A Reduced Dynamic Model for Evaluating the Impact of Man on the Environment

L.Gardini, H.Sedehi, R.Serra

TEMA S.p.A.

ABSTRACT

A representation of socio-economic systems using reduced models allows a "qualitative" type of analysis to be carried out. It is often the case, especially in long term processes, that the main interest is directed towards the asymptotic behaviour of the solutions as a function of the initial state and to evaluating the properties of stability of stationary states. In this article, after a short outline of the procedure and methodology adopted, we describe the application of these techniques in the construction and use of a dynamic model for the design of a tourist village. The model, which mainly deals with the impact of man on the environment, serves to evaluate the social and economic effects of the construction of a tourist centre in a natural environment which must be conserved.

INTRODUCTION

A typical feature of socio-economic modelling as distinct from physical modelling, is the lack of natural laws which may be relied upon for rigorous foundations (leaving aside for the time being subtle questions about what the laws of nature really are). This leads to uncertainties about the form of the basic model equations. Further uncertainties regard parameter values and the initial conditions.

These well-known observations, which are common to almost all socio-economic models (with the exception of a few rather trivial examples) lead to the conclusion that families of models, rather than individual ones, have to be considered.

This situation is not dramatic, as long as one is dealing with structurally stable models. However, most models do not seem to belong to the set of structurally stable systems. The well known phenomena of sensitive dependence on initial conditions, bifurcations, chaos (Allen et al., 1982; Nicolis and Prigogine, 1977; Haken, 1977, 1983; Cvitanovic, 1984; Serra et al., in press) invite caution in the use of such models.

This does not mean, of course, that models are useless. It means, however, that in order for them to be useful rather than misleading, they must be carefully handled and their results properly interpreted. It is clear that such observations could make the penetration of system dynamics into industrial environments more difficult. It is always easier to sell a reliable tool, rather than problems. However, a growing number of managers and staff assistants are realizing the need for a better understanding of the dynamic behaviour of the systems they are working with. A key step in this direction has been the observation, on several occasions, of counter-intuitive transient behaviour of complex systems by J. Forrester and co-workers (Forrester, 1968). There are also examples of counterintuitive dynamic behaviour in the asymptotic regime, which are worthy of study.

A question naturally rises about the "correct" modelling level. Recent developments in hard and soft sciences has led to the so-called "science of complexity" (Jantsch, 1980), (Capra,1982). Here, it will suffice to say that, in the science of complexity, there is no single, privileged viewpoint, or modelling level. The need is rather that of integrating information coming from different perspectives, which are all partial and incomplete. System dynamics can provide a field of application for this approach, since different models are needed for answering different questions about the same physical system. A major requirement is non-contradiction among the different models, while it seems unrealistic, in general, to require that they all be deduced by an overall global model via projection techniques.

We discuss here a model for a tourist settlement in a natural environment. This problem has been studied by our group for a tourist village in Sardinia, and a model was described in a preceding system dynamics conference (Sedehi et al., 1983). The model considered was rather a detailed one, describing the interaction between different natural species, different kinds of tourists, the structures available in the village and their ageing, and the economic-financial subsystem. The main goal of the model described here is a study of the long term behaviour of the system.

In this case it seems particularly interesting to analyse possible complex asymptotic behaviours. However, such an analysis is much better performed in models with only a few variables. We present here a simplified version of the model, and discuss its behaviour.

The need for reduced descriptions has been stressed previously (Serra et al., 1984), and we will not go into the details here. The basic idea is, that since our brain can work with a limited number of variables, a projection of some kind is always required, even if we use a complicated model with a lot of interacting dynamic variables. This projection is often implicitly done by the system dynamist, but in some cases one can devise formal projection techniques. This is the case when there are variables whose dynamics differ considerably, and the corresponding projection techniques are known as adiabatic elimination procedures.

Another point deserves consideration. In the case of the tourist village it is natural to consider a discrete map rather than a continuous dynamic system, because the main cooperative phenomenon is the transmission of information among tourists. The tourists who visited the village one year will express their opinion, positive or negative, to their friends, and this will influence the number of visitors in the following year. There is a natural clock, and the natural unit of time is the year. An analogous situation exists for natural species, if their reproduction cycle is annual. We are thus led to a model of the type:

$$N(t+1) = f(N(t), \text{other variables}) \tag{1}$$

rather than

$$N(t) = f(N(t), \text{other variables}) \tag{2}$$

It is well known that complicated dynamic behaviours (e.g.,chaos) are much more common in discrete maps than in continuous systems.

It is to be stressed, in this respect, that although the conceptual framework of system dynamics is continuous in time, most applications integrate the dynamic system with the simple Euler algorithm, thus transforming it into a discrete map of the type of Eq. (1).

