregate dollar value of capaital divided by the aggregate production capacity measured in dollars per year. Likewise the aggregate lifetime of capital can be defined as the aggregate dollar value of capital stock divided by the aggregate depreciation rate in dollars per year. However, prices are not included in the model, and even if they were, they would change over time, making it difficult to interpret the results. The only way around this is to aasume that relative prices of different capital types are constant. Moreover, units of capital can be defined in such a way as to allow direct addition of the physical capital stocks to obtain aggregates. (see Kampmann, 1984.)
A series of simulation experiments were made with a Two-sector and a Five-sector model, respectively, in which all sectors were assumed to have a Cobb-Douglas production function, thus relegating the issue of capital substitution to future studies. Given the Cobb-Douglas production functions and the number of sectors, *n*, one can completely describe the system by the following parameters:

- The coefficient of capital type i in sector j's production function, RVS(i,j); i=1,...,n; j=1,...,n. (When production cost is minimized, this parameter is equal to the flow value share of capital i in sector j's production cost, hence the acronym RVS for "Reference Value Share")

- The capital/output ratio for sector j, FOR(j); j=1,...,n (In the model, the more general name "Factor/Output Ratio" was used, hence the acronym FOR(j))

- The flow share of sector j in final demand, IVSED(j); j=1,...,n (The acronym stands for "Initial Value Share of End Demand")

- The average lifetime of capital type i, AL(i); i=1,...,n

Appendix 1 contains a computer listing of the two-sector model. For a more detailed explanation and description of the models, see Kampmann (1984).

SOME SIMULATION RESULTS

A couple of the many experiments performed with the model are described and reproduced below to give an impression of the range of possible behavior. The experiments centered around two major themes:

- How will variations in input-output structure (including the capital intensity of individual sectors) affect behavior, assuming the lifetime of each capital type is the same? This corrensponds to variations in the paramters RVS(i,j), IVSED(j) and FOR(j)

- How will diversity in capital lifetimes affect behavior in systems with identical (flow) input-output structures? This corresponds to varying the parameters AL(i)

While the experiments show a wide range of behavior, there are certain features which persist throughout. The examples below illustrate some of these features, which are summarized under the conclusions below.

Variations in the Input-Output Structure.

If all capital has the same lifetime and all sectors have the same overall capital intensity, how will a disaggregate input-output system behave differently from a simple one-sector system? Figure 1 shows an example of what can happen when the input-output structure of the system is varied.

It turns out that the symmetry of the system is of crucial importance for the result. A symmetrical input-output system is defined as one where an increase in orders for any capital product will require an increase in each sector's gross output of the same proportion for all sectors. This condition is satisfied if any given sector's cost share is the same everywhere, i.e. if:

RVS(i,j) = IVSED(i) for all i,j.

A symmetrical system would behave exactly as the simple model if household orders were perturbed from equilibrium in the same proportion. Even when incoming orders are not exactly proportional at all times, the interdependence implied by the input-output structure results in a very strong entrainment, bringing all sectors to fluctuate uniformly. This entrainment is illustrated in Figure 1(a), which shows a simulation of a symmetrical two-sector system where each sector has an equal share of end demand and uses the two capital types in equal proportions in the production function, i.e.

$$RVS(i,j) = \begin{matrix} .5 & .5 \\ .5 & .5 \end{matrix} \qquad IVSED(j) = .5 \quad .5$$

At time 10, the household demand for sector 1's product only is increased 5 percent. If both household demands were stepped uniformly, the response would be identical to a one-sector system, but even still, the two sectors move quickly into phase to give a fluctuation pattern virtually identical to that of the simple model.

In figure 1(b) and (c), the symmetry is broken by changing the shares of the two sectors in final demand, IVSED(j). The split is changed from fifty-fifty in Figure 1(a) to 2:1 in (b) and the extreme in (c), where Sector 1, henceforth called sector "F" for "Final" thus delivers all final demand, while Sector 2, henceforth called sector "I" for "Intermediate", produces only intermediate goods.

The result is a more complex pattern of fluctuation. In Figure 1(b) the long wave breaks into a combination of a 70 year and a smaller 35 year cycle, and Figure 1(c) shows a double cycle with the longer period of about 50 years.

Moreover, the relative amplitude of fluctuation will be different for each sector and for each cycle. The change in relative fluctuation comes from the change in the composition of the incoming orders in each sector. Thus, when sector "F" holds a larger share of household demand, the "self-ordering" component of its incoming orders is relatively less than in sector "I". It is not surprising that the sector whose orders thus fluctuate the most also shows the most fluctuation in production and capacity.

The difference in relative amplitudes of the two sectors also explains the double cycle: After the initial disturbance, the "I" sector will expand relatively more above the equilibrium capacity than the "F" sector. In the subsequent downturn, there will therefore be ample availability of sector "I"'s product, and when demand meets capacity in sector "F", sector "I" will still have unused capacity. As sector "F" expands again, the excess of the "I" product will reduce the strength of the self-ordering mechanism that fuels the expansion, and production will catch up faster with demand. In Figure 1(b), demand eventually catches up with capacity in sector "I", but at such a late stage that the new peak is significantly lower than the previous one. Both sectors will therefore have less excess capacity in the next downturn, setting the stage for a larger expansion in the following upturn. In 1(c) the disparity is so pronounced that sector "I" has unused capacity even at the peak of the intermediate cycle. Through the duration of the long cycle downturn, sector "F" therefore effectively behaves like a one-sector model with a capital/output ratio half of normal, since only half of sector "F"'s orders for capital fall upon itself. Such a system would show a very lightly damped cycle of about 20 years (see Kampmann, 1984, App.1). The intermediate cycle observed in 1(c) is thus an internal dynamic of sector 1.

