HOW CAN ONE EXPLICITLY REPRESENT THE FUZZY NATURE OF SOCIAL CAUSAL RELATIONS IN FORMAL MODELS? FUZZY SET SIMULATION MODELS

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Abstract

This paper introduces and discusses the concept of verbally formulated simulation models. Such models can operate with linguistic values as 'high', 'rather high', 'low' and 'not low', etc. as inputs. The output will be similarly verbally formulated. The simulation procedure is based on a fuzzy set-theoretic semantical model of a fragment of the English language, which converts verbal expressions into numerical quantities. The paper applies one particular semantical model in a simulation example.

Verbal models may be more believable, or significant, than conventional system dynamic models, in that they adequately represent the fuzzy knowledge of the system which is modeled. The cost of this significance is loss of precision in model output.

Verbal models are also easier to test for sensitivity to parameter-, state- and input values than traditional models. Therefore, a comprehensive understanding of the model's behavior patterns is more readily obtained. The realm of successful applications of verbal models seems, however, to be restricted to systems with variables which are not physically measurable, but whose values are only available through human intuition.

Finally, verbal models may successfully be incorporated in conventional system dynamic models if technically feasible. Such a prosedure would allow for an adequate handling of non-quantifiable data.

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THE POINT OF DEPARTURE

When traditional simulation models of social phenomena are formulated, causal relations are represented as precise mathematical functions. Such is the case even when the modeler has only a vague idea about their nature, a condition which is most often true. To compensate for this shortcoming, the modeler must interpret simulation outputs in quite liberal terms. That is, he has to "add fuzz" which was removed in the model to the output.

The source of this difficulty is in part at least that the modeler is restricted by the language he has chosen, namely a computer language, in which he can only communicate via hard facts, - on other words, numbers.

To avoid the artifical step of translating vague ideas with an inappropriate exactitude, the modeler should instead be allowed to formulate his models in natural language. In this way, the models might become truer representations of the kind of information which is available about the real system. The question still remains, however, whether such models can be used to make inferences without the modeler once again having to retreat to precise mathematical formulations.

Contention

Verbally formulated dynamic models may contain so much information that a system can be devised to demonstrate their implications by direct simulation. That is, the modeler does not have to reduce the inherent fuzziness to arrive at useful conclusions.

Qualifications

To justify this contention, the modeler must restrict himself to a suitable class of verbal models. Such a class can be defined by specifying a vocabulary and a generative grammer in the sense of Chomsky (1965).

We must also assume that the modeler and model users can use natural language slightly more precisely and consistently than everyday language. Otherwise, inferences from verbal models would tend to become too fuzzy to be informative.

To illustrate the direction of our thoughts, assume the following premises:

x is rather low, but
y is slightly higher than x.

Then, it may be possible to show that by stipulating consistent sementical usage, we are led to conclude that:

y is more or less sort of low.

Moreover, a computer can actually be forced to "calculate" the conclusion regarding y's value for any value of x.

IMPLEMENTATION

To implement these ideas -- that is, to make the verbal models deductive, we have to:

i) Define a class of verbal models by specifying a vocabulary and a grammer.
ii) Define a sementical model of the meaning of the individual members in the vocabulary.
iii) Implement the syntactical-semantical system in a computer language.

S y n t a x

The concept of a verbal assignment statement can be rigorously defined by a generative grammer. Although this discussion offers only one unpretentious implementation of the idea, its general merits will become clear in light of the ensuing semantics:

Let us begin by specifying the vocabulary:

	Lexical category	Symbol		Vocabulary
1)	Primary term	T	→	high, low, unknown, undefined
2)	Primary relation	R	→	higher, lower, similar, opposite
3)	Connective	C	→	and, or, nor
4)	Value hedge	VH	→	very, not,neither,indeed,sortof, rather,moreorless,atleast
5)	Relation hedge	RH	→	very, not,neither,indeed,slightly, somewhat,moreorless,considerably
6)	Relation evaluator	E	→	of,than,to
7)	Model variable	X	→	(any name of a model variable)

The arrows indicate that the symbol represents any of the vocabulary members in the corresponding category. In other words, each symbol can generate any of the words.