MODEL DESCRIPTION

We refer the reader to Sedehi et al. (1983) for a detailed discussion of the model. Its main features are summarized below.

The number of visitors at time t, N_t, is determined by two factors: advertising and the number of visitors in the preceding years and their satisfaction. Satisfaction, in turn, is determined by the state of the environment and by crowding.

Let us start from the environment, which will be represented by M interacting species. We choose a simple standard model in population biology (Goel et al., 1971), i.e.:

$$\dot{A}_j = \frac{1}{T_j} A_j - \frac{1-g_j}{T_j} A_j^2 - \sum_{\substack{k \\ k \neq j}} \frac{c_{jk}}{T_j} A_j A_k - \frac{a_j}{T_j} N A_j \qquad (3)$$

The last term takes into account the interaction between man and the j-th species, and may be positive or negative (for instance, tourist settlements often cause a reduction in animal or vegetable species but might lead to an increase in parasitic species which grow on waste). The coefficients c_{jk} may also take positive or negative values according to the competitive or cooperative nature of the interaction between the j-th and k-th species. The coefficients will vanish if there is no interaction.

Now, in order to simplify the preceding model, let us suppose that the dynamics of a single species, say the first, is much slower than the dynamics of the other species interacting with it. Then we can apply the so-called direct adiabatic elimination procedure, which has been described elsewhere (Serra et al., 1984). The procedure consists of setting

$$\dot{A}_j = 0 \qquad j = 2, \dots, M \qquad (4)$$

The fast variables, according to this approximation, instantaneously adjust themselves to their equilibrium values, corresponding to a given value of the slow variable. The equilibrium values of the fast variables turn out to be linear functions of A_1 and N:

$$A_{j\infty} = q_j - r_j A_1 - p_j N \qquad (5)$$

The time evolution of the slow species is therefore given by:

$$\begin{aligned} \dot{A}_1 &= K_1 A_1 - K_2 A_1^2 - K_3 A_1 N \\ K_1 &= \frac{1}{T_1} \left[1 - \sum_j c_{1j} q_j \right] \\ K_2 &= \frac{1}{T_1} \left[1 - g_1 - \sum_j c_{1j} r_j \right] \\ K_3 &= \frac{1}{T_1} \left[a_1 - \sum_j c_{1j} p_j \right] \end{aligned} \qquad (6)$$

We thus reach the following important conclusion: if the time scale of a species is widely separated from the time scale of the other interacting species, we can use a Verhulst model for the slow species alone, instead of the whole set of Lotka-Volterra-Verhulst equations.

From now on we will suppose that these conditions are met, and we will therefore consider only one species, which will be regarded as representative of the state of the natural environment. We will now introduce three models for tourist-environment interaction, whose behaviour will be discussed in the next section.

Let us first of all consider the simplest model, which will be called "model 1c" - c meaning continuous:

$$\begin{aligned} \dot{A} &= \gamma A - \gamma A^2 - \beta N A \\ \dot{N} &= (s-1) N + Pu \end{aligned} \tag{7}$$

If the satisfaction coefficient $s>1$, then we have a positive tendency to the growth in the number of visitors, N. Another (constant) term which contributes to the growth in the number of tourists is advertisig (Pu). We will suppose that the effect of man on the A species is negative, therefore $\beta>0$.

The form of the satisfaction coefficient is given below (Eq.10). We suppose that the quantities A and N are normalized. The former is normalized by dividing the original equation by the equilibrium number of type A individuals in a condition where tourists are absent. This explains why the coefficients of the linear and quadratic terms are equal: it is a consequence of normalization. The tourist number is normalized by dividing the original equation by the maximum number of tourists which can be accommodated in the village.

As discussed in the introduction, it is more correct to consider the analogous discrete model which will be called Model 1d:

$$\begin{aligned} A(t+1) &= (1+\gamma) A(t) - \gamma A^2(t) - \beta N(t) A(t) \\ N(t+1) &= s(t) N(t) + Pu \end{aligned} \tag{8}$$

This model considers a homogeneous tourist population, whereas it is known that people differ strongly with respect to destructiveness towards nature and sensitivity to nature and crowding. We will therefore investigate the differences between the previous model and another model, whose discrete version (Model 2d) is given below:

$$A(t+1) = (1+r)A(t) - r A^2(t) - \beta_1 A(t) N1(t) - \beta_2 A(t) N2(t)$$

$$N1(t+1) = s_1(t) N1(t) + Pu_1 \qquad (9)$$

$$N2(t+1) = s_2(t) N2(t) + Pu_2$$

The corresponding continuous model (2c) can be written in a straightforward way. An N1 type of tourist may be considered as being very sensitive and very careful towards the environment, with N2 tourists being very sensitive towards overcrowding.