Figure 1
Variations of Input-Output Structure in a Two-Sector System

The sectors are named "F"inal, "I"ntermediate and "H"ousehold, respectively. In all three runs, the initial equilibrium is perturbed by stepping up household orders for product "F" 5% at time 10. In the symmetrical system in (a), where sector "F" and "I" hold equal shares of household demand, strong entrainment results in behavior almost identical to the simple model. But as the split of end demand between product "F" and "I" is changed in (b) and (c), the long wave breaks into a double cycle, due to a change in the strength of the self-ordering effect in the two sectors.

Figure 1(a)

Cost Share		in Sector F	I	H
of Product	F	.5	.5	.5
	I	.5	.5	.5

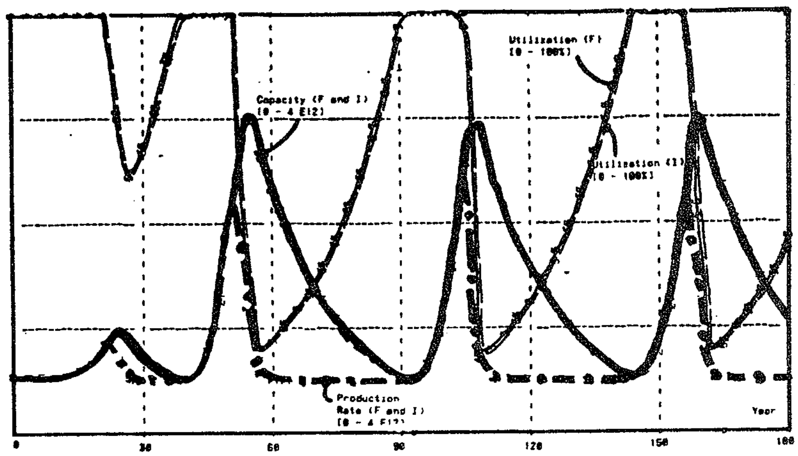


Figure 1(b)

Cost Share		in Sector F	I	H
of Product	F	.5	.5	.67
	I	.5	.5	.33

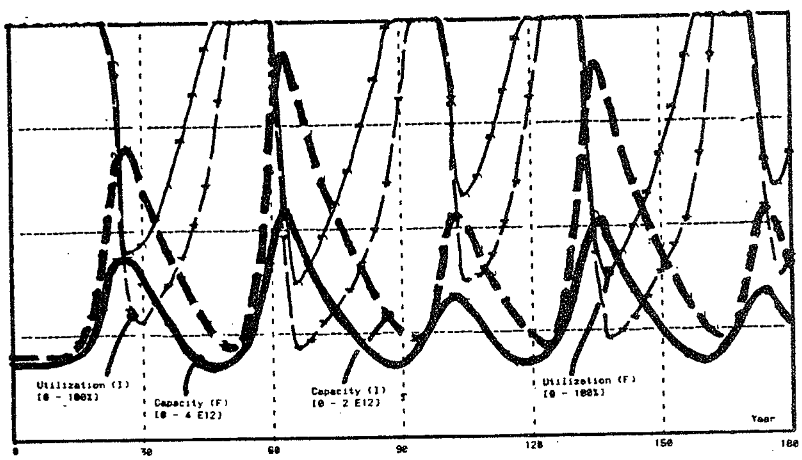
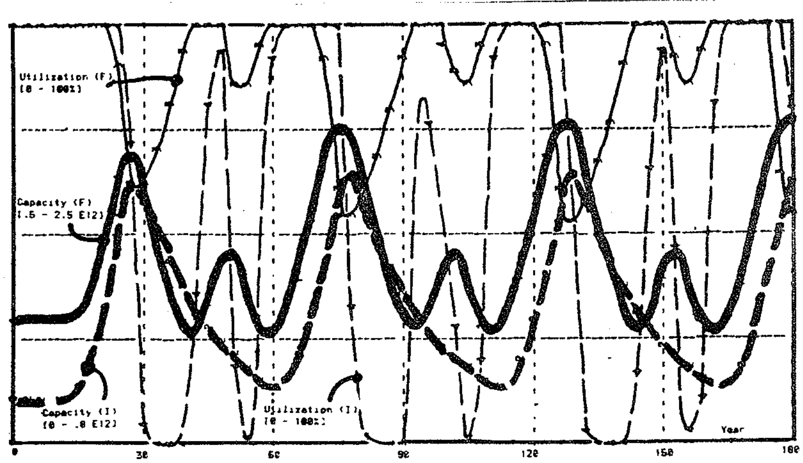


Figure 1(c)

Cost Share		in Sector F	I	H
of Product	F	.5	.5	1
	I	.5	.5	0



In the above variations in input-output structure, the capital/output ratio was kept the same for each sector. In the simple model, this parameter is very important because it determines the strength, or the gain, of the self-ordering loop. An increase in this parameter will increase the self-ordering effect, causing fluctuations of longer period and higher amplitude. In a disaggregate system, however, there is not one but many self-ordering loops, and they will in general not be of equal gain if the capital intensity of production varies significantly among sectors, or if capital lifetimes differ.