On a deeper level than the lexical categories lies the concept of composite expressions:

	Composite expression			
8)	Composite term	CT	→	CT C CT, VH CT, T
9)	Composite relation	CR	→	CR C CR, RH CR, R

This concept means that a composite term, CT, can be written as, for instance, CT C CT; then, if desired, as CT C VH CT; then optionally as T C VH T. Now, considering rules (1), (3) and (4), we may finally write'high or rather high' which is an example of a completed composite term. The reader can also easily convince himself that 'neither slightly higher nor considerably lower' is an example of a grammatical composite relation.

On a still deeper level, lie the truth- and value expressions:

	Semantical category	Symbol		Resultant string or symbol
10)	Value expression	V	→	CT, CR E X
11)	Truth value expression	N	→	N C N, X is V

Here, 'is' is a member of the vocabulary. A value may therefore be written directly as a composite term, or in a form corresponding to 'higher than X'. That is, CR E X, where X is a model variable. A truth value will be expressed in the form exemplified by 'X is low' and by 'X is low and Y is not high'.

An assignment statement is concerned with linguistically expressed values. The deep structure of a linguistic value is as follows:

12) Linguistic value L → L or L, V if N, V

'or' and 'if' are members of the vocabulary. As a result, a linguistic value may be written as a disjunction of two linguistic values, as a conditional value expression ('high if C is low'), or as an unconditional value expression.

Finally, the structure of an assignment statement is that of assigning a linguistic value to a dependent variable. Therefore:

13) Assignment statement S → X ← L

where the left-arrow symbolizes value assignment.

Production rules (1) through (13) now define precisely what will be understood with a grammatically correct assignment statement.

D e f i n i t i o n o f V e r b a l M o d e l s

A set of grammatically correct assignment statements describing causal relations between a set of variables will be called a verbal model. Figure 1, for example, shows the structure of a model of bureaucratic patterns in industrial organizations, inspired by Gouldner (1954).

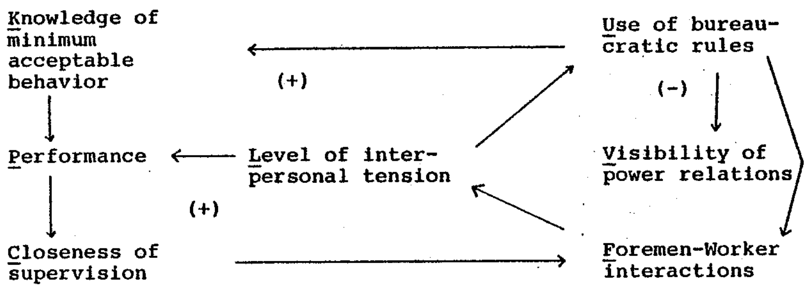

Figure 1. Structure of Gouldner's model.

Below, the causal relations among the variables in Gouldner's model are specified in the form of a verbal model. The letters representing the variables correspond to the underlined ones in figure 1:

U_t ← somewhat higher than U_{t-1} if L_{t-2} is very high or rather high, or
equal to U_{t-1} if L_{t-2} is neither low nor very high, or
slightly lower than U_{t-1} if L_{t-2} is low or rather low

K_t ← very similar to U_t

L_t ← very similar to V_{t-1}

C_t ← (description of foremen's behavior patterns)

F_t ← opposite of U_t if C_t is low or rather low, or
very similar to C_t if C_t is not low

V_t ← equal to F_t if U_t is not higher than U_{t-1}, or
considerably lower than F_t if U_t is higher than U_{t-1}

We leave as an exercise for the reader to show that the model is grammatical. The model in this very form, becomes a computer simulation model once the meaning of each word has been specified. The input and output values are supposed to be given in the form of composite terms, such as 'high', 'rather high' and 'not very low'.

S e m a n t i c s

To be able to draw inferences from a verbal model -- to find the implied dynamic behavior when the initial state and exogenous variables are known -- the modeler must know the meaning of the words which are used in the model's specification. This meaning will normally refer to a system of consistent semantical usages which a group of native speakers can easily accept if perceived as a norm.