The satisfaction coefficient s is the sum of two contributions, one coming from the state of the environment and the other from crowding. The two contributions are weighted by a coefficient k ≤ 1 :

$$s(t) = s_{max} \left[K f(A(t)) + (1-K) g(N(t)) \right] \qquad (10)$$

f(A), which measures the satisfaction due to the natural environment, is assumed to have a logistic shape, while g(N), sensitivity to the presence of other people, should be bellshaped (indeed, people generally do not like to be too lonely and do not like overcrowding). Two simple such functions are the following:

$$f(A) = \frac{1}{1 + Re^{-qA}} - \frac{1}{1+R} \qquad (11)$$

$$g(N) = \exp\left\{ -\frac{1}{2} \left[\frac{N-\mu}{\sigma} \right]^2 \right\} \qquad (12)$$

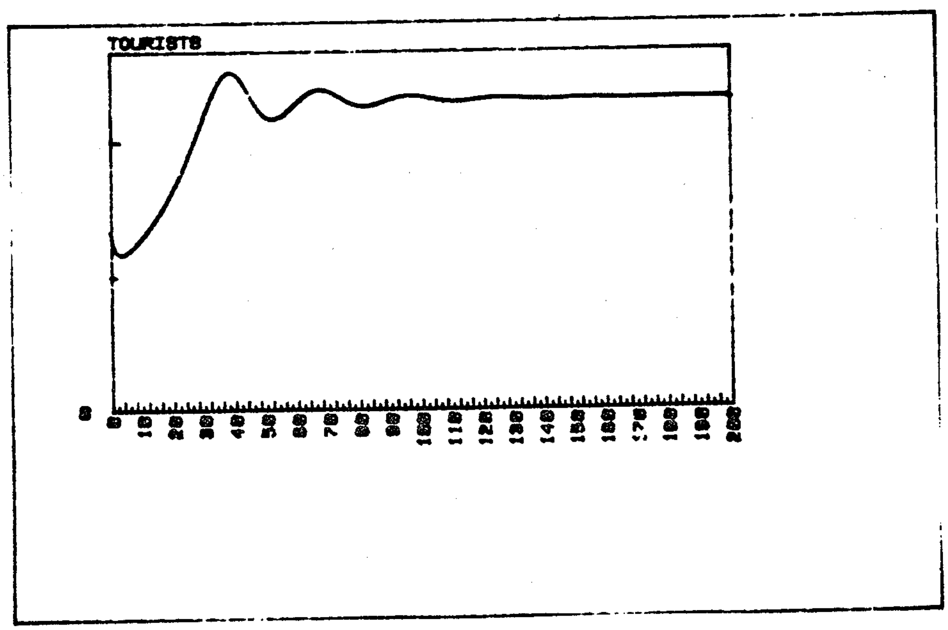
In the following section we will discuss some of the dynamic behaviour characterising the above models.

DYNAMIC BEHAVIOUR OF THE MODEL

Asymptotic behaviour will first be discussed, followed by a short account of transient phenomena as reduced models are particularly interesting for long-term analysis.

The discussion begins with Model 1, considering in particular the dynamic features of its discrete version (Eq. 8), and briefly mentioning those of the continuous model (Eq. 7). Taking first the case where Pu=0, (i.e., no advertising) attention will be focused on the cooperative phenomena due to the spread of information about tourists' satisfaction.

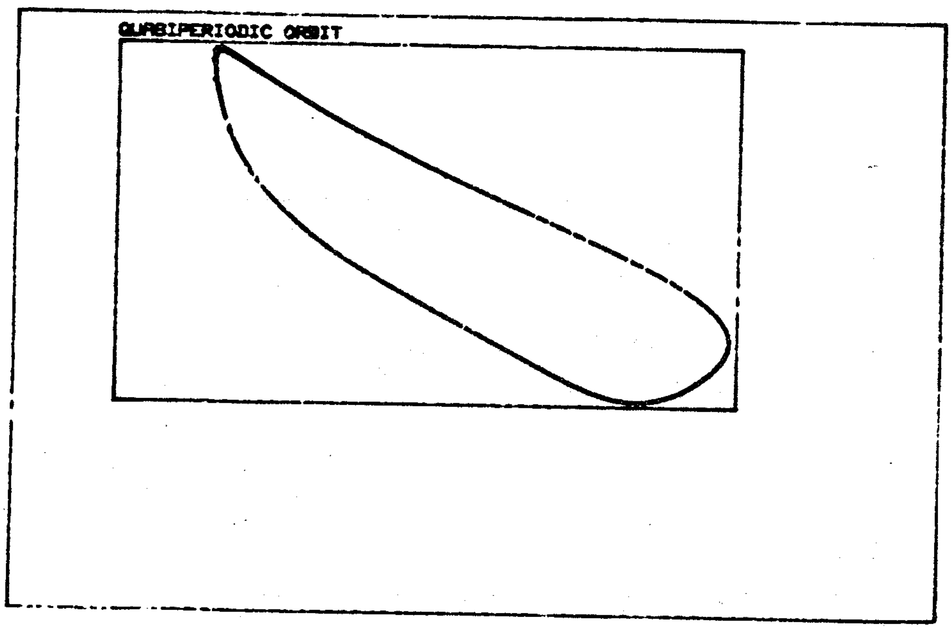
Starting from a "reasonable" set of parameter values, (see Note on Parameters), the following behaviour was observed as the parameter was changed. There was always an equilibrium point P* (N=0, A=1), stable for $0 < \tau < 2$, which corresponded to the unperturbed situation (no tourists, completely natural environment). The basin of attraction of this point was, however, small (the basin of attraction is the set of initial conditions which lead the system to the given attractor in the asymptotic time limit). There were two further equilibrium points, one of which was always unstable. The third equilibrium point was stable for sufficiently low values of τ (fig.1) and its basin of attraction much wider (in the phase plane) than the other one, including "realistic" values of the population N.