In the particular case where the system is symmetric, as defined above, a difference in capital intensity has no effect on behavior. Only when non-symmetry is allowed will there be any effect. It is possible, for instance, to increase the capital intensity in all sectors while keeping aggregate capital intensity constant by increasing the less capital intensive sectors' share of end demand. The effect is similar to increasing the capital/output ratio in the simple model: the period and amplitude of fluctuation both increase.

On the other hand, even quite extreme variations in capital intensity have not nearly the effect a change in the capital/output ratio has in the simple model. Variations by a factor of 10 of the sectoral capital/output ratios produce fluctuations with a period between 28 and 60 years and only moderately different amplitudes. Moreover, the qualitative behavior is the same as in the simple model.

Variations in Capital Lifetimes

Table 1 summarizes the result of varying the lifetimes of the two kinds of capital in a model with the same completely symmetric input-output flow structure as in Figure 1(a) above. The variation is done so as to keep the aggregate equilibrium average lifetime at 20 years as in the simple model. In this particular input-output configuration, the equilibrium aggregate lifetime is simply the arithmetic mean of each capital lifetime.

In all cases, the outcome is still a very regular limit cycle with the same basic characteristics as that of the simple model. However, because of the faster dynamics inherent in the short-lived capital stocks, there is now the possibility of an additional shorter cycle. The double cycle occurring in the previous example (Figure 1(b) and (c)) had its root in the change in the relative strength of self-ordering for the two sectors (a "gain" component). The intermediate cycle now results from the difference in the time it takes for excess capacity to depreciate (a "delay" component).

As Table 1 testifies, there is no simple relationship between the diversity of lifetimes and the period and amplitude of fluctuation. The period stays in a range from about 40 to about 65 years, and the amplitude generally declines as the diversity of lifetimes increases, but there are several exceptions to that rule. These irregularities are due to the non-linear frequency entrainment of the two sectors (see Kampmann, 1984, App.4).

The last run of Table 1 is reproduced in Figure 2. Here, sector "S" for Short and sector "L" for Long produce a product with a lifetime of 5 and 35 years, respectively. The system shows a combination of two modes, a long cycle of a 66 year period, and a shorter, damped cycle of about 15 years. The