A description of the meaning of each word in the vocabulary will be called a semantical model. A possible set of basic principles for this kind of model is outlined below, and suggestions are offered as to how these principles can be applied to the special vocabulary outlined here. The principles were designed by Zadeh (1965, 1972, 1973, 1975).

Fuzzy sets as representation of meaning.

Imagine the variable 'degree of cooperation'. Although this variable is difficult to measure physically, people have no problem in offering a verbal estimate of its value in a given situation. They might even be willing to express the value as a composite term. The range of all subjectively conceivable degrees of cooperation will then constitute the present universe of discourse.

We shall interpret the meaning of a linguistic value (which includes composite terms) as a fuzzy subset of the universe of discourse. (See figure 2.)

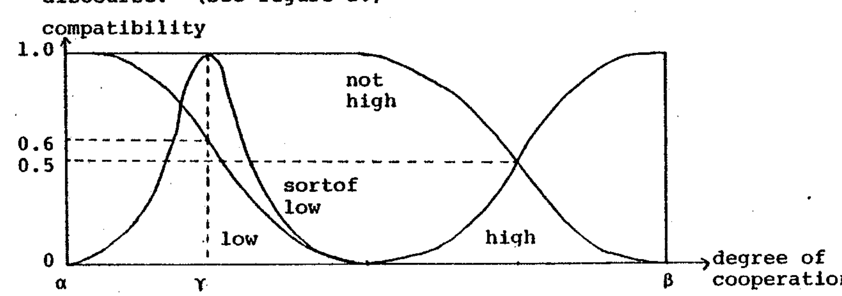

Figure 2. The meaning of linguistic values is represented as fuzzy sets.

The interval $\alpha-\beta$ represents a presumed mental picture of the universe of discourse. α and β are the lowest and highest conceivable levels, respectively. The curve labeled 'low' represents the subjective compatibility of different levels of cooperation, with the label 'low'. Therefore the level γ, for instance, is compatible with the value 'low' to a degree of 0.6. Another way of making this point, is by saying that the degree of membership of γ in the set of 'low' degrees is 0.6. A curve specifies a semantic interpretation of a linguistic value by defining a fuzzy subset of the universe of discourse, that is an interval with unsharp boundaries. The term 'sortof low' is less approximate than 'low', and 'low' is less approximate than 'not high'. These interpretations should be understood to be an approximate model of what people in general mean when they use such terms. Correspondingly, the fuzzy set interpreting a linguistic value will be called the meaning of this value.

Representation of fuzzy sets.

The meaning of linguistic terms is more conveniently represented as number strings rather than continous functions. Assume that the membership functions in Figure 2 are sampled at, perhaps, 11 equidistant points. Then this sample may approximately represent the functions:

low	↔	1	1	.7	.3	.1	0	0	0	0	0	0
sortof low	↔	0	0	.1	1	.3	0	0	0	0	0	0
not high	↔	1	1	1	1	1	1	.9	.7	.3	0	0

Representation of sharp, or numerical values.

There is no problem at all in representing precise numerical values in the same system. In this case, the universe of discourse would be the real line. The appropriate membership function vanishes everywhere except at the correct point, where its value is 1.0.

The meaning of composite relations.

The meaning of verbally expressed relations between two variables can be represented as two-dimensional fuzzy sets. In our system, such a procedure would correspond to matrices filled with memberhip values. Interpretation is analogous to analysis of linguistic values. (For further information, see Zadeh 1973).

Fuzzy inference.

The most important problem to be solved, is that of fuzzy inference. Consider again the following problem:

Premises: X is rather low
Y is slightly higher than X

Conclusion: Y is ?

How can we infer the value of Y? Zadeh contends (1975) that if we are given the meaning of 'rather low' and 'slightly higher' in terms of fuzzy sets, then the fuzzy set corresponding to Y's value can be computed using the principle of fuzzy compositional inference. Application of this principle corresponds mathematically to calculating the matrix product between the two entities. The only difference is that the operations of addition and multiplication must be substituted for by maximization and minimization, respectively.

Linguistic approximation.

When the matrix-multiplication is performed, the result will be a fuzzy set. Since the set of composite terms is not closed under this operation, the modeler will in general have to find the appropriate label for Y's value by searching for that composite term with meaning closest to the computed fuzzy set. This operation, called linguistic approximation, is readily computerized, for instance, using the mean square error as a fitness criterion.