A(0) = 0.8   N(0) = 0.1 τ = 0.6

Figure 1

As τ is increased this point becomes unstable, and a supercritical Hopf bifurcation takes place: a stable limit cycle appears, which includes the (now unstable) equilibrium point. The term "limit cycle" is correct for the continuous model, while for the discrete model it would be more proper to speak of quasiperiodic behaviour. The phase plane plot of this cyclic attractor is shown in fig. 2 for the discrete case (Eq.8). The analogous one for the continuous model (Eq.7) is qualitatively the same, in shape and dimensions (not reported here).



A(0) = 0.8 N(0) = 0.1 τ= 0.7

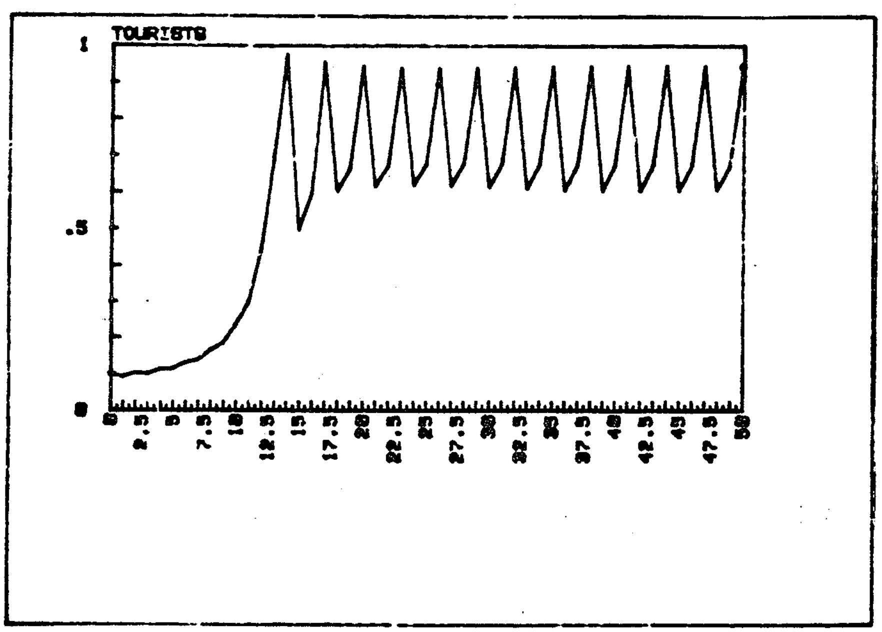
Figure 2

This indicates a close relationship between the two models, which persists for greater values of the parameter. In fact, on increasing τ, another bifurcation occurred in both models: the unstable equilibrium point became stable via a subcritical Hopf bifurcation (the stable limit cycle dimensions diminished around the unstable equilibrium point and reduced towards it giving a stable equilibrium point). The asymptotic behaviour is qualitatively the same for the continuous and discrete models: orbits tending towards the equilibrium points.

On increasing τ, the equilibrium point always remains stable in the continuous case, while in the discrete case another threshold value is found, after which quasi periodic behaviour is observed.

This phenomenon is due to the different kinds of bifurcations for continuous models and maps. We note here that in the bifurcations mentioned above the eigenvalues relating to the equilibrium point cross the unitary circle in the complex plane with positive real parts for the map and cross the imaginary axis for the continuous model, while in this last bifurcation the eigenvalues cross the unitary circle with the negative real parts in the map having no analogue in the continuous model where a complex eigenvalue with a negative real part gives a stable equilibrium point.