Figure 2

Introducing a Split in Capital Lifetimes in a Two-Sector Model

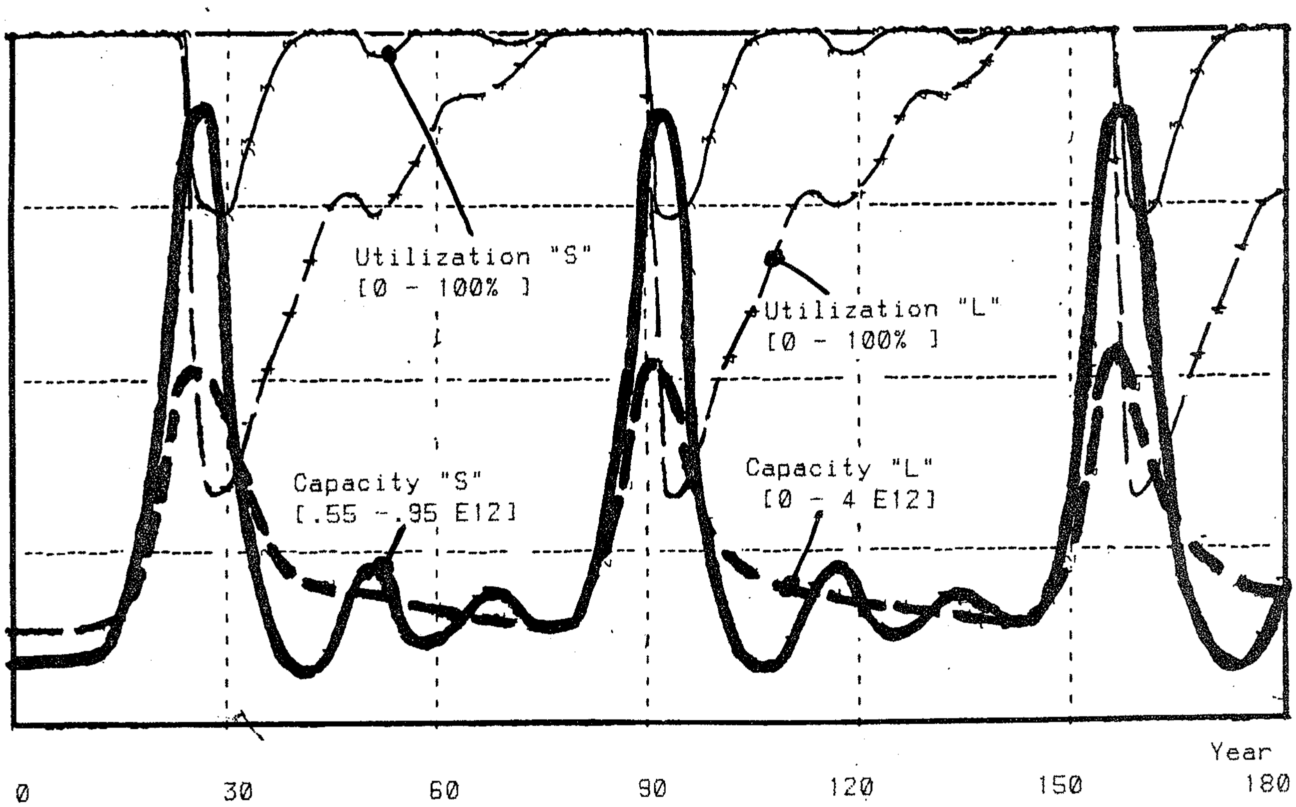


Table 1

Results of Varying Average Lifetimes of Capital in Two-sector Model

Run #	AL(1) years	AL(2) years	Period[a] years	Peak Value[a] PC(1) % of Base Case	PC(2)
Base[b]	20	20	49	100	100
19	18	22	50	87	98
20[c]	16	24	45/31	71/32	93/39
21[d]	14	26	63	72/66[e]	95
22[d]	12	28	61	57	92
23[c,d]	10	30	60/49	49/35	92/57
24[d]	5	35	66	31	73

a) Limit cycle steady state values

b) The base run is identical to run 5

c) The cycle is not uniform; every other peak is larger. Both peak values are shown, and where the time distance between peaks alternates in a similar manner, both of these are shown in the "period" column.

d) In addition to the long cycle, there is a short intermediate fluctuation in PC(1).

e) Every other peak in PC(1) is larger, and both peak values are shown. PC(2) shows a uniform cycle.

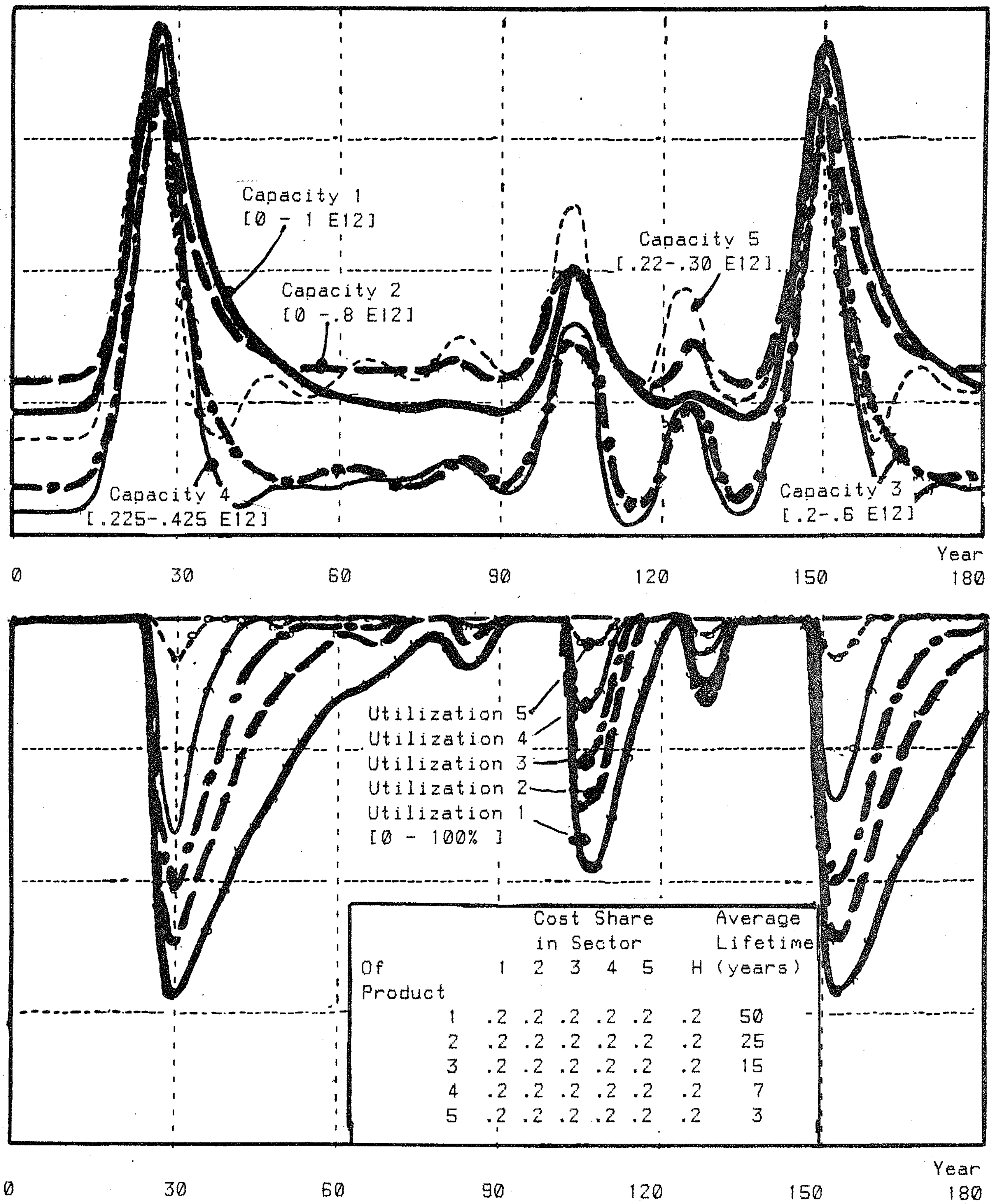
	Cost Share in Sector S	L	H	Average Lifetime (years)
of S	.5	.5	.5	5
Product L	.5	.5	.5	35

Regardless of whether the flow input-output structure of a system is symmetrical, introducing substantial differences in capital lifetimes can result in composite cycles. The figure shows the last case of Table 1, where sector "L" and sector "S" produce capital that has a 35 and a 5 year average lifetime, respectively. Sector "L" fluctuates the most in the long cycle, while only sector "S" exhibits the shorter cycle.