Semantical interpretation of individual words.

Following the just stated principles, a model of the meaning of each word in the vocabulary can now easily be proposed.

The primary terms are assumed to be constant fuzzy sets:

high	↔	0	0	0	0	0	0	.1	.3	.7	1	1
low	↔	1	1	.7	.3	.1	0	0	0	0	0	0
unknown	↔	1	1	1	1	1	1	1	1	1	1	1
undefined	↔	0	0	0	0	0	0	0	0	0	0	0.

The primary relations are similarly represented by square matrices. The hedges are assumed to be operators on the meaning (fuzzy set) to their right in an expression. They are therefore functions with fuzzy sets as arguments. The results of a particular definition of some of the hedges applied to 'high' would be:

moreorless high	↔	0	0	0	0	0	0	.3	.5	.8	1	1
very high	↔	0	0	0	0	0	0	0	.1	.5	.9	1
rather high	↔	0	0	0	0	0	0	0	0	1	.2	0
not very high	↔	1	1	1	1	1	1	1	.9	.5	.1	0 .

(For further discussion on hedges, see Zadeh 1972 , Lakoff 1972, and Wenstøp 1975).

The connectives 'and' and 'or' are defined as operators that take minimum and maximum values, respectively, of the two membership functions appearing on either side in an expression. For example:

high or low	↔	1	1	.7	.3	.1	0	.1	.3	.7	1	1
not high or low	↔	0	0	.3	.7	.9	1	.9	.7	.3	0	0
not high, and not low	↔	0	0	.3	.7	.9	1	.9	.7	.3	0	0

(Bellman & Gierts (1973) have shown that these definitions of the connectives are the only possible definitions if certain natural axioms are to be satisfied. The definitions reduce to classical Boolean ones in the non-fuzzy case.)

The relation evaluators perform fuzzy inference, and are therefore interpreted as matrix-product performers. The same case applies with the word 'is', which appears exclusively in expressions as 'X is high', and therefore calculates truth values.

Finally, 'if' is assumed to perform the minimum operation. This definition of 'if' can be shown to reduce to that of classical propositional calculus in the non-fuzzy case (Wenstøp, 1975).

This concludes a brief, sketchy outline of a semantical system based on fuzzy set theory.

V a l i d a t i o n o f a s e m a n t i c a l m o d e l

Before a semantical model can be put into use, --that is, used in simulation models, the modeler has to ensure that its operational characteristics are acceptable by normal intuition-based standards. This confidence can be gained by presenting a potential user with a list of pairs of linguistic values which, according to the model, have approximately the same meaning. The subject will then be asked to react in one of two ways according to his judgement:

i) reject the model as fundamentally counterintuitive
ii) accept the model as a semantical norm

If he chooses the second alternative, the model has been deemed acceptable.

Some examples of pairs of linguistic values are shown below. They are derived in the following way: A particular semantic model (Wenstøp 1975) has been implemented in the APL computer language. The model includes a routine, LABEL, that performs linquistic approximation. When a grammatical linguistic values is written on the terminal, the computer will automatically compute its meaning in the form of a fuzzy set. This feature is made possible by correspondence of the syntax of APL to the Germanic languages (Iverson, 1962). When a linguistic value is preceded by LABEL, the result will be a composite term whose meaning is approaches that of the linguistic value.

In the following list, input to the computer is indented. The computer response is flushed left. Parentheses are used to remove ambiguity.