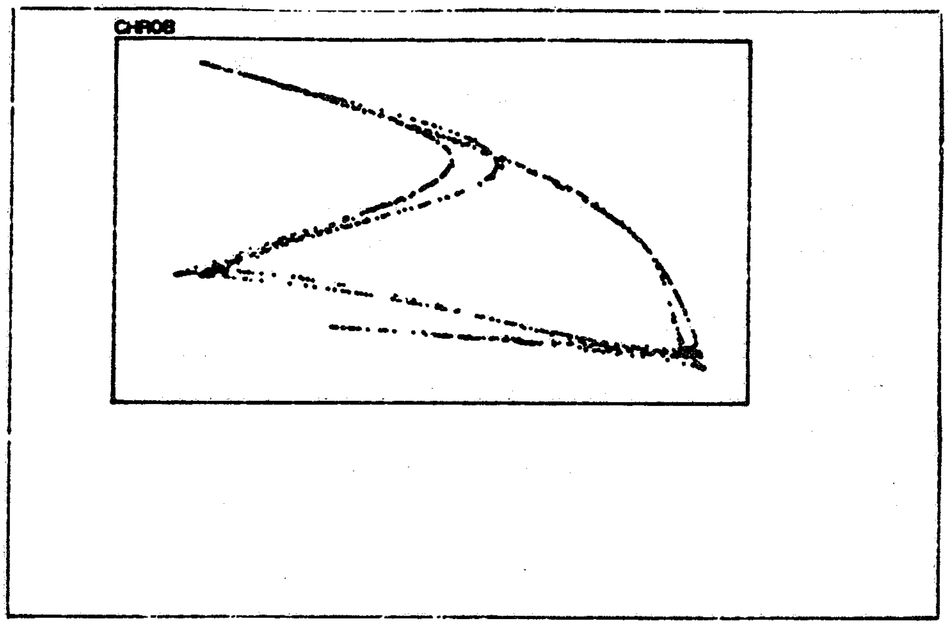
After another threshold, a strictly periodic behaviour is observed, with period three (fig. 3). In the phase plane the attractor is composed of three distinct points, between which the system's representative point continuously jumps.



$A(0) = 0.8 \quad N(0) = 0.1 \quad \tau = 2.1$

Figure 3

After another threshold, the period doubles, and a six year cycle is found, which is followed by a twelve year cycle, and so on, leading to chaos (fig. 4). This sequence of bifurcations reminds one of the well-known Feigenbaum route to chaos (Cvitanovic, 1984; Guckenheimer and Holmes, 1983).



A(O) = 0.7 N(O) = 0.4 τ = 2.5

Figure 4

The series of threshold values has not yet been analysed in sufficient detail to categorically state that it follows Feigenbaum's rule. However, a first estimate of such a sequence suggests a convergence rate quite similar to the well-known value for flip bifurcations of minimal maps of the interval.

On further increasing τ ($\tau > 2.71$) the chaotic behaviour drastically changes. In all examples considered, the tourist population, after a chaotic transient, tends to zero, only the chaotic behaviour of the first variable A surviving.

In an attempt to explain this, it is to be noted that the equilibrium point, say P*, stable for $\tau < 2$ with a "small" basin of attraction becomes unstable via a flip-bifurcation at $\tau = 2$ and two new stable equilibrium points appear near P*. At about $\tau = 2.5$ they become unstable and 4 stable equilibria appear. On further increasing τ, a full sequence of period doubling bifurcations occurs reaching a chaotic behaviour in A, with the variable N=0. This sequence of bifurcation values is different from the previous one and thus, two distinct routes to chaos take place on increasing τ and, at high values, the latter seems to dominate.