Figure 3

Variations in Capital Lifetimes in a Five-Sector System

Large differences in capital lifetimes in a five-sector model typically leads to rather irregular fluctuations, where long-lived sectors are hit the worst.



Of Product	Cost Share in Sector 1	2	3	4	5	H	Average Lifetime (years)
1	.2	.2	.2	.2	.2	.2	50
2	.2	.2	.2	.2	.2	.2	25
3	.2	.2	.2	.2	.2	.2	15
4	.2	.2	.2	.2	.2	.2	7
5	.2	.2	.2	.2	.2	.2	3

fluctuations in overall production are a great deal less than in the simple model, but vary significantly between the two sectors; sector "S" shows relatively less fluctuation than sector "L". Moreover, only sector "S" capacity exhibits the short cycle.

The result is thus quite parallel to the difference between sector "I" and sector "F" in figure 1, though for different reasons. The excess of factor "L" takes too long to depreciate for sector "L" to show a short cycle. In contrast, the excess of factor "S" falls quickly in the long cycle downturn and sector "S" then operates essentially like a one-sector system with a factor/output ratio of 1.5 and an average lifetime of 5 years. The result is the damped 15 year cycle. When the excess of factor 2 has finally been eliminated, self-ordering regains its full strength, causing a new major upsurge in demand and production.

To get an impression of the possible range of behavior in a more complicated input-output system, runs were performed with a five-sector model, letting the average lifetimes vary widely while keeping the aggergate lifetime equal to 20 years. As an typical example, Figure 3 shows the response of a system where all five sectors have equal shares in both end demand and in each others production functions, but where the lifetime varies between products from 50 years in sector 1 to 50 years in sector 5.

The behavior is much less regular than in simpler systems. One sees a number of occasional major surges in production and capacity, and a shorter, more regular fluctuation. The sectors with the longest lived products fluctuate uniformly more in large surges than the sectors with shorter-lived products. The long-lived sectors remain depressed for longer periods because the the excess of the factor they produce takes longer to decay. During this period, the other sectors, like in the previous examples, behave like a system of production with shorter lived products and with a lower overall capital/output ratio than the full system, giving a short, damped fluctuation. However, as the excess of the long-lived products is eliminated, the strength of self-ordering increases, and the system becomes more prone to self-created major surges in demand. Thus, the short cycle fluctuations start to get larger, and sooner or later all sectors simulataneously expand in a traditional long wave peak.

SUMMARY AND CONCLUSIONS

The simulation experiments have produced a wide range of behavior, but also some recurrent features:

1) Like the simple system, all the disaggregate systems tested show major fluctuations in production and capacity that persist without continuous exogenous disturbances, and that are much too large and long to be explained by simple adjustment delays.

2) A disaggregate system tends to exhibit a combination of major expansions and contractions, occurring over a long time period, and a shorter, more regular oscillation. While the small cycles, if they occur, may only affect some sectors, the major surges occur in all sectors simulaneously.

3) The major fluctuations in production are persistently more violent in

sectors whose incoming orders come in the main from the production system rather than from consumers. And the fluctuations in a sector are larger the longer the lifetime of its product. Thus, the more a sector has the characteristics of a capital producer, the more vulnerable that sector is to long wave fluctuations.

4) Variations in the input-output structure tend to change the behavior from a uniform to a compound cycle. The period of the long cycle tends to fall in the 30 to 70 year range. The more the household/non-household split of incoming orders is the same for each sector, the closer the behavior will be to a one-sector system. Moreover, the sectors will quickly entrain their fluctuations. Thus, a group of sectors who have largely the same mix of production capital and whose incoming orders vary roughly in proportion can be well approximated by a single sector.

5) Variations in sector capital intensity can change the period of fluctuation, but not dramatically. The higher the symmetry of the system, the less discrepancies in sector capital intensity will alter the behavior.

6) With very large variation in capital lifetimes, the major fluctuations can be quite irregular. Indeed, it becomes difficult to speak of a single "cycle"; rather, self-ordering creates a potential in the economy for large self-created surges in demand and production.

The most fundamental feature of the simple model is its inherent tendency to create very large long term fluctuations in capacity and production that persist without continuous outside triggering. This feature has reappeared in all the simulations of the disaggregate model, and it is evident that the surges and collapses of demand are created in the disaggregate model by the same basic self-ordering mechanism. Based on the evidence so far, one must therefore conclude that the self-ordering hypothesis retains its validity for a disaggregate system.