LABEL SIMILAR TO LOW
MOREORLESS LOW

LABEL OPPOSITE OF LOW
MOREORLESS HIGH

LABEL HIGHER THAN LOW
NOT VERY LOW

LABEL (SLIGHTLY HIGHER) THAN LOW
RATHER LOW

LABEL (SLIGHTLY HIGHER) THAN RATHER LOW
SORTOF LOW

LABEL (SOMEWHAT HIGHER) THAN RATHER LOW
NEITHER HIGH NOR LOW

LABEL(CONSIDERABLY HIGHER) THAN SORTOF LOW
RATHER HIGH

LABEL (NOT HIGHER) THAN SORTOF LOW
(MOREORLESS LOW) OR RATHER LOW

LABEL HIGH OR NOT HIGH
UNKNOWN

LABEL HIGH OR NOT LOW
NOT LOW

LABEL HIGH AND NOT HIGH
(SORTOF HIGH) OR RATHER HIGH

LABEL (NOT HIGH) AND NOT LOW
NEITHER HIGH NOR LOW

LABEL MOREORLESS VERY HIGH
HIGH

Simulation of verbal models

Assume the successful formulation of a verbal model. Further assume the availability of an acceptable APL-implemented semantical model. The modeler is then in the following situation: When coupled with the semantical model, the verbal model itself is a discrete-time simulation model. The values of exogenous variables and the initial state may be given either numerically or in the form of composite terms. Values of output variables will be computed as fuzzy sets, but reported linguistically if so desired.

Two modes of simulation.

When verbal models are simulated, the output typically becomes more fuzzy over time. This result is a natural consequence of verbally-formulated relations generally implying increased entropy. For instance, if we assign:

y ← similar to x

then, on computation, y's value will be more fuzzy than that of x. If x is 'high', then y will become 'moreorless high'. As a result, in practice, the following development of the value of a variable can be observed: 'high', 'moreorless high', 'moreorless high or sortof high', 'not low', not very low', 'unknown'. The meaning of 'unknown' is completely fuzzy; it conveys no information at all. Of course, the development of value sharpness with time is completely reasonable, given the inherent fuzziness in the model formulation.

On the other hand, if the purpose of simulation is not prediction, but rather investigation of principal modes of behavior, then the tendency toward augmented fuzziness must be removed. This operation can achieved by restoring the values of the variables to complete sharpness after each period of simulation. The modeler should replace the fuzzy value with a numerical value corresponding to the point with the highest membership in the fuzzy set. Since this model, in a sense, constitutes revitalization of the model after each time period, the method may be called the <u>revitalization mode</u> of simulation. In contrast, if a model not revitalized, is in the <u>natural mode</u> of simulation.

Natural mode simulation example

To illustrate the nature of verbal model simulation, let us examine one simulation based on a semantic model (Wenstøp 1975). The principal operational characteristics of the semantical model have already been exhibited here.

Consider the previously discussed model of bureaucracy patterns. Assume the following problem: Which of three foremen reaction patterns is most efficient when the system starts from a rather unfavorable state where all initial values are 'rather high', except performance which is 'rather low'?

i) The hard line:

C_t ← very high

ii) The common sense line:

$C_t \leftarrow$ considerably higher than C_{t-1} if P_t is lower than P_{t-1} and P_t is low, or
equal to C_{t-1} if P_t is not lower than P_{t-1} and P_t is low, or
slightly lower than C_{t-1} if P_t is not low

iii) The soft line:

$C_t \leftarrow$ low

In patterns i) and iii) closeness of supervision, C, is not at all dependent on worker performance, P, whereas ii) encompasses the natural tendency of foremen to tighten up when things are going badly.

When any one of the three response patterns is included in the model and the semantical system is involed, a complete simulation model has been created and the modeler can derive its consequences.

Simulation 1, the hard line. Output.

Period 1

Use is very high
Performance is very low
Supervision is very high

Period 2

Use is very high
Performance is atleast low
Supervision is very high

This state eventually proves to be stable. The foremen's behavior demonstrably worsens rather than improves the situation.

Simulation 2, the common sense line. Output.

Period 1

Use is very high
Performance is rather low
Supervision is very high

Period 2

Use is very high

This state also eventually proves stable. The results are therefore identical to the results of simulation 1.

Simulation 3, the soft line. Output.

Period 1

Use is very high
Performance is very low
Supervision is low

Period 2

Use is very high
Performance is atleast very high
Supervision is low

Period 3

Use is very high
Performance is atleast very high
Supervision is low

Period 4

Use is rather high
Performance is atleast very high
Supervision is low

Period 5

Use is softof high
Performance is atleast rather high
Supervision is low

Period 6

Use is neither high nor low
Performance is atleast sortof high
Supervision is low

Period 7

Use is atleast sortof low
Performance is atleast sortof high
Supevision is low

This state eventually proves stable as well. Consequently by staying away from the workers, the foremen have managed to improve performance and reduce the use of bureaucratic rules. Performance initially deteriorated, however, before rising to a higher level. The model therefore points to the interesting conclusion that lenient treatment of workers is best in a bad situation, but some time must pass before the results appear.