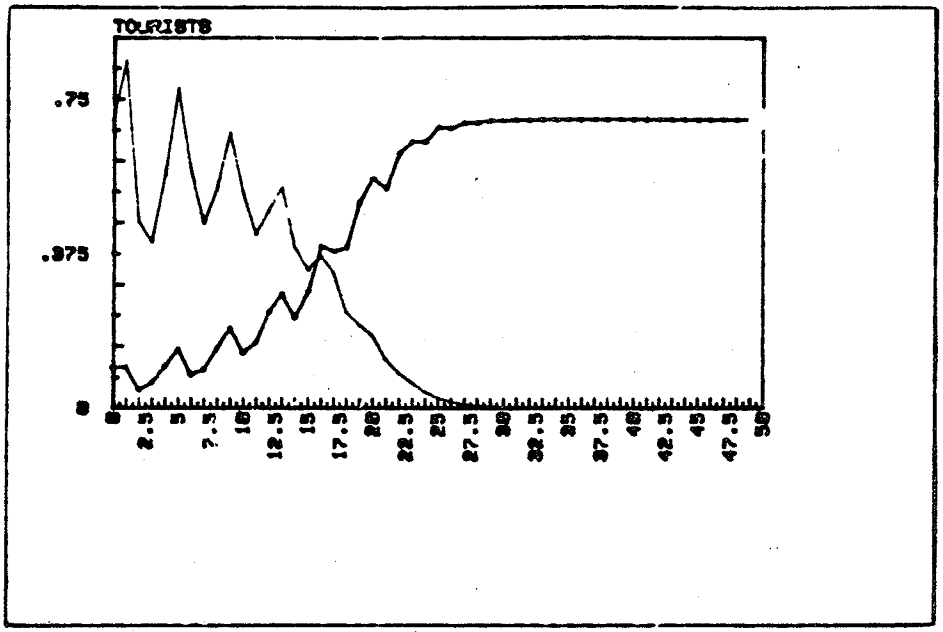
Variations in β while keeping τ fixed were also tried. Table 1 shows the model's asymptotic behaviour for a fixed value of τ. As β increases, we pass from a stable point to a three year cycle, to a quasi periodic behaviour, and back again to a stable point. In general, it seems that, by varying β, we can obtain a set of dynamic behaviours, whose "upper limit of complexity" is determined by the value of τ.

Value	Type of Attractor
$0.4 < \beta < 0.5$	Stable equilibrium point
$0.55 < \beta < 0.7$	Stable periodic orbit of period 3
$0.80 < \beta < 1.1$	Quasiperiodic orbit
$1.20 < \beta < 3.0$	Stable equilibrium point

Table 1 $\tau = 2$

A low value of the advertising variable (Pu=.1) is then introduced. In this case there is only one equilibrium point for the model (Eq.8) and the tourist population value, say N_{eq}, increases with increasing τ (and is greater in value than the corresponding one for the previous case, Pu=0, as expected). The dynamic behaviour is similar to that discussed above. Some differences detected at high τ-values are worth noting. As before the range of τ-values over which there is chaotic behaviour is quite wide (here about 2.3-2.75) but on further increasing τ we now observe a phenomenon which we shall call "reorganization". In fact, periodic orbits of period 9 and 3 are observed. However, the transient to reach this "order" has chaotic dynamics and persists over a long period which increases with increasing τ.

Let us now consider model 2d, where interaction between two different kinds of tourist is taken into account. Let us again increase τ while keeping the other parameters fixed (see Note on Parameters). We consider first the no advertising case: at sufficiently low τ values (fig. 5), we have a stable fixed point which corresponds to the complete extinction of one of the two kinds of tourists (N1=0).