A conspicuous difference between the simple and the disaggregate model occurs in the shape of fluctuations. While the simple model shows a completely regular cycle with a well defined period, the disaggregate version generally generates more complex and sometimes quite irregular patterns. Such results suggest that the long wave should be sought of as an inherent (endogenous) ability in the economy to create large fluctuations, rather than as a regular cycle.

On the other hand, there are several factors which may shape the irregular fluctuations of an input-output system into a more coherent pattern. Foremost among these is probably the effect of substitution in production capital. For instance, if a factor is in excess supply, its delivery time and/or price will be low, causing firms to shift to a more intensive use of that factor. The resulting increase in orders will draw down the excess supply faster--an important way to bring long-lived sectors more into phase with the rest of the economy. The most important next step in further research will therefore be to introduce a more sophisticated ordering rule in the model that reflects relative availability of factors and the elasticity of substitution.

Work on the National Model suggests that there are several structures other than capital self-ordering involved in the long wave (Sterman, 1984a). It is conceivable that these structures, involving prices, interest rates, wages and other factors, would mold the fluctuations of the capital sector into a more regular pattern. It would be interesting to test this hypothesis by introducing in the National Model a disaggregate capital structure that in isolation would give highly irregular fluctuations.

Another next step could be to examine the role of non-linearities in the disaggregate model, and compare it to the simple model. In some simulations (Kampmann, 1984), a change in non-linearities which is sufficient to stabilize the one-sector system does not stabilize a disaggregate system. The self-ordering theory could thus be further refined, and possibly strengthened.

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Equation Listing of the Two-sector Model, KON2