E P I L O G U E: V E R B A L M O D E L S I N A S Y S T E M D Y N A M I C S E N V I R O N M E N T

Verbal models, as defined here, may be a supplement to the type of models now employed in the system dynamics approach, since verbal models can also handle numerical values. Verbal assignment statements may therefore appear as elements in a system dynamics model. Although technical advances to do so have yet to be solved, the principle is clear enough. However, APL is presently the only computer language which makes it possible for verbal expressions themselves to appear as program statements. To leave APL would therefore hamper the ease with which verbal models can now be applied.

M a k i n g v e r b a l m o d e l s m o r e g e n e r a l

One problem in conventional system dynamic models is the selection of parameter values. A comprehensive study of the impact of different value sets on model behavior may easily become a formidable task if little is known a priore.

With verbal models, the problem is reduced somewhat since the models can be formulated at different levels of precision. Consider for instance, 'very similar to Y' versus 'moreorless similar to Y'. The two relations are essentially the same, except that the latter is more general than the former. The same is true with 'higher than Y' versus 'not lower than Y'.

The validity of verbal model appearently can be enhanced simply by increasing the fuzziness in model formulation. This approach will, of course, also make simulation output more fuzzy; so the process therefore has to stop before the model ceases to be informative.

F u z z y s c e n a r i o s

In system dynamics, the difficulties encountered in arriving at a comprehensive understanding of model behavior are avoided by focusing on so-called scenario analysis. A scenario is an intinitively identifyable state of affairs and/or mode of policy implementation by decision-makers. One selected set of state values and input values is supposed to be representative of a given scenario. The model is then simulated and the results said to be characteristic of the scenario.

A corresponding verbal model can be simulated, not only from an initial-state point examplifying a scenario, but from a state intuitively describing the scenario itself with all its fuzzy ramifications. This representation is achieved simply by assigning to the state- and input variables suitable fuzzy values. Notice again, however, that the output will become correspondingly fuzzy.

T h e t r a d e - o f f: P r e c i s i o n v e r s u s s i g n i f i c a n c e

By allowing fuzz to enter a model in order to increase confidence in its ability to represent the real system, the cost is loss of output precision. The two extremes are readily appearent: With mathematical functions representing causal relations and numbers as input, the output will be exact numbers. Certainly, however, the model does not exactly represent the real system. On the other hand, with enough fuzz introduced in to the model, all output variables will have the value 'unknown'. Certainly, this output can yield no false information.

Somewhere between these two extremes, lies the optimal tradeoff between significance and precision.

When should verbal models be employed

Some particular difficulties pertaining to verbal models should not be disregarded. The most important concerns the subjective nature of verbally-expressed values. The modeler should view these values as relative to a psychological continuum which is a presumed mental picture of the universe of discourse (Torgerson, 1958). Such an impression turns out to be a necessity anyway when the variables in question are by no means physically measurable. But if they are measurable, the verbal approach severely restricts, or at least en-cumbers, complicated causal relations which may have to be represented in such cases. A psychological continuum is simply too simplistic to allow for mathematical manipulations, for example: We can appreciate that 'x is rather high' and that 'y is low', but thereafter the path to inferring the linguistic value for z, if $z = 2x^2/y$, is dubious.

Verbal models may prove superior to convential ones only in situations where the variables in question are not susceptible to entry in mathematical formulations, as is most often the case when they are physically measurable.

On the other hand, significant variables which are only measurable through human observation, for instance, by responses to questionnaires influence the real system in a way that depends on the perception of these values by human subjects in the system. With respect to such values, verbal description yields full information -- exactly the kind of data needed. Since the modeler can use this information without tampering with it in his verbal simulation models, the verbal method must be superior in non-quantifiable cases.

Verbal assignment statements may actually turn out to be valuable as elements of larger system dynamics models, covering instances where human behavior plays the prominent role.

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