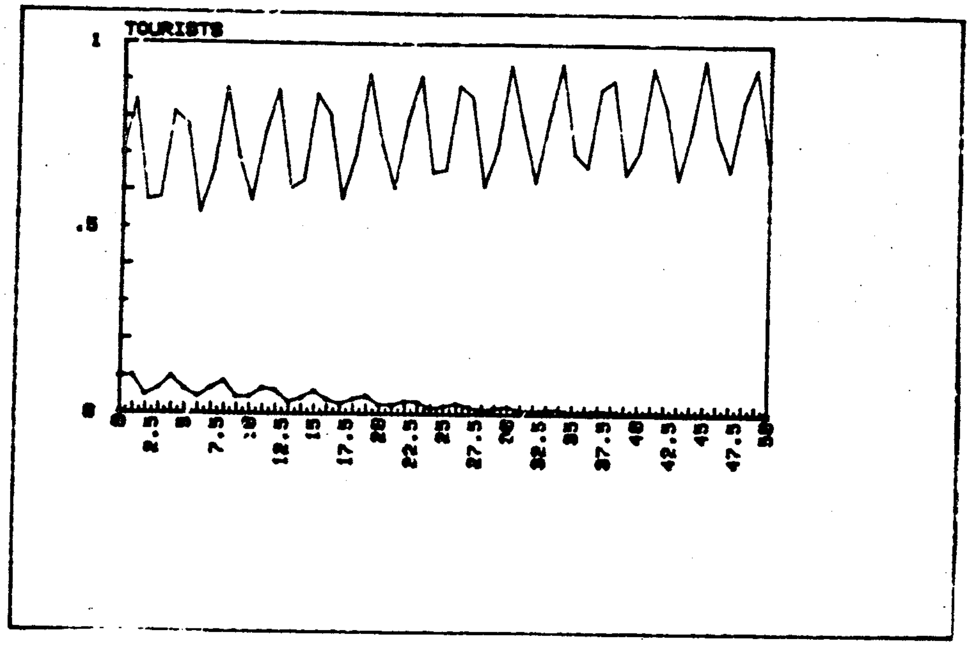


A(0) = 0.8   N1(0) = 0.7   N2(0) = 0.1 τ = 1.5

Figure 5

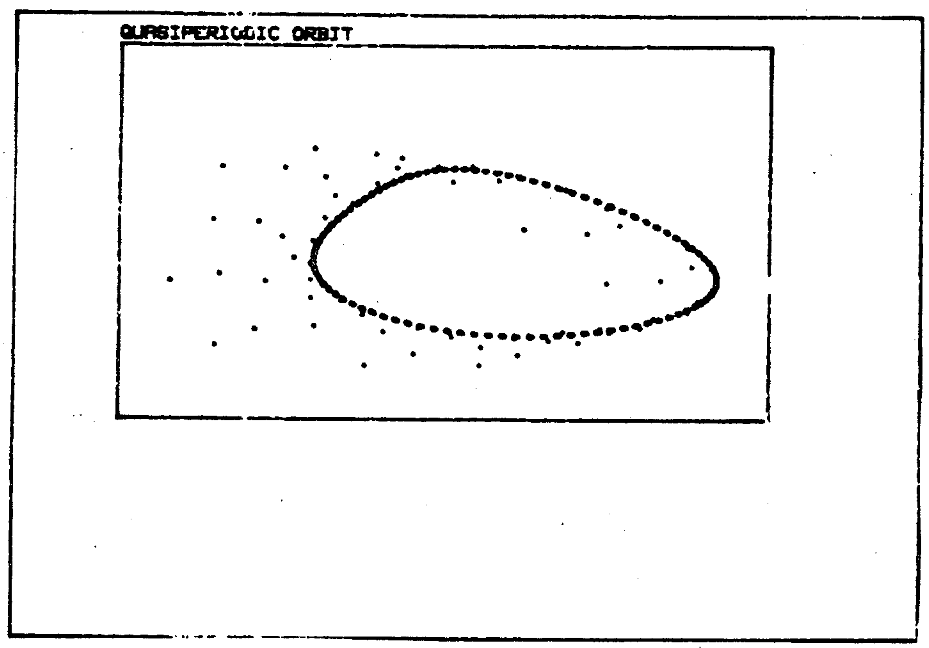
As τ reaches a threshold value, we observe two kinds of dynamic behaviour, completely different in character. Until the population N1 reaches a characteristic value, $N1^*$ say, (which may depend on the A value), i.e. with an initial condition of $N1 < N1^*$, the orbits tend towards the equilibrium point with N1=0 and the second population dominates, while with $N1 \geq N1^*$ the dynamics is reversed; now in fact, it is the second population which tends to zero and a quasiperiodic behaviour survives between N1 and A. (fig. 6 and fig. 7).

On further increasing τ, the characteristic value $N1^*$ decreases and, as above, two different dynamics are detected: either $N1 \rightarrow 0$ and (N2,A) tend to a periodic orbit of period three, or $N2 \rightarrow 0$ and (N1,A) tend to a quasiperiodic orbit. These dynamics can be explained as follows: depending on which of the two populations dominates, the asymptotic behaviour (which is reached after only a few iterations) is the same as that of model 1d (Eq.8) with the corresponding τ and β values.