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NOTE
NOTE
* DISAGGREGATED TWO-SECTOR KONDRATIEFF MODEL
NOTE
NOTE   DEFINITION OF SECTORS
NOTE
I      TF=2
I      TP=2
FOR    F=1,TF/P=1,TP
NOTE
NOTE   PDUCTION SECTORS
NOTE
NOTE PRODUCTION RATE
NOTE
A      PR.K(P)=PC.K(P)*CU.K(P)
A      CU.K(P)=TABHL(CUT,IP.K(P)/PC.K(P),0,2,.2)
T      CUT=0/.3/.55/.75/.9/1/1/1/1/1/1
A      IP.K(P)=B.K(P)/NDD(P)
NOTE
NOTE PRODUCTION CAPACITY
NOTE
A      PC.K(P)=S1*PC1.K(P)+S2*PC2.K(P)+S3*PC3.K(P)
A      PC1.K(P)=RP.K(P)*MIN(S.K(1,P)/RS.K(1,P),S.K(2,P)/RS.K(2,P))
A      PC2.K(P)=RP.K(P)*EXP(SPC.K(P))
A      PC3.K(P)=RP.K(P)*((RVS(1,P)*S.K(1,P)/RS.K(1,P))+
X      (RVS(2,P)*S.K(2,P)/RS.K(2,P)))
A      SPC.K(P)=RVS(1,P)*LOGN(S.K(1,P)/RS.K(1,P))+
X      RVS(2,P)*LOGN(S.K(2,P)/RS.K(2,P))
NOTE
NOTE STOCK AND SUPPLY LINE OF FACTOR
NOTE
L      S.K(F,P)=S.J(F,P)+DT*(A.J(F,P)-D.J(F,P))+PEVS.J(F,P)
A      D.K(F,P)=SDL(F,F)*S.K(F,P)/AL(F)
A      A.K(F,P)=SDL(F,P)*SL.K(F,P)/AT.K(F)
L      SL.K(F,P)=SL.J(F,P)+DT*(O.J(F,P)-A.J(F,P))+PEVSL.J(F,P)
NOTE
NOTE ORDERS FOR FACTOR
NOTE
A      O.K(F,P)=SDL(F,F)*S.K(F,P)*FO.K(F,P)
A      FO.K(F,P)=TABXT(FOT,IFO.K(F,P),-.1,.5,.05)
T      FOT=0/0/0/.05/.1/.15/.2/.25/.3/.35/.38/.4/.4
A      IFO.K(F,P)=(D.K(F,P)+CSL.K(F,P)+CS.K(F,P))/S.K(F,P)
A      CSL.K(F,P)=(DSL.K(F,P)-SL.K(F,P))/TASL(F)
A      DSL.K(F,P)=PAT.K(F)*D.K(F,F)
A      PAT.K(F)=NAT(F)*EATSL.K(F)
A      EATSL.K(F)=TABXT(TEATSL.AT.K(F)/NAT(F),0,3,.5)
T      TEATSL=0/.5/1/1.5/2/2.5/3
A      CS.K(F,P)=(DS.K(F,P)-S.K(F,F))/TAS(F)
NOTE
NOTE DESIRED STOCK OF FACTOR
NOTE
A      DS.K(F,P)=RS.K(F,P)*RDRS.K(F,P)
A      RDRS.K(F,P)=TABXT(TRDRS,IS.K(F,P)/RS.K(F,P),-.5,7.5,.5)
T      TRDRS=0/0/.5/1/1.5/2/2.5/3/3.5/4/4.5/5/5.4/5.7/5.9/6/6
A      IS.K(F,P)=IPC.K(P)*RS.K(F,P)/RP.K(P)
A      IPC.K(P)=EIO.K(P)+CB.K(P)
A      CB.K(P)=(B.K(P)-IB.K(P))/TAB(P)
A      IB.K(P)=NDD(P)*EIO.K(P)
L      EIO.K(P)=EIO.J(P)+(DT/TAIO(P))(IO.J(P)-EIO.J(P))+PEVEO.J(P)
NOTE
NOTE   COUPLING EQUATIONS
NOTE
A      IO.K(P)=SUMV(O.K(P,*),1,TF)+HO.K(P)
A      B.K(P)=SUMV(SL.K(P,*),1,TF)+HSL.K(P)
A      DD.K(F)=B.K(P)/PR.K(P)
A      AT.K(F)=DD.K(F)
NOTE
NOTE   HOUSEHOLD SECTORS
NOTE
L      HSL.K(P)=HSL.J(P)+DT*(HO.J(P)-HA.J(P))+PEVHSL.J(P)
A      HO.K(P)=SSHO.K*RO(P)*(1+STEP(FSHO,TSHO)+STEP(FSHOP(P),TSHO))
A      SSHO.K=CLIP(1,0,TIME.K,0)
A      HA.K(P)=HSL.K(P)/AT.K(P)
NOTE
NOTE   PARAMETERS
NOTE
N      NAT(F)=NDD(F)
N      SDL(F,P)=CLIP(1,0,RVS(F,P),1E-9)
T      NDD(*)=1.5/1.5
T      RVS(*,1)=1/0
T      RVS(*,2)=0/1
T      TASL(*)=3/3
T      AL(*)=20/20
T      TAS(*)=3/3
T      TAB(*)=1.5/1.5
T      IVSED(*)=.5/.5
T      TAIO(*)=2/2
N      RO(P)=THO*IVSED(P)
C      THO=1E12
C      FSHO=.05
C      TSHO=1E9
N      S1=MAX(0,1-MAX(SALT-1,1-SALT))
N      S2=MAX(0,1-MAX(SALT-2,2-SALT))
N      S3=MAX(0,1-MAX(SALT-3,3-SALT))
C      SALT=2
T      FSHOP(*)=0/0
T      FOR(*)=3/3
NOTE
NOTE   PROCEDURE TO INITIALIZE IN EQUILIBRIUM
NOTE
N      S(F,P)=1E-2
N      SL(F,P)=1E-2
N      HSL(P)=1E-2
N      EIO(P)=1E-2
L      INS.K(F,P)=INS.J(F,P)+SDL(F,P)*(DINS.J(F,P)-INS.J(F,P))
N      INS(F,P)=1E-9
A      DINS.K(F,P)=RVS(F,P)*DIP.K(P)*FOR(P)*AL(F)/ESAL(P)
A      DIP.K(P)=SUMV(INO.K(P,*),1,TF)+RO(P)
A      INO.K(F,P)=INS.K(F,P)/AL(F)
A      PEVS.K(F,P)=PULS.K*DINS.K(F,P)
A      PEVSL.K(F,P)=PULS.K*NAT(F)*INO.K(F,P)
A      PEVEO.K(P)=PULS.K*DIP.K(P)
A      PEVHSL.K(P)=PULS.K*RO(P)*NDD(P)
A      PULS.K=STEP(1,-DT)-STEP(1,0)
A      RP.K(P)=1E-9+STEP(DIP.K(P),0)
A      RS.K(F,P)=1E-9+STEP(INS.K(F,P),0)
NOTE
NOTE   AGGREGATES
NOTE
N      ESAL(P)=SUMV(CESAL(*,P),1,TF)
N      CESAL(F,P)=RVS(F,P)*AL(F)
A      EFOR.K=STEP(AFOR.K,0)
A      EAL.K=STEP(AAL.K,0)
A      AFOR.K=AS.K/APC.K
A      AAL.K=AS.K/AD.K
A      AS.K=SUMV(SAS.K(*),1,TF)
A      AD.K=SUMV(SAD.K(*),1,TF)
A      SAS.K(P)=SUMV(S.K(*,P),1,TF)
A      SAD.K(P)=SUMV(D.K(*,P),1,TF)
A      APC.K=SUMV(PC.K(*),1,TF)
A      SAL.K(P)=SAS.K(P)/SAD.K(P)
A      APR.K=SUMV(PR.K(*),1,TP)
NOTE
NOTE   SIMULATION CONTROL PARAMETERS
NOTE
A      PLTPER.K=CLIP(PLPER,IPLPER,TIME.K,TCP)
C      PLPEP=1
C      IPLPEP=0
C      TCP=0
A      PRTPER.K=CLIP(PRPER,IPRPER,TIME.K,0)
C      PRPER=0
C      IPRPER=0
N      TIME=-NITR*DT
C      NITR=10
SPEC   LENGTH=0/DT=.125
```

APPENDIX 1: LIST OF VARIABLES

Variable List

Symbols for units:
% - Dimensionless
U - Units of product
YR - Year

A	ARRIVAL OF FACTOR (U/YR)
AAL	AGGREGATE AVG. LIFETIME (YR)
AD	AGGREGATE DEPRECIATION (U/YR)
AFOR	AGGREGATE FACTOR/OUTPUT RATIO (YR)
APC	AGGREGATE PRODUCTION CAPACITY (U/YR)
APR	AGGREGATE PRODUCTION RATE (U/YR)
AS	AGGREGATE STOCK OF FACTOR (U)
AT	ARRIVAL TIME (YR)
B	BACKLOG (U)
CB	CORRECTION FOR BACKLOG (U/YR)
CS	CORRECTION FOR STOCK (U/YR)
CSL	CORRECTION FOR SUPPLY LINE (U/YR)
CU	CAPACITY UTILIZATION (%)
CUT	CU TABLE
D	DEPRECIATION OF FACTOR (U/YR)
DD	DELIVERY DELAY (YR)
DS	DESIRED STOCK (U)
DSL	DESIRED SUPPLY LINE (U)
EATSL	EFFECT OF ARRIVAL TIME ON DESIRED SUPPLY LINE (%)
EIO	EXPECTED INCOMING ORDERS (U/YR)
F	FACTOR INDEX (%)
FO	FRACTIONAL ORDERS (%/YR)
FOT	FO TABLE
HA	HOUSEHOLD ARRIVALS (U/YR)
HO	HOUSEHOLD ORDERS (U/YR)
HSL	HOUSEHOLD SUPPLY LINE (U)
IB	INDICATED BACKLOG (U)
IFO	INDICATED FRACTIONAL ORDERS (%/YR)
IO	INCOMING ORDERS (U/YR)
IP	INDICATED PRODUCTION (U/YR)
IPC	INDICATED PRODUCTION CAPACITY (U/YR)
IS	INDICATED STOCK (U)
O	ORDERS FOR FACTOR (U/YR)
P	PRODUCT INDEX (%)
PAT	PERCEIVED ARRIVAL TIME (YR)
PC	PRODUCTION CAPACITY (U/YR) (1:LEONTIEFF 2:COBB-DOUGLAS 3:INF. SUBST.)
PR	PRODUCTION RATE (U/YR)
RDRS	RATIO OF DESIRED TO REFERENCE STOCK (%)
S	STOCK OF FACTOR (U)
SAD	SECTOR AGGREGATE DEPRECIATION (U/YR)
SAL	SECTOR AVERAGE LIFETIME (YR)
SAS	SECTOR AGGREGATE STOCK (U)
SFC	SUM OF FACTOR CONTRIBUTIONS (%)
SL	SUPPLY LINE OF FACTOR (U)
SSHO	SWITCH TO START HOUSEHOLD ORDERS (%)
TEATSL	TABLE FOR EATSL
TF	TOTAL FACTORS (%)
TP	TOTAL PRODUCTS (%)
TRDRS	TABLE FOR RDRS

Parameters:

AL	AVERAGE LIFETIME OF FACTOR,PRODUCT (YR)
FOR	FACTOR OUTPUT RATIO (YR)
FSHO	FRACTION STEP IN ALL HOUSEHOLD ORDERS (%)
FSHOP	FRACTIONAL STEP IN HOUSEHOLD ORDERS FOR SPECIFIC PRODUCT (%)
IPLPER	INITIAL PLOT PERIOD (YR)
IPRPER	INITIAL PRINT PERIOD (YR)
IVSED	INITIAL VALUE SHARE OF END DEMAND (%)
NAT	NORMAL ARRIVAL TIME (YR)
NDD	NORMAL DELIVERY DELAY (YR)
PLPER	PLOT PERIOD (YR)
PRPER	PRINT PERIOD (YR)
RO	REFERENCE HOUSEHOLD ORDERS (U/YR)
RVS	REFERENCE VALUE SHARE (%)
TAB	TIME TO ADJUST BACKLOG (YR)
TAIO	TIME TO AVERAGE INCOMING ORDERS (YR)
TAS	TIME TO ADJUST STOCK (YR)
TASL	TIME TO ADJUST SUPPLY LINE (YR)
TCP	TIME TO CHANGE PERIOD (YR)
THO	TOTAL HOUSEHOLD ORDERS (U/YR)
TSHO	TIME TO STEP HOUSEHOLD ORDERS (YR)

Variables for equilibrium initialization and special parameters:

CESAL	COMPONENT IN ESAL (YR)
DINS	DESIRED INITIAL STOCK (U)
DIP	DESIRED INITIAL PRODUCTION (U/YR)
EAL	EQUILIBRIUM AGGRAGATE FACTOR LIFETIME (YR)
EFOR	EQUILIBRIUM FACTOR/OUTPUT RATIO (YR)
ESAL	EQUILIBRIUM SECTOR AGGREGATE LIFETIME (YR)
INO	INITIAL ORDERS (U/YR)
INS	INITIAL STOCK (U)
NITR	NUMBER OF ITERATIONS IN INITIALIZATION (%)
PEVEO	PULSE OF EQUILIBRIUM VALUE OF EIO (U/YR)
PEVHSL	PULSE OF EQUILIBRIUM VALUE OF HSL (U)
PEVS	PULSE OF EQUILIBRIUM VALUE OF S (U)
PEVSL	PULSE OF EQUILIBRIUM VALUE OF SL (U)
PULS	PULS FUNCTION (%)
RP	REFERENCE PRODUCTION (U/YR)
RS	REFERENCE STOCK (U)
S1,S2,S3	SWITCHES FOR CHOICE OF PRODUCTION CAPACITY
SALT	SWITCH FOR ALTERNATIVES IN PRODUCTION CAPAICITY
SDL	SWITCH FOR DELIVERY LINK