A(0) = 0.8   N1(0) = 0.7   N2(0) = 0.1 τ = 1.8

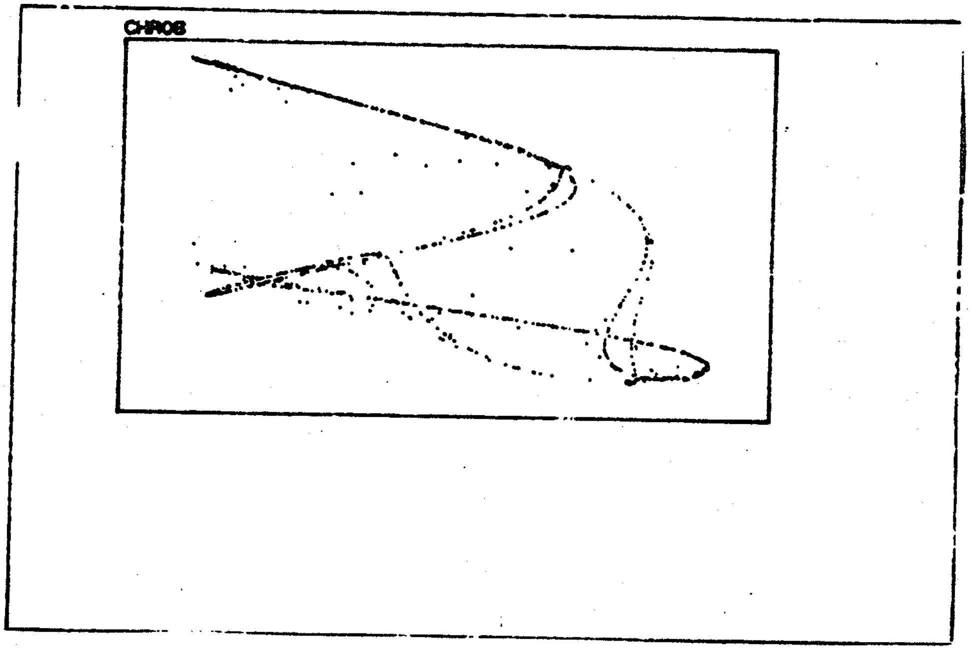
Figure 6



A(0) = 0.8   N1(0) = 0.7   N2(0) = 0.1 τ = 1.8

Figure 7

Thus, on further increasing τ, the dynamics of the surviving population N1 is a quasiperiodic orbit of equilibrium point, while for the dominating population N2, a chaotic behaviour is soon reached (fig. 8), as expected.



A(0) = 0.6 N1(0) = 0.2 N2(0) = 0.2 τ= 2.5

Figure 8

An interesting behaviour is detected at high values of τ. At τ=2.8 neither of the two populations dominates, both tending towards zero and the dynamic of A is chaotic. Again, the dynamics of the bidimensional model (Eq.8) can explain this phenomenon.

Finally, we consider the same model 2d in the case where a constant level of advertising is allowed. The main difference from the previous case is that now we no longer have the extinction of one of the two populations. Both populations survive due to the constant flow via advertising channels. However, the dynamic behaviours are quite similar to those observed in the other models. Let us briefly outline this qualitative behaviour at various τ values. Also in this case we have first (for small τ values) a stable equilibrium point with population N2 prevailing over N1. On increasing τ, a first bifurcation occurs and at τ=2 a periodic orbit of period three is detected.

In two of these states N2 prevails (but the difference between the two populations is less than before) and in the third one N1 prevails (although by only a small amount). On increasing the value of τ a full sequence of period doubling bifurcations takes place and a chaotic behaviour is observed for high values (τ>2.4).

CONCLUSIONS

A brief comment will be made on the asymptotic analysis carried out, although still incomplete. First of all, how relevant is asymptotic analysis? In the first place, it can be seen that a typical transient lasts for 10-30 years. Asymptotic analysis is therefore relevant for long-term planning. There are, however, some exceptions, discussed in the previous section, where a chaotic transient lasts so long that the more regular asymptotic behaviour becomes apparent only after centuries: it is clear that in this case asymptotic analysis is of no direct practical use.

Second, we remark that a major goal of simulation models is to understand how things will go, rather than actually predict accurate values. So asymptotic analysis can tell us if corrective actions are to be taken, such as more advertising, for instance.

The use of reduced models requires caution. We have seen formal projection techniques at work in the case of widely separated time scales. However, we often find that such formal techniques are inapplicable, as in the case of the two different populations of tourists. We have shown that the frequently adopted procedure of taking average values can be misleading. Indeed, if one population dominates there is no sense in averaging its characteristic parameters with those of an extinguishing population, for asymptotic analysis. We have also seen an example of a delicate relationship between dynamic behaviour and the dominance relation between two groups of tourists, which requires further analysis. Let us also remark the importance of initial conditions in strongly cooperative models such as those considered.

We have also shown an illustrative example of a well-known property, namely that discrete-time models give rise to a much wider set of dynamic behaviour than continuous-time models. System dynamists usually think in terms of continuous models, but there are instances (such as the present one) where it is better to use discrete maps. This, however, does not conflict with the basic principles of the system dynamics method.

A further point which needs be mentioned is the extreme sharpness of the thresholds encountered. The most striking phenomenon discussed, namely the change in dynamic behaviour accompanied by a change of prevailing population takes place over a very narrow range. This sharpness can be observed with respect both to the parameter values and to the initial conditions.

Lastly, it is to be remarked that the "counterintuitive behaviour of complex systems" is a concept which refers not only to their transients, but also to their asymptotic patterns.

NOTE ON PARAMETERS

For all simulations a set of fixed parameters was chosen which are as follows:

Model 1d (A-N)

Smax=1.8 , q=10 , R=1000 ,μ=0.5 ,σ=0.25 , k=0.5 , β =0.8 , Pu=0

Model 2d (A-N1,N2)

Smax=1.8 , q=10 , R=1000 ,μ=0.5 ,σ=0.25 , k1=0.7 , k2=0.3 ,β1=0.6 , β 2=1 Pu=0

The variable parameter in both models ,mentioned above, for all results reported is τ . Together with the initial conditions of the variables the value of τ is noted under each figure.

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